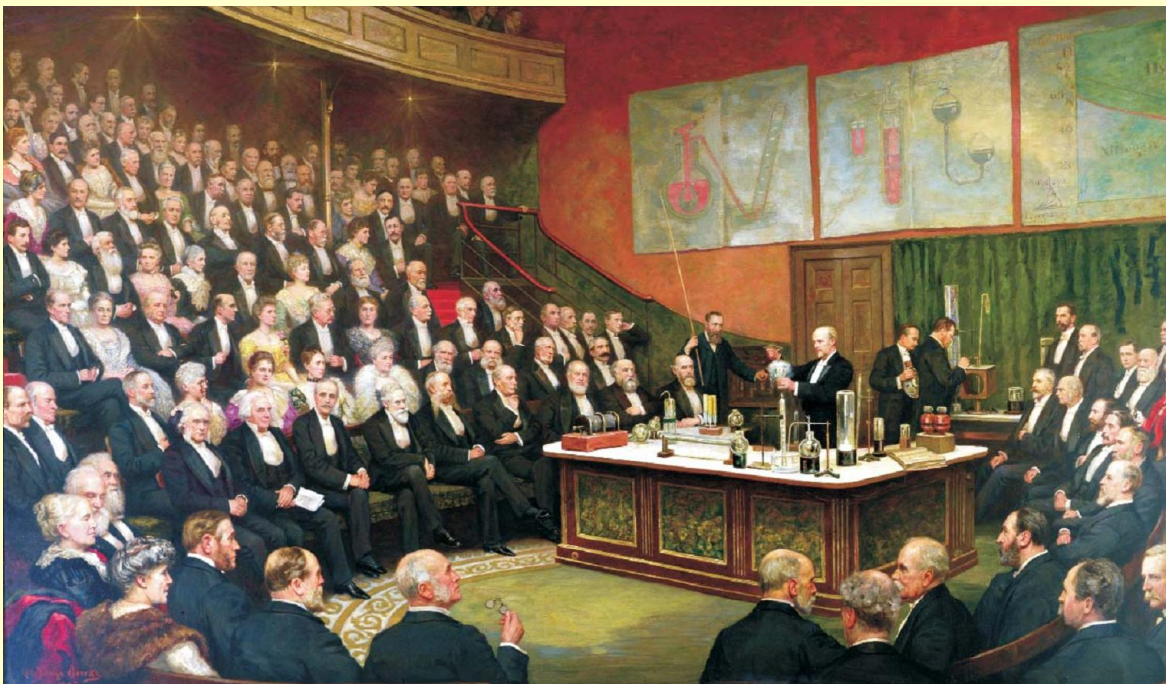


§ 5 超导体的电磁性质

本节主要内容：

1. 超导体特性之一：零电阻
2. 超导体特性之二：完全抗磁性 (Meissner 迈斯纳效应)
3. 超导体的电动力学性质
4. 超导环的磁通俘获和磁通量子化现象

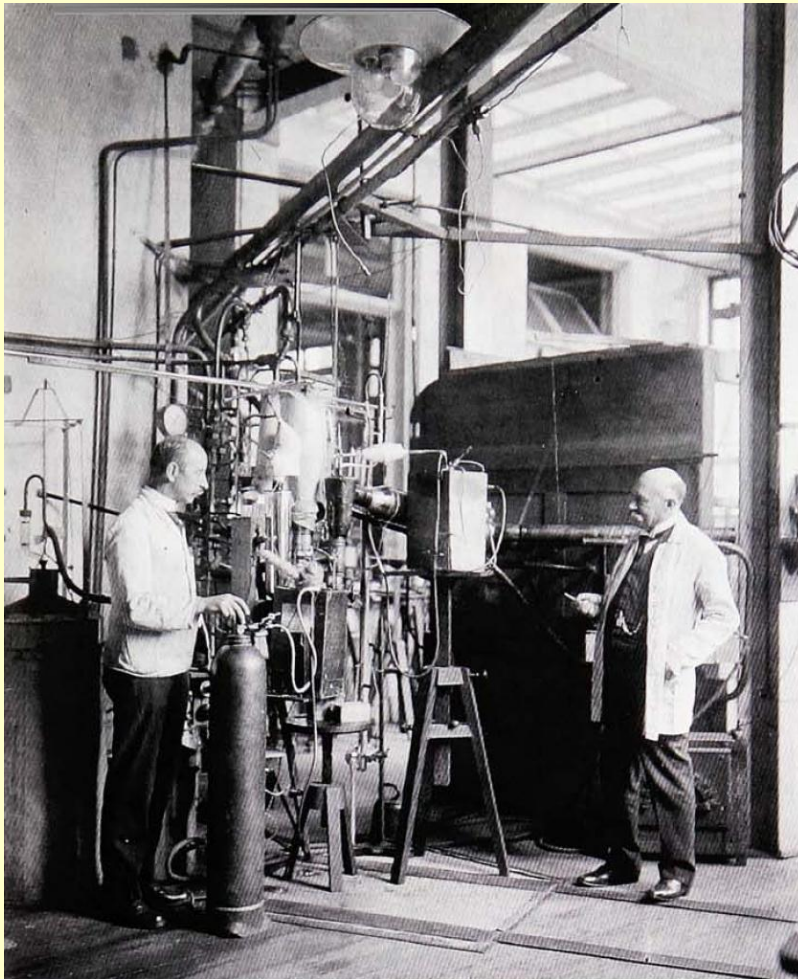
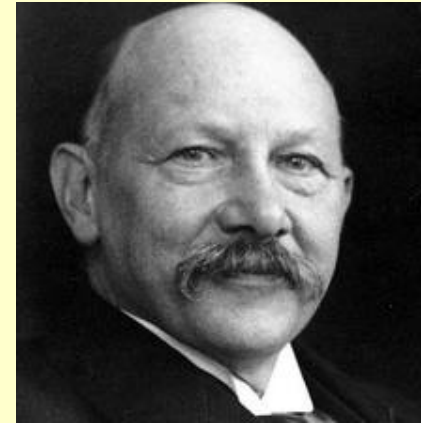
气体液化与低温环境的获得



- 1892年，发明了杜瓦瓶（中间抽真空，内胆涂有银的双层玻璃瓶）
- 1899年，杜瓦（James Dewar）在伦敦皇家研究所成立100周年庆典上，展示氢气（ H_2 ）的液化实验

水银超导体的发现

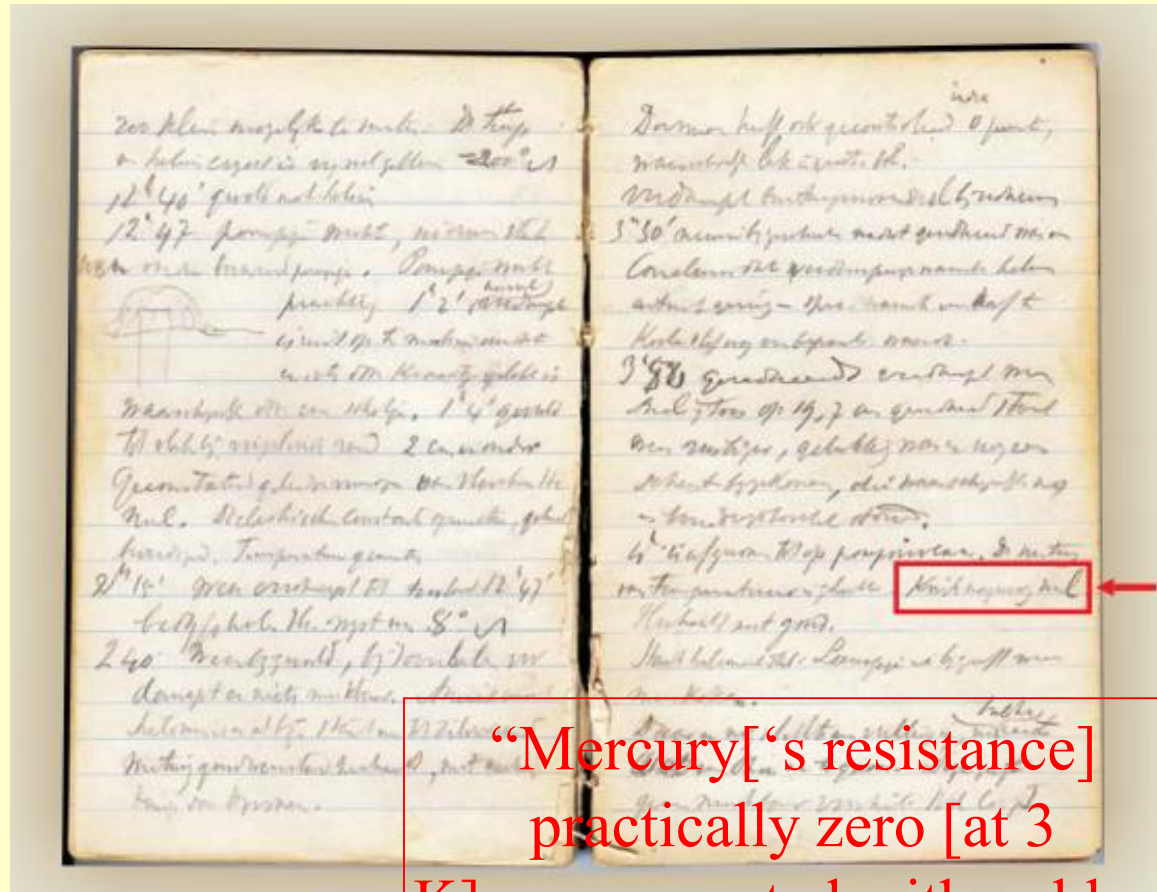
Heike Kamerlingh Onnes
(1853-1926)
Dutch Physicist、



- 1882年，进入Leiden大学，研究低温气体；
- 1908年，将液体的温度降低到大约1K，成功将氦气液化；
- 1911年，开始研究金属在极低温下的性质；
- 1912年，发现了水银的超导电性，
- 1913年，获Nobel奖

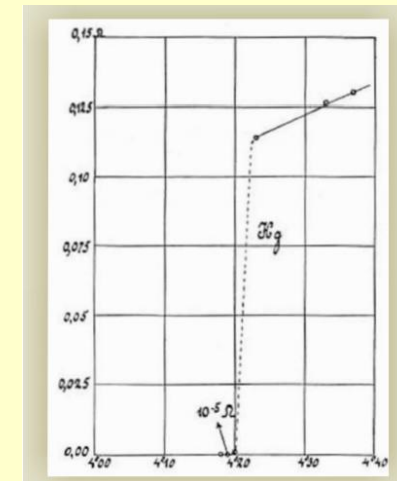
The discovery of superconductivity

Notebook 56, 8 April, 1911



“Mercury[\'s resistance] practically zero [at 3 K]repeated with gold...”

