§2 电磁波在半无限绝缘介质分界面上的反射、折射

1. **反射、折射定律**

   入射角、反射角和折射角的关系；

2. **菲涅耳（Fresnel）公式**

   入射波、反射波和折射波振幅、位相的关系

3. **布儒斯特（Brewster）定律**

4. **折射、反射系数**

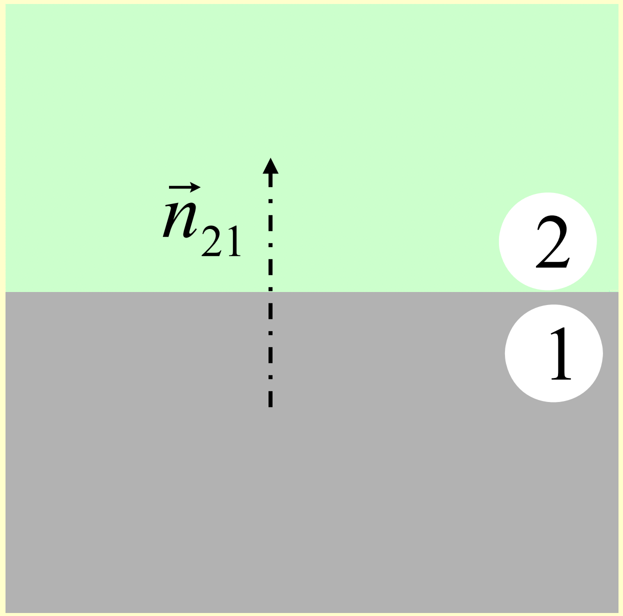
5. **全反射现象**

1、半无限绝缘介质的分界面上的反射和折射定律

由于是绝缘介质构成的
分界面，因此分界面：

① 没有自由电荷面分布

② 没有传导电流面分布



$$\vec{n}_{21}\cdot\left(\vec{D}_2-\vec{D}_1\right)=0$$

$$\vec{n}_{21}\times(\vec{E}_2-\vec{E}_1)=0$$

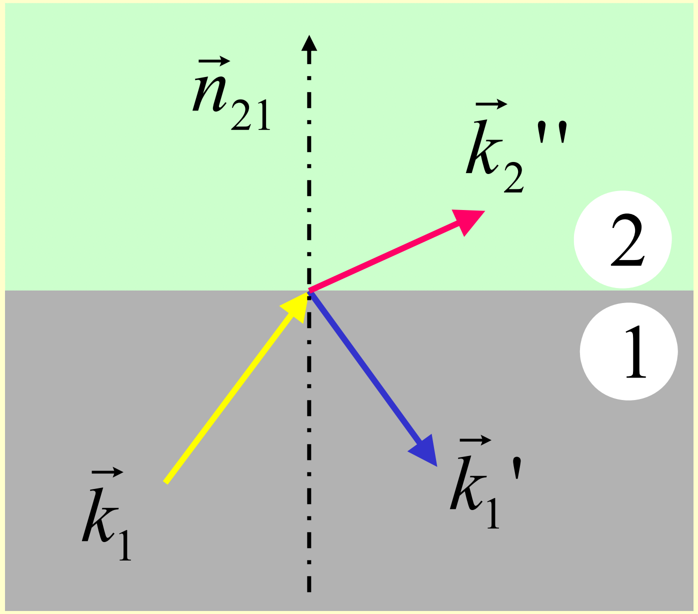
$$\vec{n}_{21}\cdot\left(\vec{B}_2-\vec{B}_1\right)=0$$

$$\vec{n}_{21}\times(\vec{H}_2-\vec{H}_1)=0$$

$$\left\{\begin{aligned}&\vec{n}_{21}\cdot\left(\vec{D}_2-\vec{D}_1\right)=\sigma_f\\&\vec{n}_{21}\cdot\left(\vec{B}_2-\vec{B}_1\right)=0\\&\vec{n}_{21}\times(\vec{E}_2-\vec{E}_1)=0\\&\vec{n}_{21}\times(\vec{H}_2-\vec{H}_1)=\vec{\alpha}_f\end{aligned}\right.$$