Notebook 57, 26 October, 1911



The historic plot. Superconducting transition at 4.2k in mercury

Meissner effect

Fritz Walther
Meissner
(1882-1974)

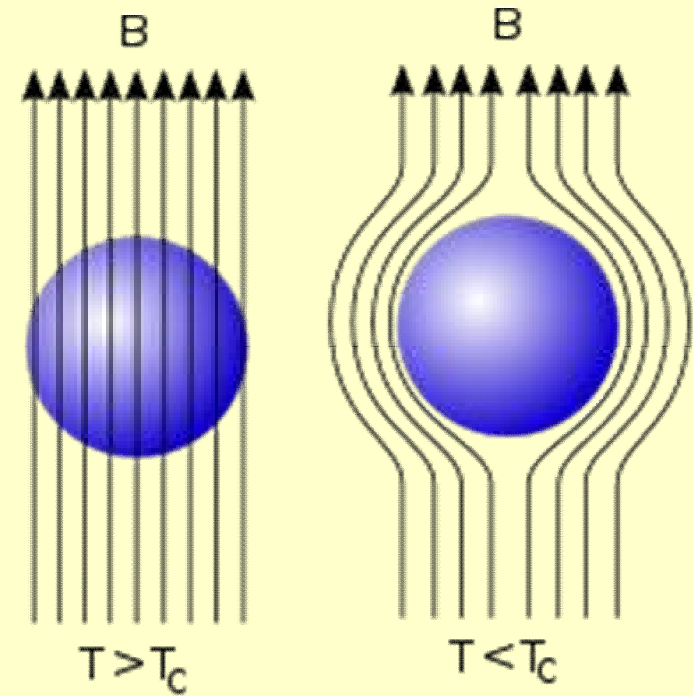


Robert Ochsenfeld
(1901-1993)



German physicists

1933



Perfect diamagnetism
below T_c

Londons' theory

Heinz
London
(1907-1970)



Fritz Wolfgang
London
(1900-1954)



German Physicists

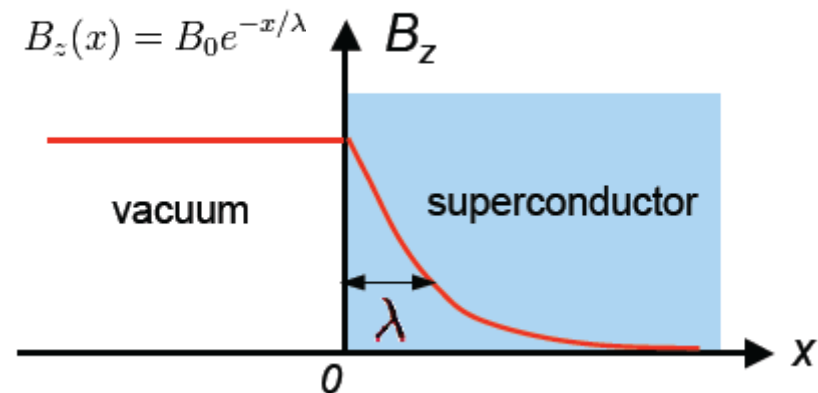
2014/11/5

Londons' Equation:
(1935)

$$\frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m} \mathbf{E}, \quad \nabla \times \mathbf{j}_s = -\frac{n_s e^2}{mc} \mathbf{B}.$$

Ampère's law: $\nabla \times \mathbf{B} = \frac{4\pi \mathbf{j}}{c}$

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}, \quad \lambda \equiv \sqrt{\frac{mc^2}{4\pi n_s e^2}}.$$



Ginzburg-Landau theory

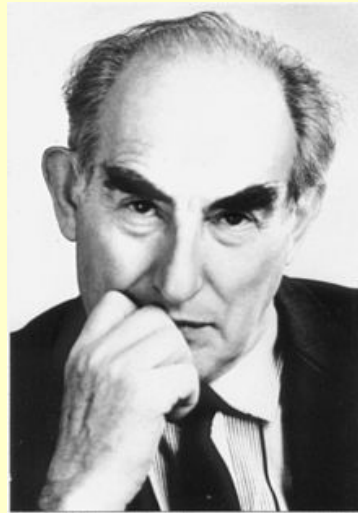
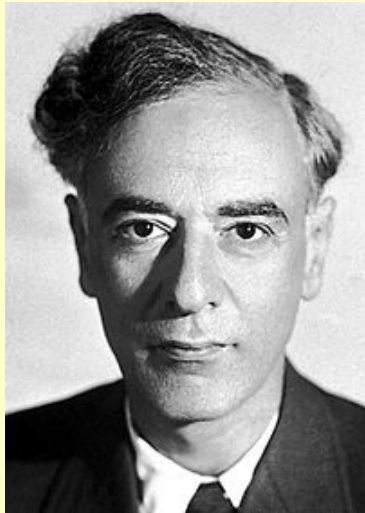
Lev Landau
(1908-1968)

Vitaly Ginzburg
(1916-2009)

1950

The free energy density:

$$F = F_n + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m} |(-i\hbar\nabla - 2e\mathbf{A})\psi|^2 + \frac{|\mathbf{B}|^2}{2\mu_0}$$



Soviet physicists

Complex order parameter



U(1) gauge symmetry broken

其它几种超导体

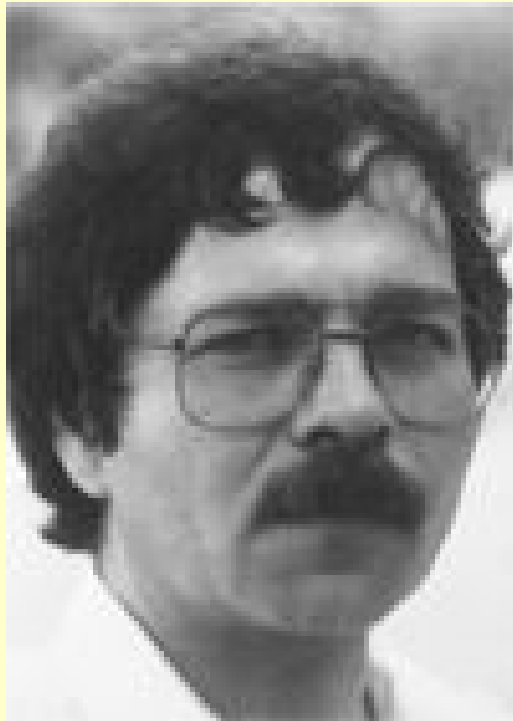
<u>元素</u>	<u>超导转变温度</u>
Al (铝)	1.2 K
In (铟)	3.4 K
Sn (锡)	3.7 K
Pb (铅)	7.2 K
Nb (铌)	9.2 K

1911

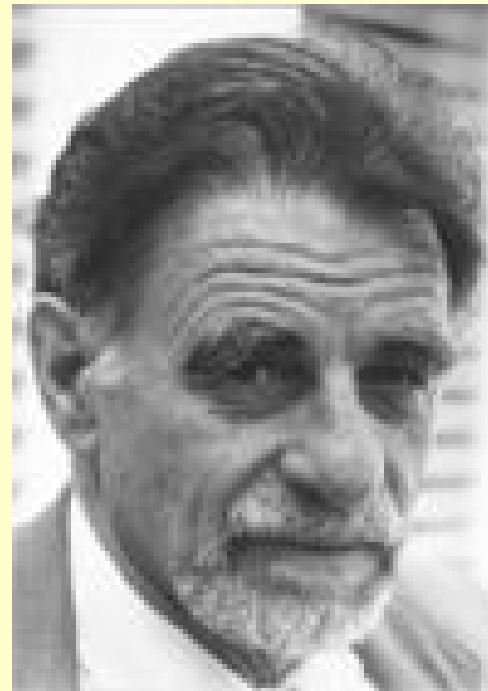


1986

1986年，Muller和Bednorz发现：陶瓷氧化物
 LaBaCuO 的转变温度可达到35K。



J. Georg Bednorz
Germany



K. Alexander Muller
Switzerland

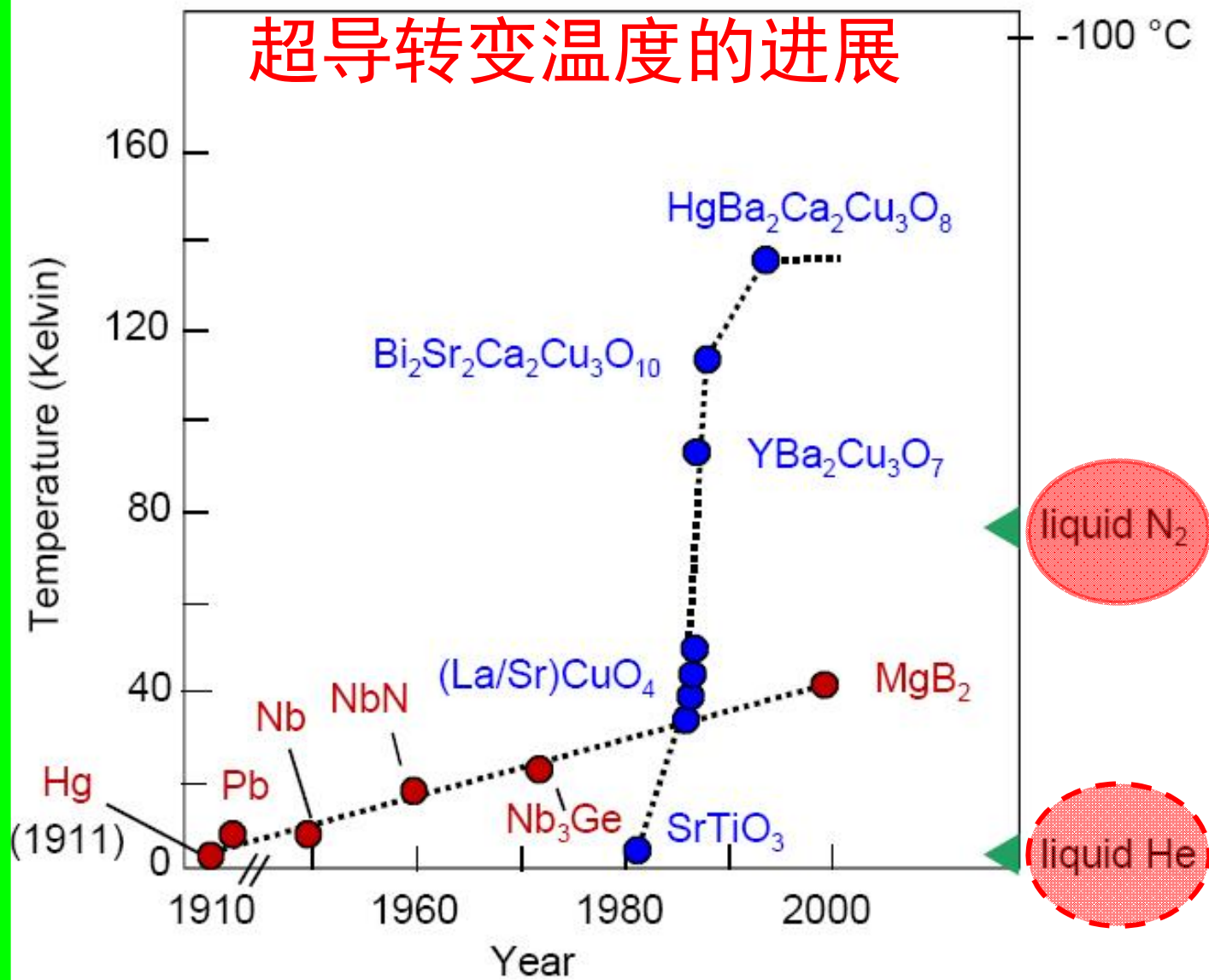
Possible high T_c superconductivity in the Ba–La–Cu–O system

J. G. Bednorz¹ and K. A. Müller¹

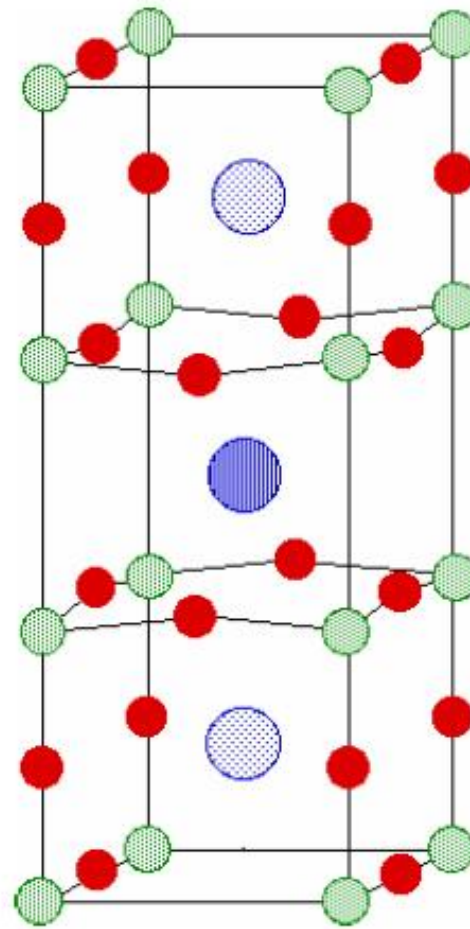
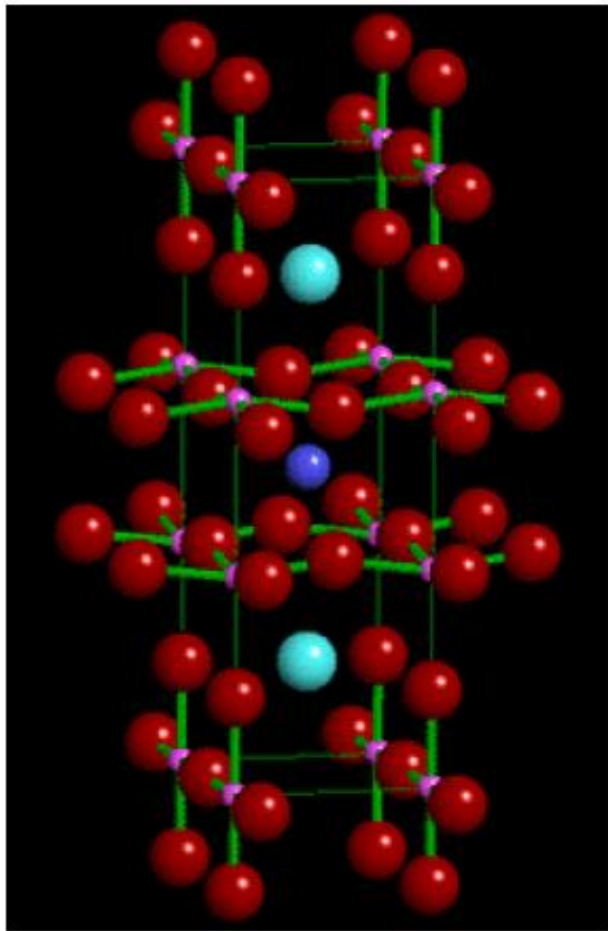
IBM Zürich Research Laboratory, Säumerstrasse 4, CH-8803 Rüschlikon,
Switzerland





Abstract Metallic, oxygen-deficient compounds in the Ba–La–Cu–O system, with the composition $\text{Ba}_x \text{La}_{5-x} \text{Cu}_5 \text{O}_5(3-y)$ have been prepared in polycrystalline form. Samples with $x=1$ and $0.75, y>0$, annealed below 900°C under reducing conditions, consist of three phases, one of them a perovskite-like mixed-valent copper compound. Upon cooling, the samples show a linear decrease in resistivity, then an approximately logarithmic increase, interpreted as a beginning of localization. Finally an abrupt decrease by up to three orders of magnitude occurs, reminiscent of the onset of percolative superconductivity. The highest onset temperature is observed in the 30 K range. It is markedly reduced by high current densities. Thus, it results partially from the percolative nature, but possibly also from 2D superconducting fluctuations of double perovskite layers of one of the phases present. (*Zeitschrift für Physik B Condensed Matter* V 64, No 2 / June, 1986, Received: 17 April 1986)

超导转变温度的进展



YBCO

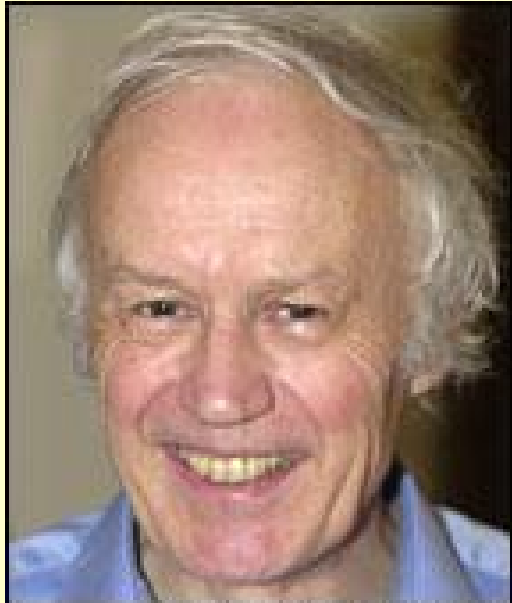


-  Barium
-  Yttrium
-  Copper
-  Oxygen

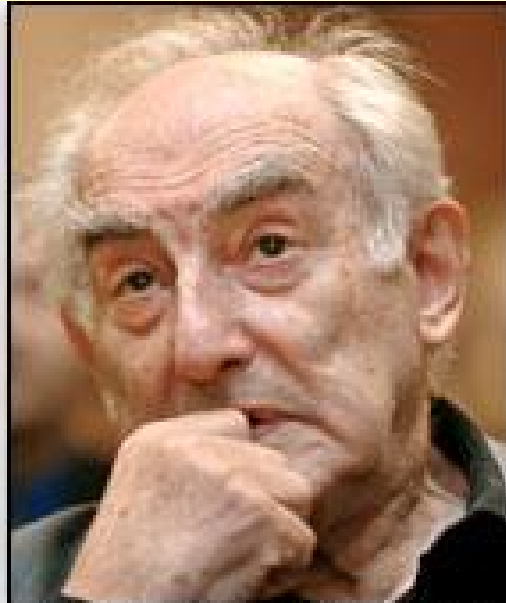
Nobel Prize Awards related to Superconductivity

- 1913: Kamerlingh Onnes discovers **zero resistance** (1911)
- 1972: Bardeen, Cooper and Schrieffer develop **“BCS” theory** (1957)
- 1973 : Ivar Giaever invents tunnel junctions (1960)
Brian Josephson invents **“Josephson Tunneling”** (1962)
- 1987: Bednorz and Muller discover **high-Tc superconductivity** (1986)
- 2003 Alexei Abrikosov predicts **Type II superconductors** (1957)

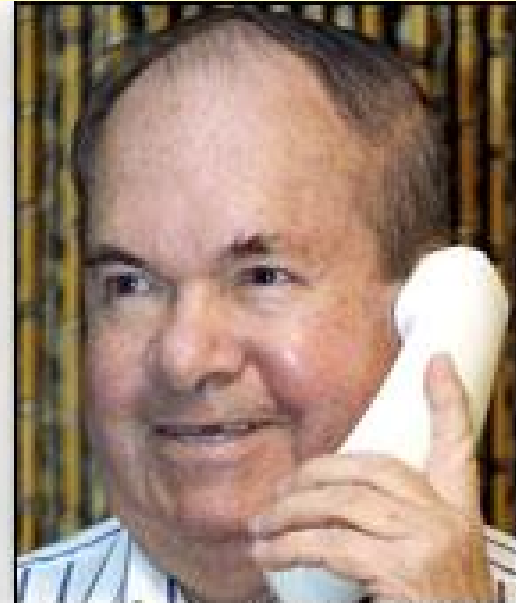
2003 Nobel Prize in Physics



Mark Cowan / Reuters



Alexander Zemlianichenko / AP



Stephen J. Carrera / AP

Discovery of New Fe-based Superconductors

J|A|C|S
COMMUNICATIONS

Published on Web 07/15/2006

Iron-Based Layered Superconductor: LaOFeP

Yoichi Kamihara,[†] Hidenori Hiramatsu,[†] Masahiro Hirano,^{†,‡} Ryuto Kawamura,[§] Hiroshi Yanagi,[§]
Toshio Kamiya,^{†,§} and Hideo Hosono^{*,†,‡}

ERATO-SORST, JST, Frontier Collaborative Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan, Frontier Collaborative Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan, and Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-4, 4259 Nagatsuta, Yokohama 226-8503, Japan

J|A|C|S
COMMUNICATIONS

Published on Web 02/23/2008

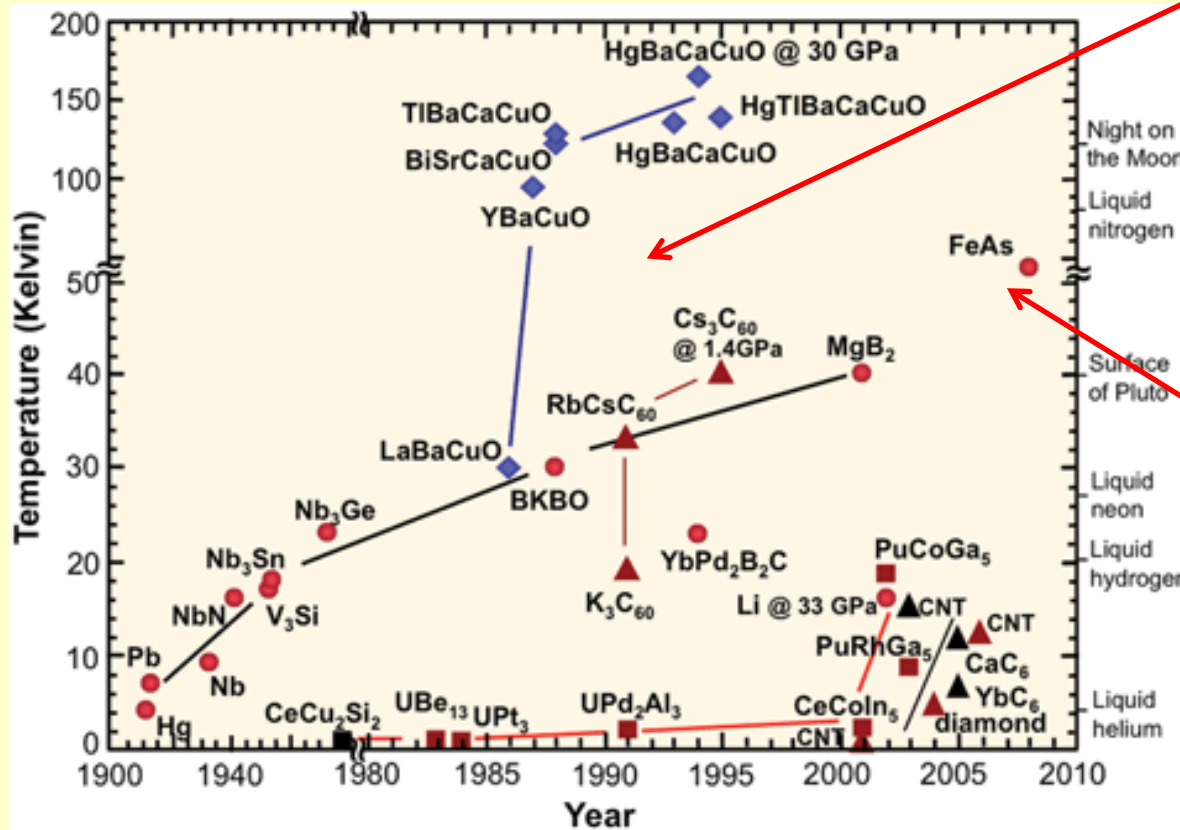
Iron-Based Layered Superconductor La[O_{1-x}F_x]FeAs ($x = 0.05-0.12$) with $T_c = 26$ K

Yoichi Kamihara,^{*,†} Takumi Watanabe,[‡] Masahiro Hirano,^{†,§} and Hideo Hosono^{†,‡,§}

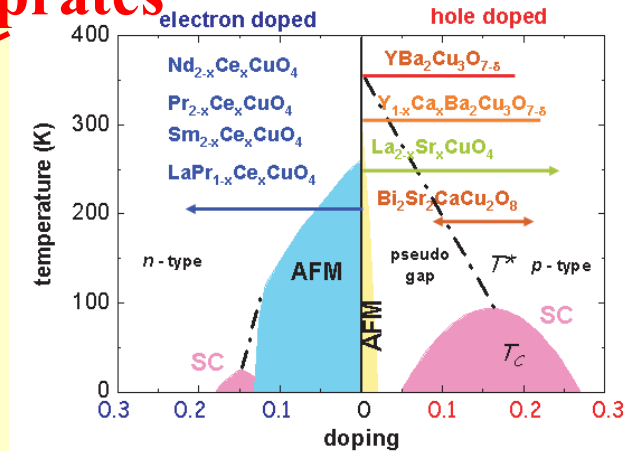
ERATO-SORST, JST, Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, Materials and Structures Laboratory, Tokyo Institute of Technology, Mail Box R3-1, and Frontier Research Center, Tokyo Institute of Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan

Received January 9, 2008; E-mail: hosono@msl.titech.ac.jp

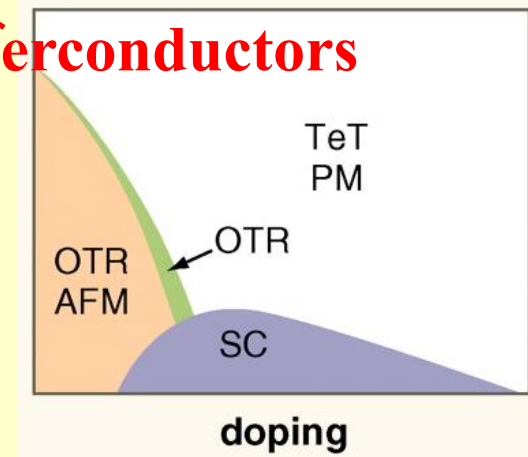
High-Tc superconductors



Cuprates

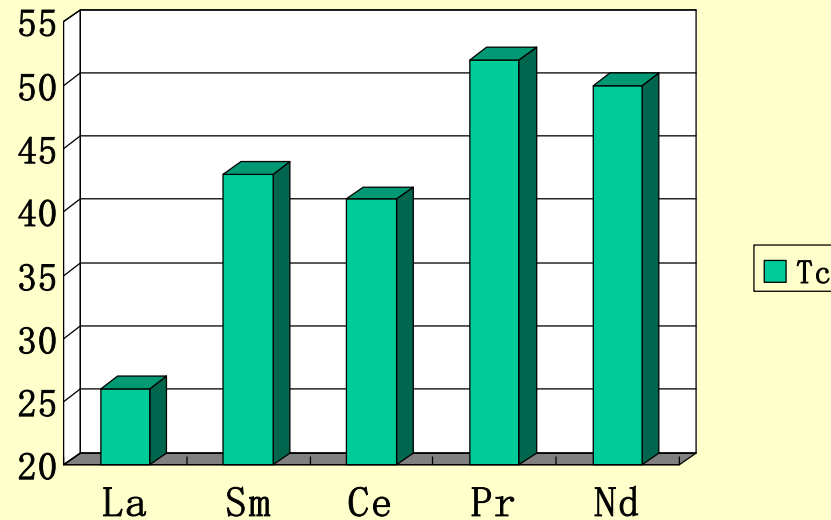


Iron-based superconductors



The Progress in Raising the Transition Temperature

LaOFeP $T_c = 4\text{K}$
La[O_{1-x}F_x]FeP $T_c = 7\text{K}$
LaONiP $T_c = 2\text{K}$



$\text{LnO}_{1-x}\text{F}_x\text{FeAs}$ (Ln=La, Pr, Ce, Sm, Nd)

X. H. Chen et al., G. F. Chen et al., Z. A. Ren et al., (陈仙辉, 中科技大)

$(\text{La}_{1-x}\text{Sr}_x)\text{OFeAs}$ H. H. Wen, et al., EPL(08)

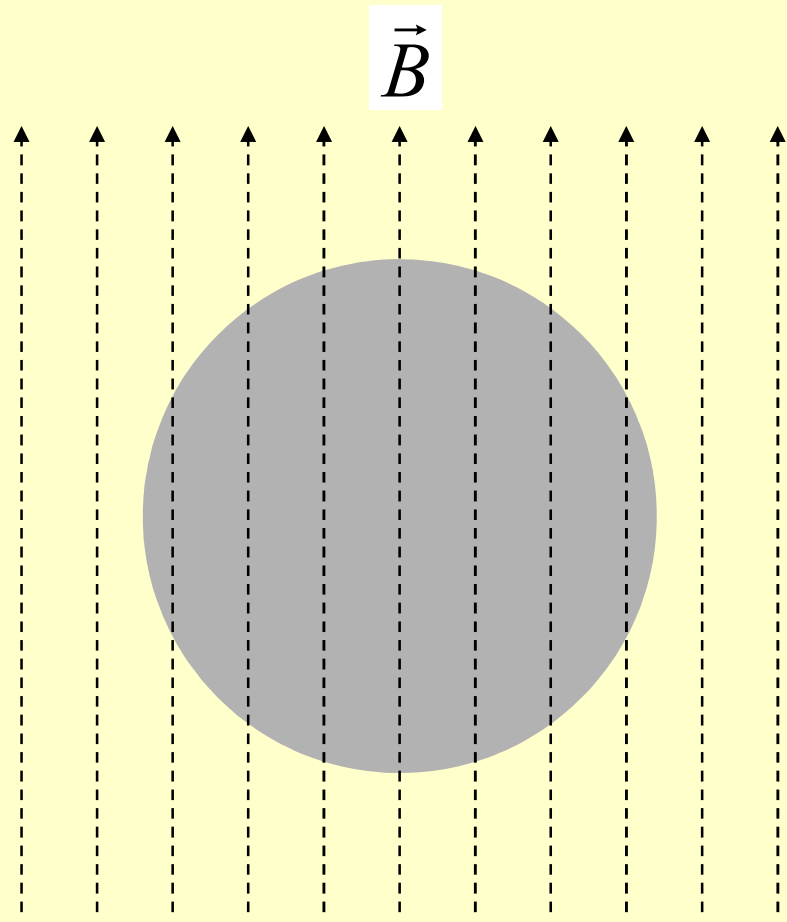
闻海虎教授, hhwen@iphy.ac.cn

超导体特性之二

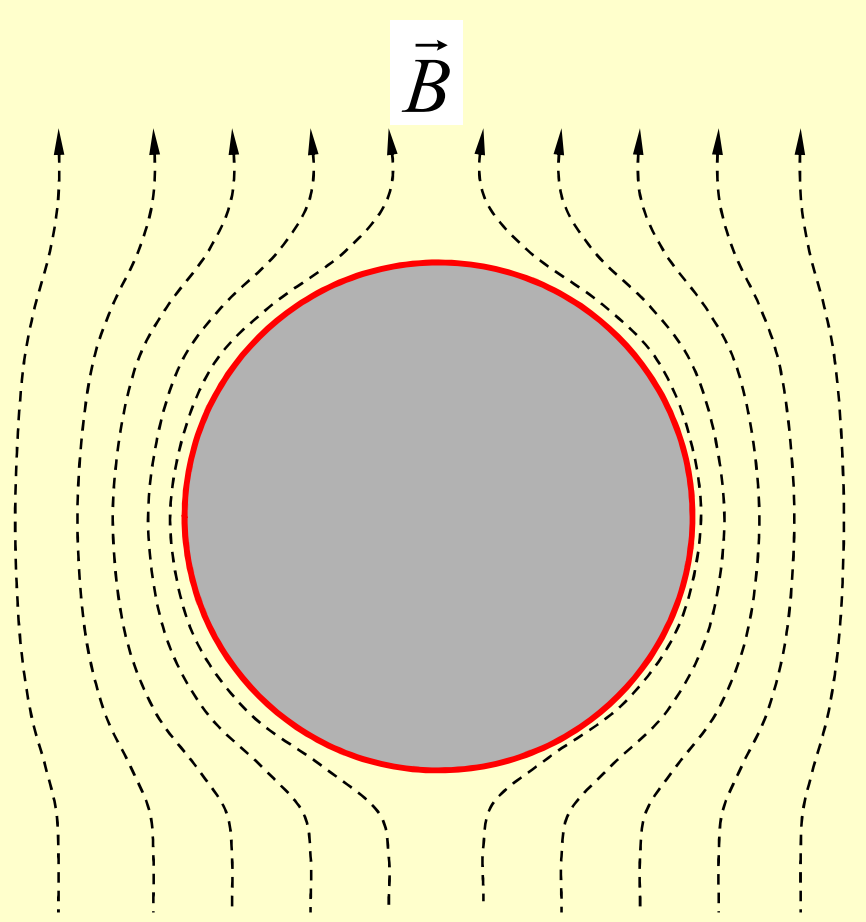
完全抗磁性 (Meissner效应)

1) 迈斯纳效应

- ① 对于处于正常态的样品，加上磁场后磁场能进入样品的内部；
- ② 当温度降低到 T_c 以下时，磁场立即被排斥在样品外，样品内部的磁感应强度 (B) 为零。

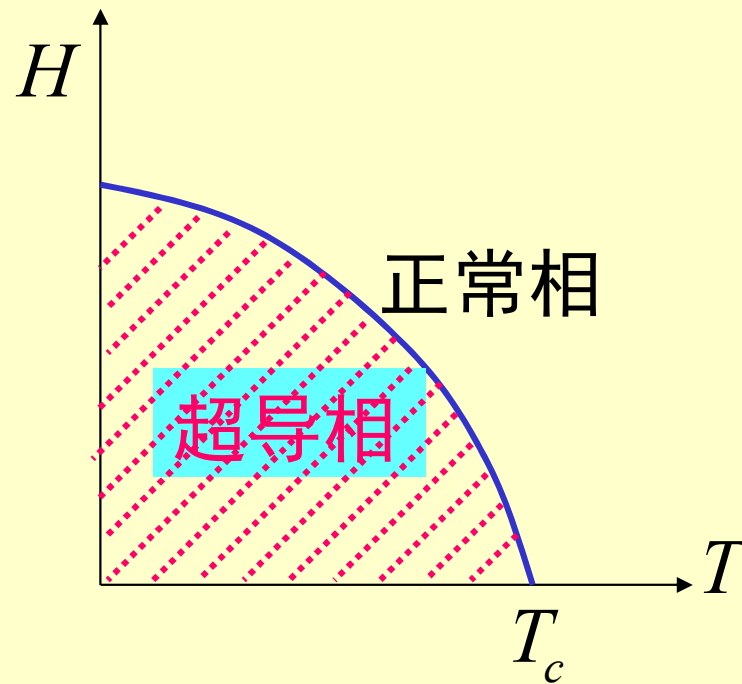


$(T > T_c(H))$



$(T < T_c(H))$

- ③ 在外磁场作用下，临界温度随磁场的增加而降低。(Onnes发现超导的数年之后，失望的发现微小的磁场就能破坏超导现象。)



$$H_c(T) = H_c(0) \left[1 - (T/T_c)^2 \right]$$

$$(\vec{J} = \sigma \vec{E})$$

2) 完全抗磁性与零电阻（超导）特性是超导体的两个独立的特性

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

例如, 根据零电阻的超导状态, 在超导体内有:

$$\vec{E} \rightarrow 0 \quad (\vec{J} \text{ 为有限值, } \sigma \rightarrow \infty)$$

$$\text{因此 } \partial \vec{B} / \partial t = 0$$

从而得出“样品内的磁场在 T_c 的前后不应发生变化”这一与实验观测相反的推测。

超导电动力学

1) 超导性本质上是一种量子效应；

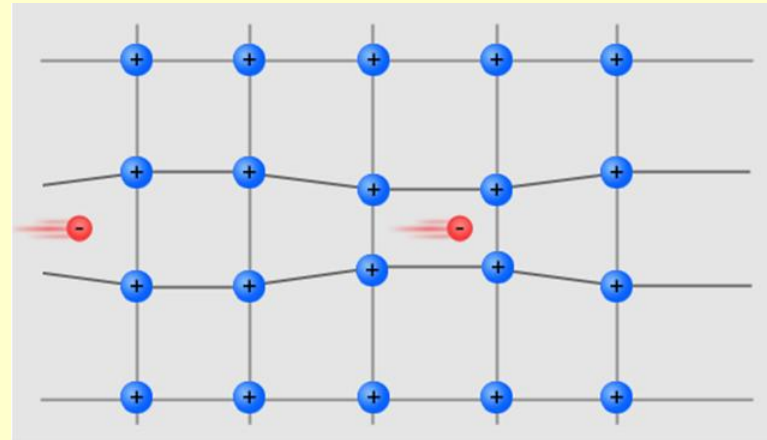
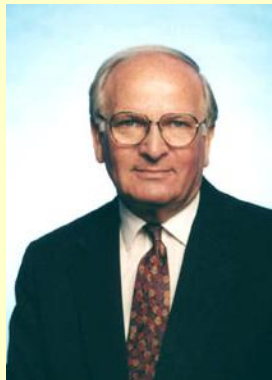
- 早期的唯象理论：伦敦方程、朗道-金兹堡方程
- 量子理论：BCS 理论
- 约瑟夫森和安德森：发现了超导波函数相位因子的重要性

BCS theory (1957)

John Bardeen
(1908-1991)

Leon Cooper
(1930-)

John Schrieffer
(1931-)



Lattice Vibration

Phonon

“Cooper Pairs”

The SC order parameter

PHYSICAL REVIEW

VOLUME 108, NUMBER 5

DECEMBER 1, 1957

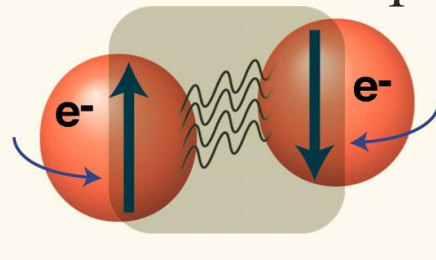
Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

A theory of superconductivity is presented, based on the fact that the interaction between electrons resulting from virtual exchange of phonons is attractive when the energy difference between the electrons states involved is less than the phonon energy, $\hbar\omega$. It is favorable to form a superconducting phase when this attractive interaction dominates the repulsive screened Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs of opposite spin and momentum, is lower in energy than the normal state by amount proportional to an average $(\hbar\omega)^2$, consistent with the isotope effect. A mutually orthogonal set of excited states in

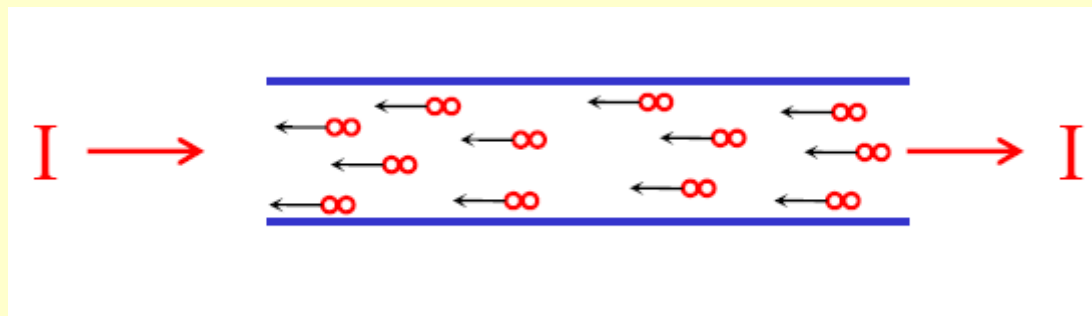
one-to-one correspondence with those obtained by specifying occupation of using the rest to form a linear combination. The theory yields a second Meissner effect in the form suggests values of specific heats and penetration variation are in good agreement an energy gap for individual-particle from about $3.5kT_c$ at $T=0^\circ\text{K}$ to zero at T_c . Tables of matrix elements of single-particle operators between the excited-state superconducting wave functions, useful for perturbation expansions and calculations of transition probabilities, are given.



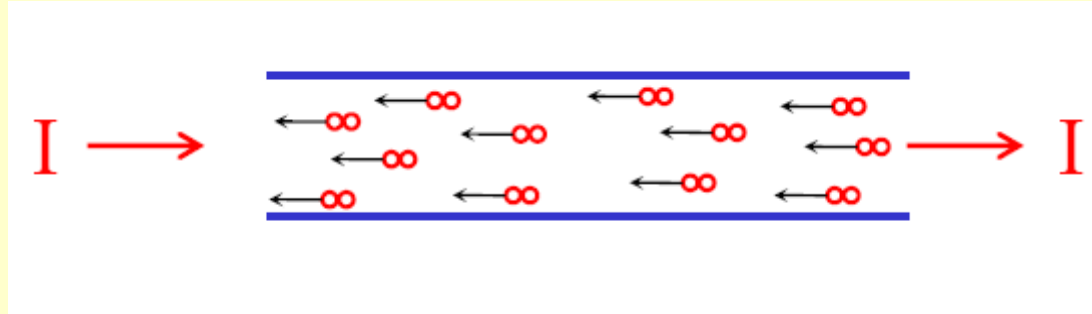
在 T_c 附近材料发生了超导相变

- ① 电子与晶格相互作用，使得两个电子之间间接吸引而成对；
- ② 超导电流就是由这些电子对的运动形成的电流；

超导体



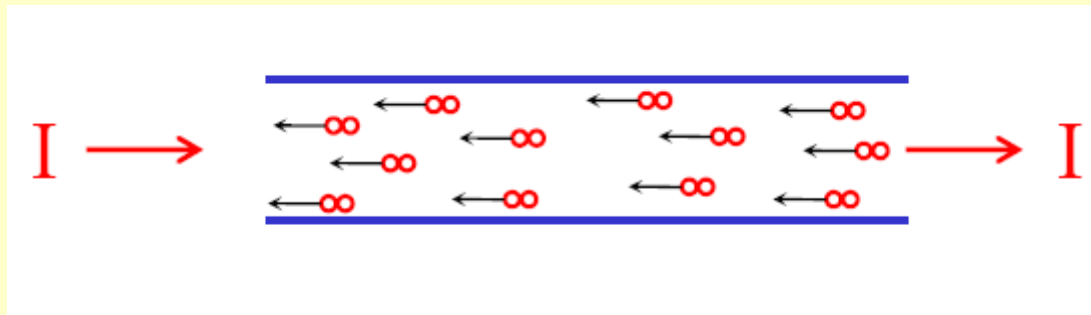
超导体



- ③ 这种束缚态存在一定的结合能 Δ ；
- ④ 在低温下，由于 $kT < \Delta$ ，热运动不会破坏库伯对；而 $kT > \Delta$ 时，库伯对被拆散；
- ⑤ 材料的超导转变温度：

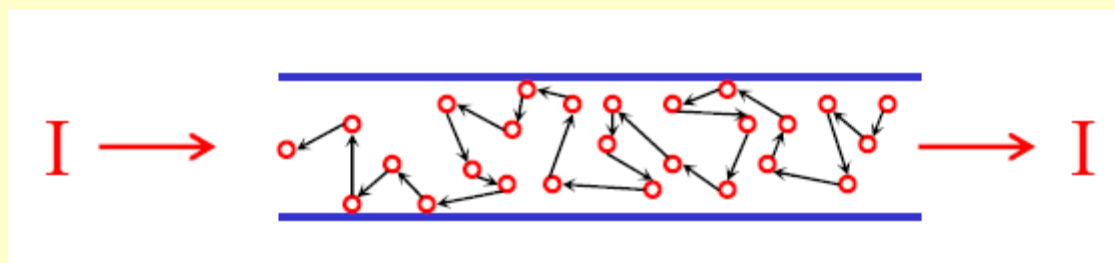
$$T_c \approx \frac{\Delta}{k}$$

超导体



- 库珀对携带超导电流，而不产生电阻；
- 超导电流不产生电势差，因而不引起能量的耗散。

常规金属



- 电子受到杂质、晶格畸变而引起的散射产生电阻；
- 电流产生电势差，引起能量的耗散。

4) 关于超导解释的早期基于经典电动力学的唯象观点:

- 在麦克斯韦方程的基础上, 通过材料电磁性质的假设, 对超导的宏观电动力学现象的给出**唯象描述**。

(1) 磁介质观点

(2) 二流体模型

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

(1) 磁介质观点:

$$\vec{M} = \chi_M \vec{H}$$

① 超导体是一种完全抗磁体;

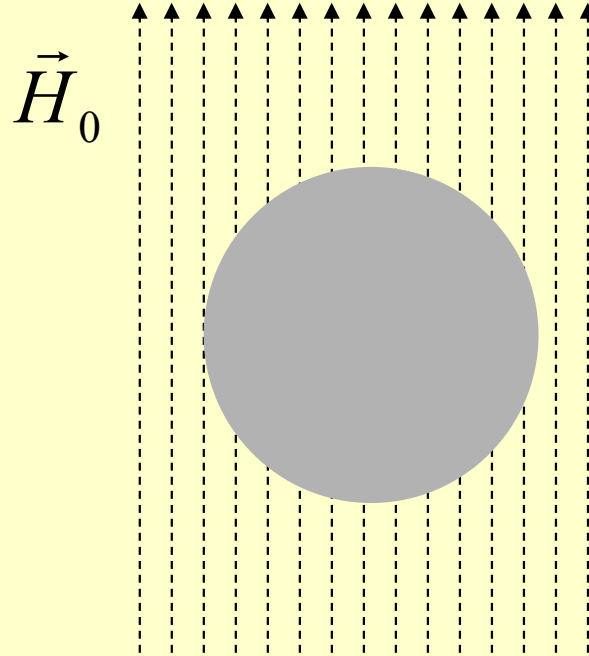
超导态: $\vec{B} = 0,$ $\vec{M} = -\vec{H}$

$$\chi_M = -1$$

$$\mu = \mu_0 (1 + \chi_M) = 0$$

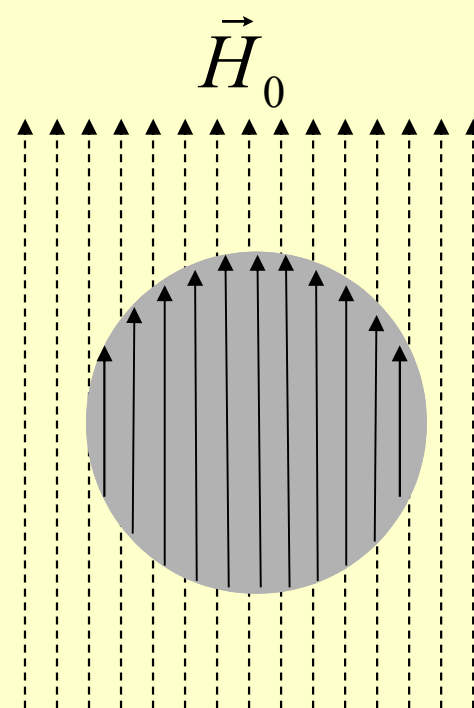
② 超导电流是磁化电流; 磁化电流以面电流的形式分布在超导体的表面。

例题：超导球体放置于均匀的外磁场中，求超导球体内外的磁场和超导面电流分布。



超导球的内外磁场都可用磁标势来描述：

- 处于外磁场中的超导球内、球外没有自由电流；
- 在这两个单连通区域内，沿任何闭合回路都没有电流环链；



$$\vec{H} = -\nabla \varphi_m$$

$$\nabla^2 \varphi_m = -\frac{\rho_m}{\mu_0}$$

$$\rho_m = -\mu_0 \nabla \cdot \vec{M}$$

$$\nabla \cdot \vec{B} = 0$$

对于各向同性介质,

$$\nabla \cdot \vec{M} = 0$$

超导球的内外磁标势满足拉普拉斯方程

$$\nabla^2 \varphi_{\text{内}} = 0, \quad \nabla^2 \varphi_{\text{外}} = 0$$

边界条件:

$$\varphi_{m1} = \varphi_{m2}$$

(区域边界没有自由电流)

$$\mu_1 \left(\frac{\partial \varphi_m}{\partial n} \right)_1 = \mu_2 \left(\frac{\partial \varphi_m}{\partial n} \right)_2$$

(对由非铁磁介质构成的分界面)

$$\varphi_{m1} = \varphi_{m2}$$

$$\mu_1 \left(\frac{\partial \varphi_m}{\partial n} \right)_1 = \mu_2 \left(\frac{\partial \varphi_m}{\partial n} \right)_2$$

超导体处于超导态时是**完全抗磁体**，其磁导率

$$\mu = 0$$

因此，超导体的第二边界条件简化为：

$$\left. \frac{\partial \varphi_{\text{外}}}{\partial n} \right|_{\text{球面}} = 0$$

球坐标系中，两个区域磁标势的通解形式：

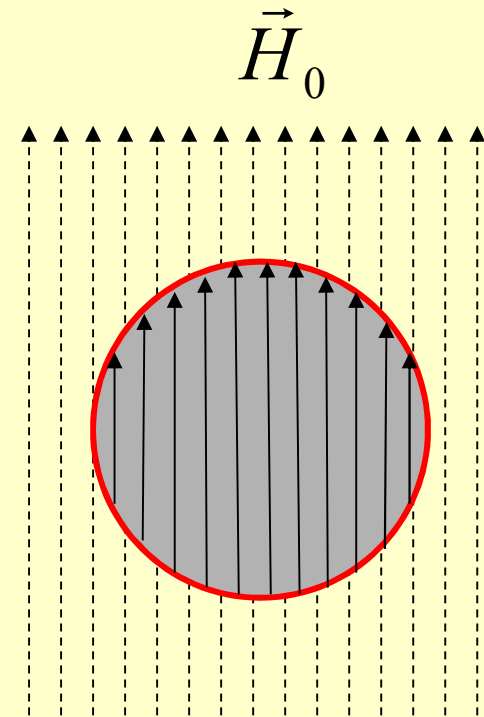
$$\varphi_{\text{外}} = \sum_n \left(a_n R^n + \frac{b_n}{R^{n+1}} \right) P_n(\cos \theta)$$

$$\varphi_{\text{内}} = \sum_n \left(c_n R^n + \frac{d_n}{R^{n+1}} \right) P_n(\cos \theta)$$

边界条件的要求如下：

(i) 在无穷远处，磁场 $\vec{H} \rightarrow \vec{H}_0$

$$\varphi_{\text{外}} \rightarrow -H_0 R \cos \theta = -H_0 R P_1(\cos \theta)$$



因此有

$$a_1 = -H_0$$

$$a_n = 0 \quad (n \neq 1)$$

(ii) 在球心处，磁标势应为一有限值，因此要求

$$d_n = 0$$

$$\varphi_{\text{外}} = \sum_n \left(a_n R^n + \frac{b_n}{R^{n+1}} \right) P_n(\cos \theta)$$

$$\varphi_{\text{内}} = \sum_n \left(c_n R^n + \frac{d_n}{R^{n+1}} \right) P_n(\cos \theta)$$

$$\varphi_{\text{外}} = -H_0 r \cos \theta + \sum_n \frac{b_n}{R^{n+1}} P_n(\cos \theta)$$

$$\varphi_{\text{内}} = \sum_n c_n R^n P_n(\cos \theta)$$

$$\varphi_{\text{外}} = -H_0 r \cos \theta + \sum_n \frac{b_n}{R^{n+1}} P_n(\cos \theta) (a_n R^n +)$$

$$\varphi_{\text{内}} = \sum_n c_n R^n P_n(\cos \theta)$$

(iii) 在介质分界面——球面上

$$\varphi_{\text{外}} \Big|_{R=R_0} = \varphi_{\text{内}} \Big|_{R=R_0}$$

$$\frac{\partial \varphi_{\text{外}}}{\partial R} \Big|_{R=R_0} = 0$$

$$-H_0 R_0 P_1(\cos \theta) + \sum_n \frac{b_n}{R_0^{n+1}} P_n(\cos \theta) \equiv \sum_n c_n R_0^n P_n(\cos \theta)$$

$$-H_0 P_1(\cos \theta) - \sum_n \frac{(n+1)b_n}{R_0^{n+2}} P_n(\cos \theta) \equiv 0$$

比较两式中 $P_1(\cos \theta)$ 前的系数，得到

$$\begin{cases} -H_0 R_0 + \frac{b_1}{R_0^2} = c_1 R_0 \\ -H_0 - \frac{2b_1}{R_0^3} = 0 \end{cases}$$

解得，

$$b_1 = -\frac{1}{2} H_0 R_0^3$$

$$c_1 = -\frac{3}{2} H_0$$

比较 $P_n(\cos \theta)$ ($n \neq 1$) 前的系数，得到，

$$\frac{b_n}{R_0^{n+1}} = c_n R_0^n, \quad \frac{(n+1)b_n}{R_0^{n+2}} = 0$$

其解为： $b_n = c_n = 0$ ($n \neq 1$)

整个空间的磁标势分布为

$$\varphi_{\text{外}} = -H_0 R \cos \theta - \frac{H_0}{2} \frac{R_0^3}{R^2} \cos \theta \quad (r > R_0)$$

$$\varphi_{\text{内}} = -\frac{3}{2} H_0 R \cos \theta \quad (r < R_0)$$

$$\varphi_{\text{内}} = -\frac{3}{2}H_0R \cos \theta \quad (r < R_0)$$

超导球内的磁场

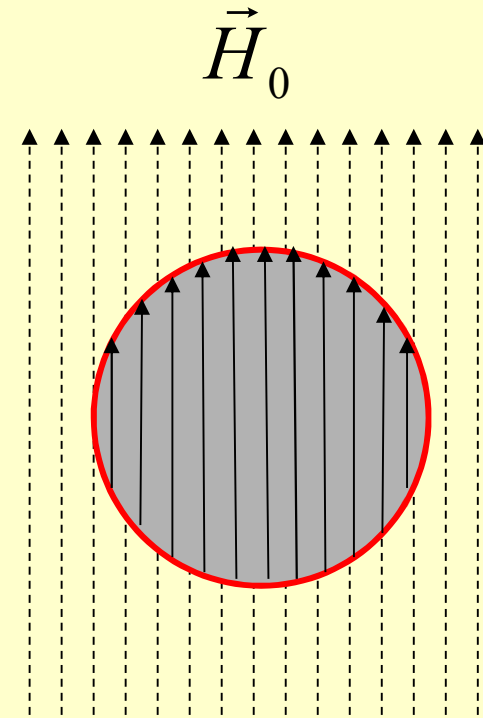
$$\begin{aligned}\vec{H} &= -\nabla \varphi_{\text{内}} = \frac{3}{2}H_0 \nabla(R \cos \theta) \\ &= \frac{3}{2}H_0 \nabla z = \frac{3}{2}H_0 \vec{e}_z = \frac{3}{2}\vec{H}_0\end{aligned}$$

超导球内的磁化强度：

$$\vec{M} = -\vec{H} = -\frac{3}{2}\vec{H}_0$$

超导球的磁矩为

$$\vec{m} = \frac{4}{3}\pi R_0^3 \vec{M} = -2\pi R_0^3 \vec{H}_0$$



$$\varphi_{\text{外}} = -H_0 R \cos \theta - \frac{H_0}{2} \frac{R_0^3}{R^2} \cos \theta$$

$$\vec{m} = \frac{4}{3} \pi R_0^3 \vec{M} = -2\pi R_0^3 \vec{H}_0$$

磁偶极子的磁势为

$$\varphi_m^{(1)} = \frac{\vec{m} \cdot \vec{R}}{4\pi R^3} = -\frac{R_0^3}{2R^3} \vec{H}_0 \cdot \vec{R} = -\frac{H_0 R_0^3}{2R^2} \cos \theta$$

超导球外的磁场是均匀外磁场与磁偶极子磁场之和。

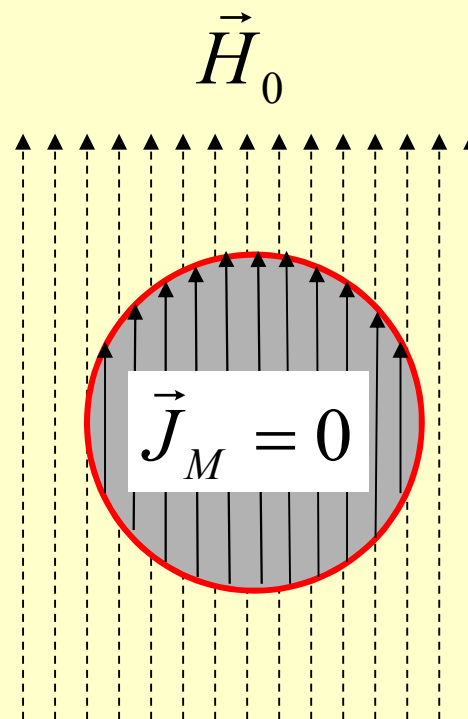
超导球表面的面电流密度

$$\vec{J}_M = \nabla \times \vec{M}$$

在超导体球内部磁化电流，

$$\vec{J}_M = 0$$

磁化电流（超导电流）以面电流形式分布在球表面。



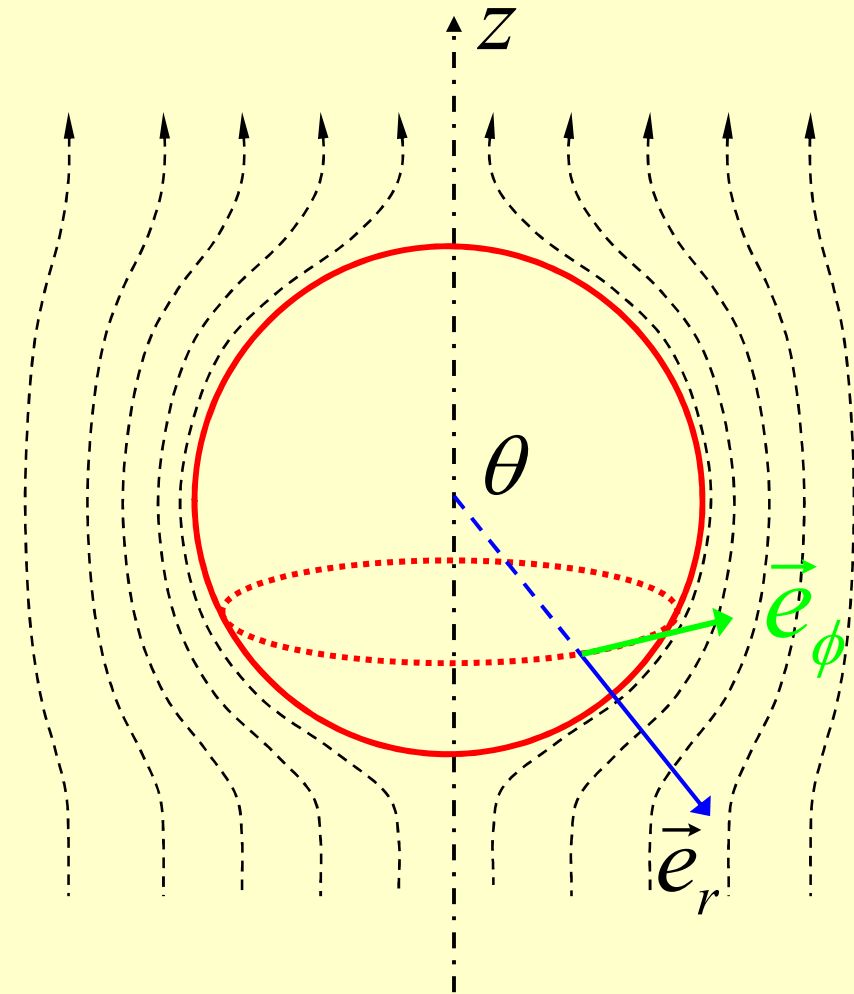
$$\vec{M} = -\vec{H} = -\frac{3}{2}\vec{H}_0$$

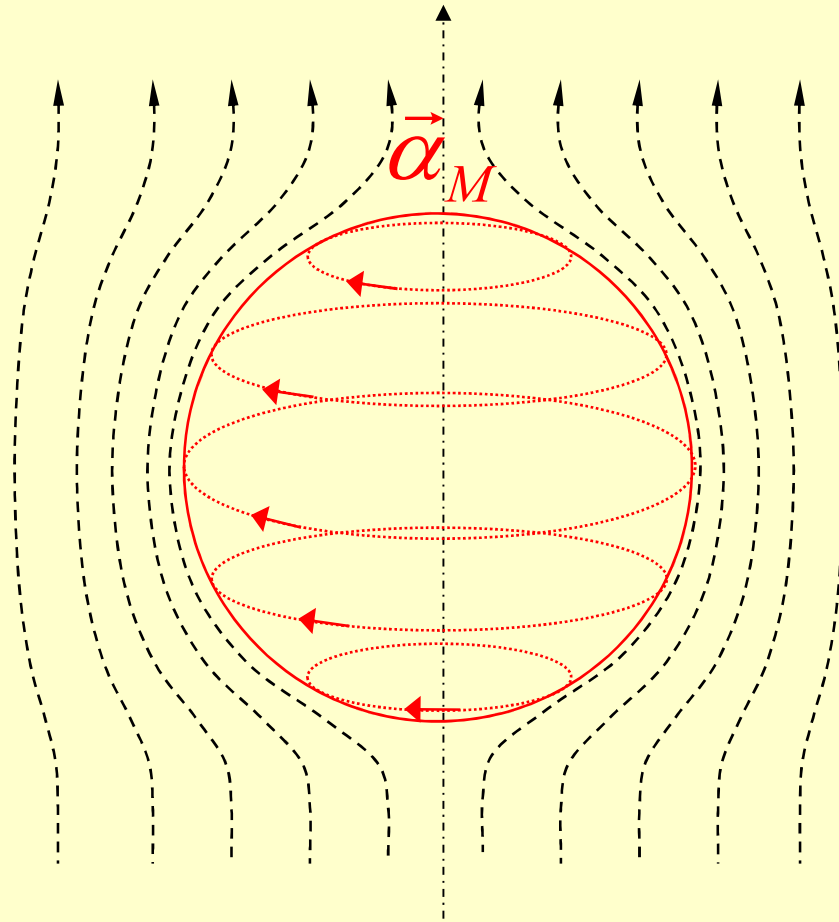
将 $\vec{J}_M = \nabla \times \vec{M}$ 积分形式

$$\int \vec{J}_M \cdot d\vec{S} = \oint \vec{M} \cdot d\vec{\ell}$$

$$\vec{n}_{21} \times (\vec{M}_2 - \vec{M}_1) = \vec{\alpha}_M$$

$$\begin{aligned}\vec{\alpha}_M &= -\vec{e}_r \times \vec{M} \\ &= \frac{3}{2}H_0 \vec{e}_r \times \vec{e}_z \\ &= -\frac{3}{2}H_0 \sin\theta \vec{e}_\phi\end{aligned}$$

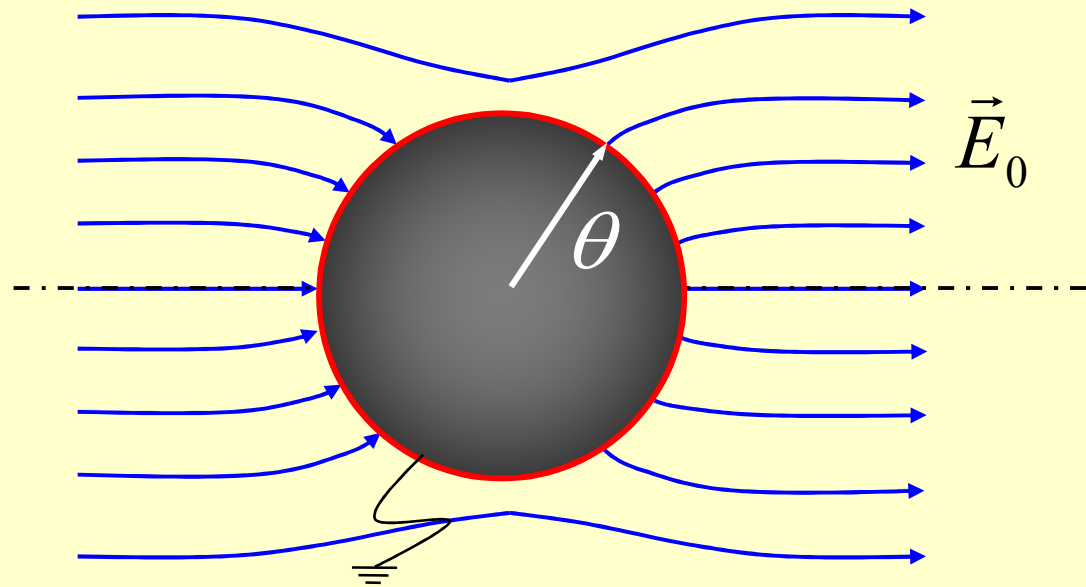




$$\vec{\alpha}_M = -\frac{3}{2} H_0 \sin \theta \vec{e}_\phi$$

- 超导球表面的面电流密度

半径为 R 的接地导体球放置于均匀外场中



导体球外空间任一点电势为

$$\begin{aligned}\varphi &= -E_0 r \cos \theta + \frac{E_0 R_0^3}{r^2} \cos \theta \\ &= -rE_0 + \frac{1}{4\pi\epsilon_0} \times 4\pi\epsilon_0 E_0 R_0^3 \cdot \frac{\cos \theta}{r^2}\end{aligned}$$

电偶极子的电势：

$$\varphi = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

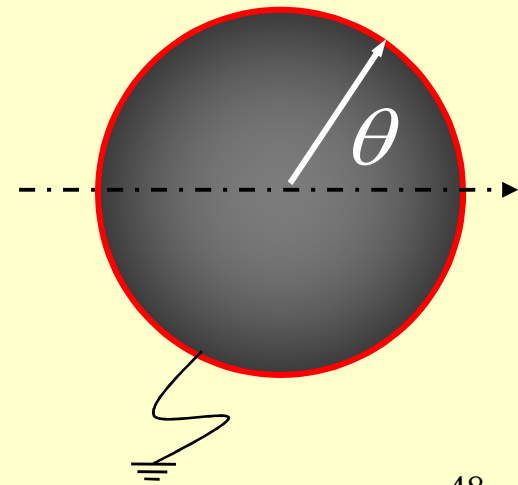
$$\varphi = E_0 \left(-r + \frac{R_0^3}{r^2} \right) \cos \theta$$

球表面感应电荷的面密度为

$$\sigma = -\varepsilon \left. \frac{\partial \varphi}{\partial r} \right|_{r=R_0} = 3\varepsilon_0 E_0 \cos \theta$$

感应电荷形成的偶极矩： $\vec{p} = \oint \vec{r} dq = \oint \vec{r} \sigma dS$

$$\begin{aligned} p &= p_x = \oint R_0 \cos \theta \cdot \sigma dS \\ &= \oint 3\varepsilon_0 E_0 R_0 \cos^2 \theta \cdot R_0^2 \sin \theta d\theta d\phi \\ &= 3\varepsilon_0 E_0 R_0^3 \times \frac{4\pi}{3} = 4\pi\varepsilon_0 E_0 R_0^3 \end{aligned}$$



电荷控制
电场强度



$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$



电流控制
磁场强度

(2) 二流体模型

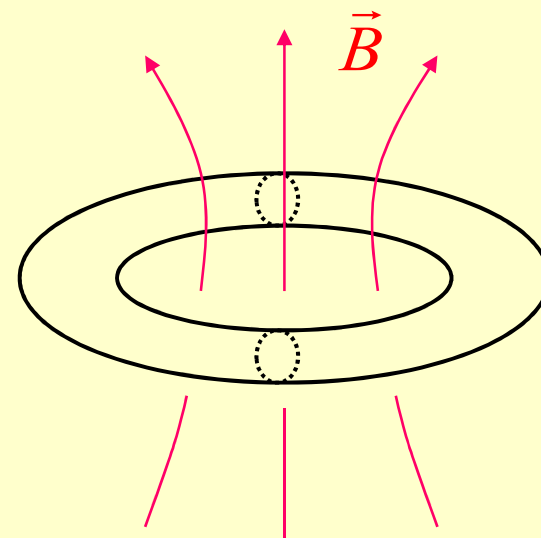
关于超导体内的磁场及超导电流的分布
——上述两种观点所得到的图象是一致的。

4、超导环的磁通俘获和磁通量子化现象

——矢势在量子效应中的可观测效应之二

1) 磁通俘获现象:

- ① 在 $T > T_c$ 时, 将一个处于正常态的超导环放置于外磁场中;
- ② 降低温度, 使得 $T < T_c$, 该环转变为**超导态**。



实验发现: 撤去外场后, 超导环的磁通仍然保持。

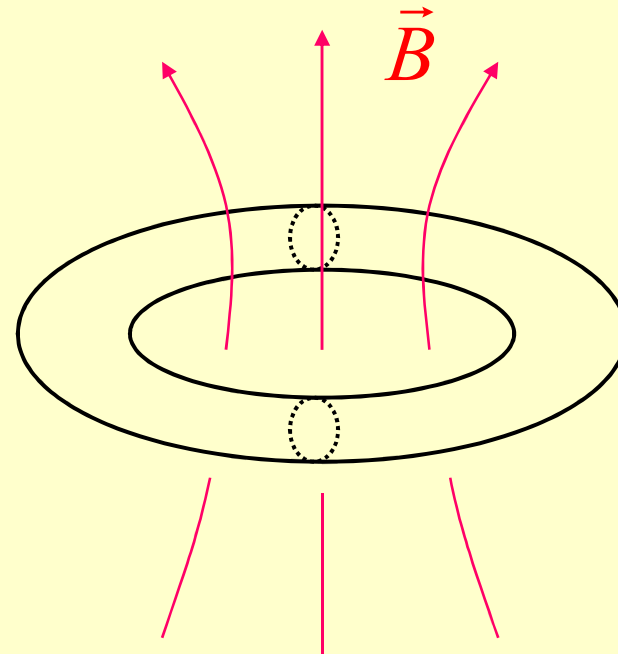
2) 采用麦克斯韦方程解释：在磁场撤去的前后，超导环中间的磁通不变

麦克斯韦方程

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

积分形式：

$$\frac{d\Phi_m}{dt} = \frac{d}{dt} \int \vec{B} \cdot d\vec{S} = -\int_C \vec{E} \times d\vec{\ell}$$

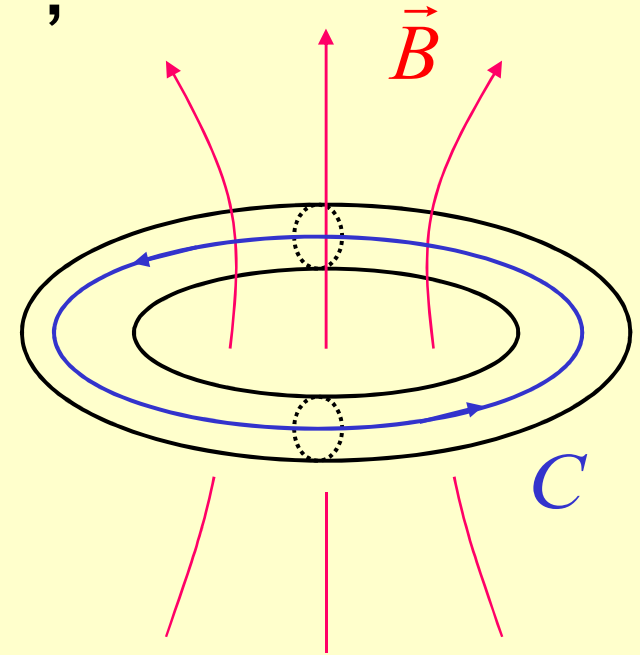


① 将积分回路选取在超导环内部（这个积分路径须离超导体的表面足够远）；

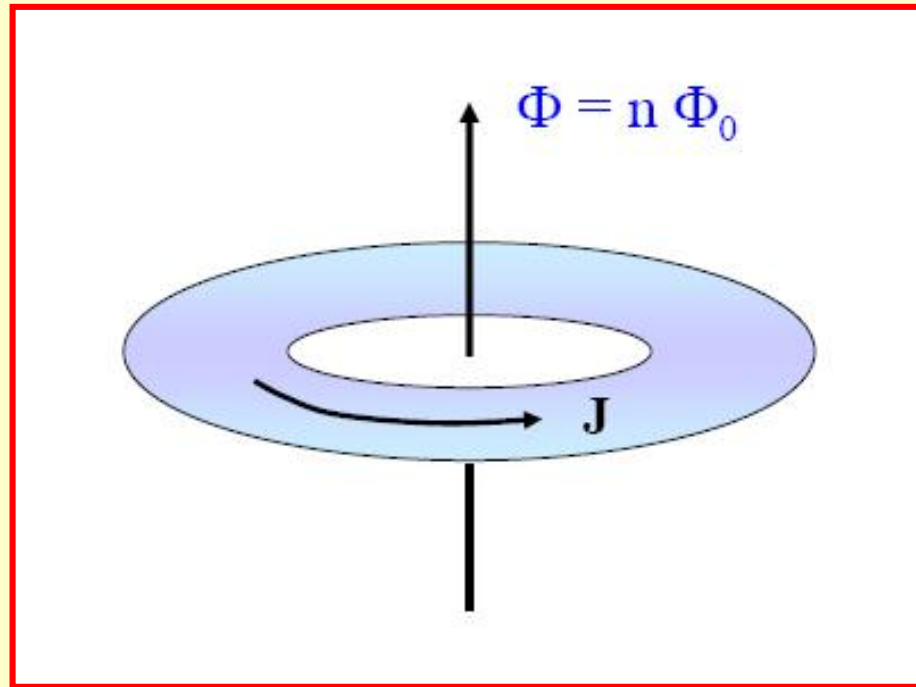
② 由于磁场撤去的前后金属环始终处于超导态，所以环内部的电场始终为零，从而得到

$$\frac{d\Phi_m}{dt} = 0$$

结论：磁通量为与时间无关的常数。



- 3) 超导环中间的磁通是量子化的：
磁通是某个磁通量子 Φ_0 的整数倍。



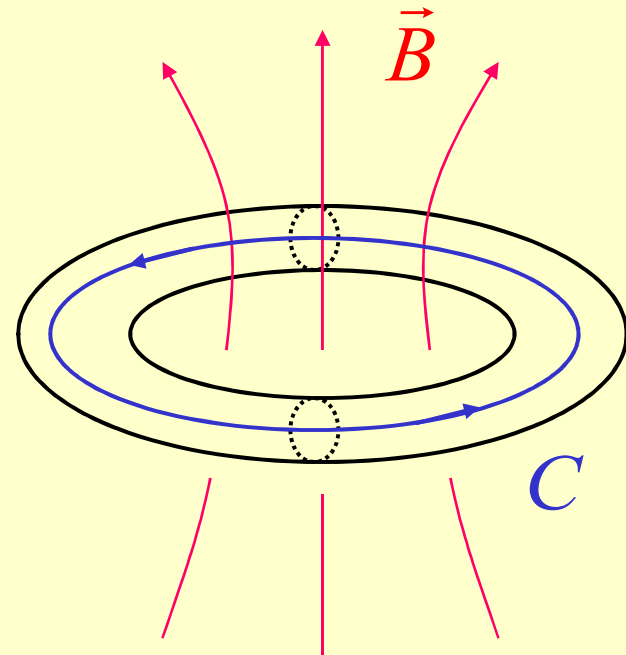
$$\Phi = n \Phi_0 \quad (n = 0, \pm 1, \pm 2, \dots)$$
$$\Phi_0 \equiv h/2e \text{ 为磁通量子}$$

分析如下：绕上述积分回路一周，电子波函数位相的变化值为

$$\Delta\phi = \frac{1}{\hbar} \oint_C \vec{P} \cdot d\vec{\ell}$$

$$\vec{P} = \vec{p} - e\vec{A}$$

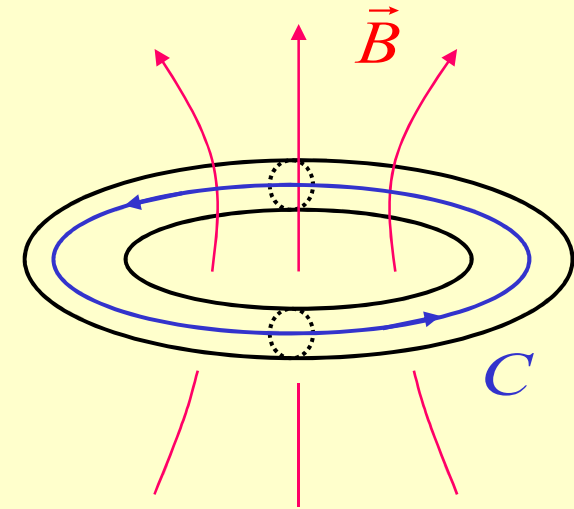
——为电子的正则动量。



超导电子是库伯电子对，正则动量为

$$\vec{P} = 2m\vec{v}_s - 2e\vec{A} = -\frac{2m}{n_s e} \vec{J}_s - 2e\vec{A}$$

$$\Delta\phi = \frac{1}{\hbar} \oint_C \left(-\frac{2m}{n_s e} \vec{J}_s - 2e\vec{A} \right) \cdot d\vec{l}$$



考虑到电子波函数的单值性，对于闭合的回路，其位相的变化值只能是 2π 的整数倍

$$\Delta\phi = \frac{1}{\hbar} \oint_C \left(-\frac{2m}{n_s e} \vec{J}_s - 2e\vec{A} \right) \cdot d\vec{l} = 2n\pi,$$

$$n = 0, \pm 1, \pm 2, \dots$$

$$\frac{1}{\hbar} \oint_C \left(-\frac{2m}{n_s e} \vec{J}_s - 2e\vec{A} \right) \cdot d\vec{\ell} = 2n\pi$$

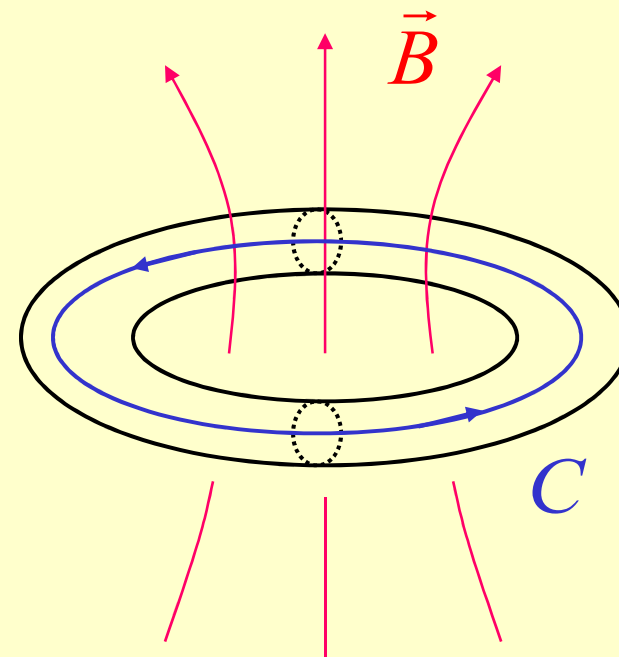
在超导状态下，超导体内部

$$\vec{J}_s = 0$$

$$\oint_C \vec{A} \cdot d\vec{\ell} = n \frac{\hbar}{e} \pi = \frac{nh}{2e}$$

$$\Phi = n \frac{h}{2e} = n\Phi_0,$$

$$(n = 0, \pm 1, \pm 2, \dots)$$



$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} \text{ Wb} \text{ 称为磁通量子}$$

磁通量子化现象

- ① 再次表明了矢势的环量在量子效应中的可观测性：
- ② 虽然在超导体的内部的积分回路上的磁场为零，但由于矢势不为零，从而影响了电子的波函数的位相，导致了磁通量子化效应。

在量子现象中, 矢势的 (环量) 可观测效应

- AB效应;
- 超导环的磁通俘获及其量子化现象

作业

第三章 习题11、12