- 上述的四个边界条件并非完全独立；

- 如果入射波为平面时谐波，则反射波和折射波也为平面时谐波。



入射面：

——入射波矢和面法线构成的面

反射、折射的基本规律：

① 反射波、折射波的频率与入射波的**频率相等**；

② 入射线、反射线和折射线处于同一平面（入射面）内；

③ 入射角等于反射角；

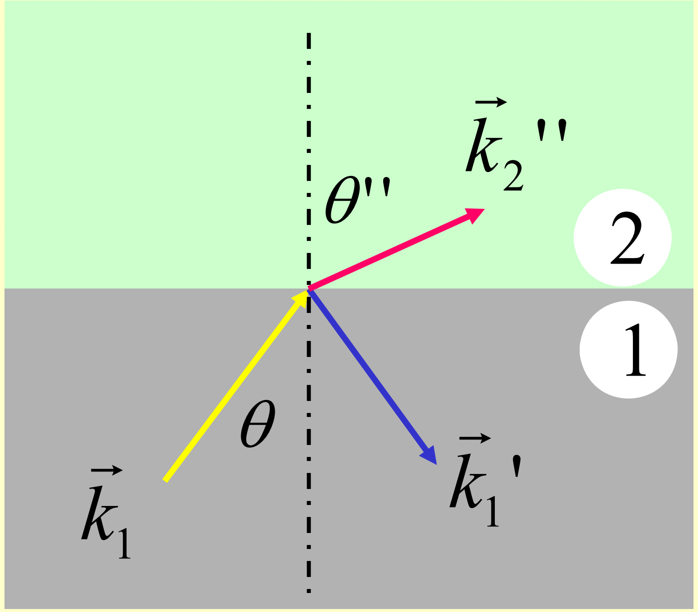
④ 折射线与入射线分布在发界面法线的两侧；

⑤ 折射定律：

$$\boxed{n_1 \sin\theta = n_2 \sin\theta''}$$

入射波（平面时谐波）：

$$\vec{E}(\vec{x},t)=\vec{E}_0\exp\left[\mathrm{i}\left(\vec{k}_1\cdot\vec{x}-\omega t\right)\right]$$



反射波：

$$\vec{E}'(\vec{x},t)=\vec{E}_0'\exp\left[\mathrm{i}\left(\vec{k}_1'\cdot\vec{x}-\omega' t\right)\right]$$

折射波：

$$\vec{E}''(\vec{x},t)=\vec{E}_0''\exp\left[\mathrm{i}\left(\vec{k}_2''\cdot\vec{x}-\omega'' t\right)\right]$$

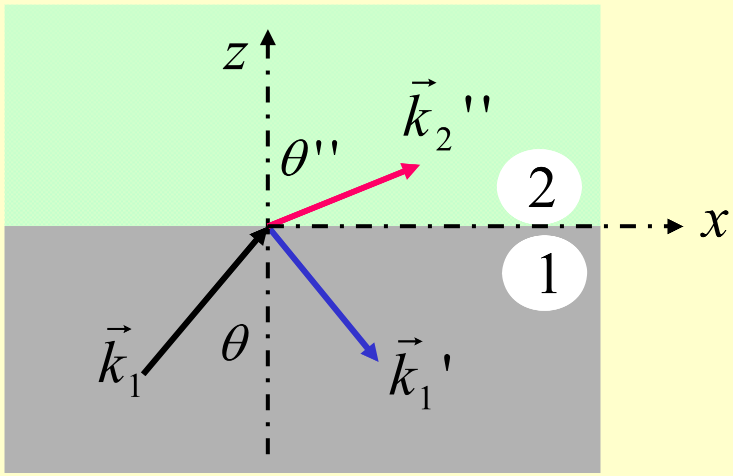
$\vec{n}_{21}\times(\vec{E}_2-\vec{E}_1)=0$ 在分界面上的每一点每一时刻都须成立

$$\vec{n}\times\vec{E}_0\exp\left[\mathrm{i}\left(\vec{k}_1\cdot\vec{x}-\omega t\right)\right]\Big|_{z=0}+\quad\vec{n}\times\vec{E}_0'\exp\left[\mathrm{i}\left(\vec{k}_1'\cdot\vec{x}-\omega' t\right)\right]\Big|_{z=0}$$

$$\equiv\vec{n}\times\vec{E}_0''\exp\left[\mathrm{i}\left(\vec{k}_2''\cdot\vec{x}-\omega'' t\right)\right]\Big|_{z=0}$$

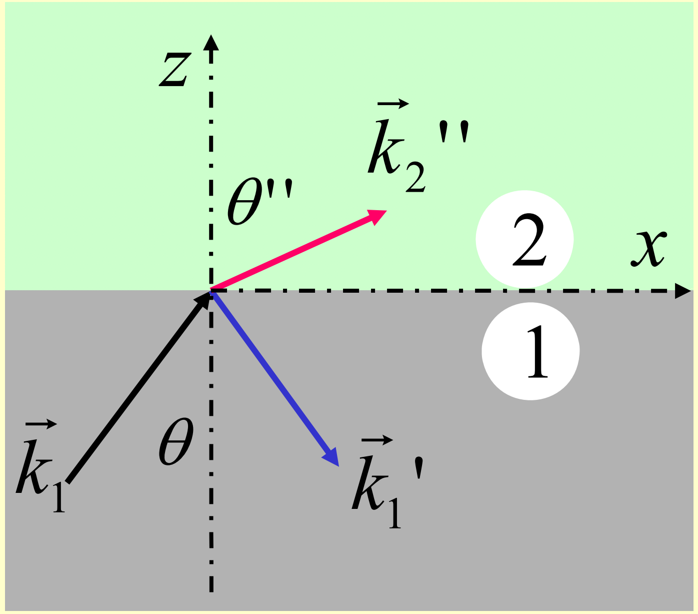
在分界面上， x，y 是两个独立的变量，因此

$$\begin{cases}\omega=\omega'=\omega''\\ k_{1x}=k_{1x}'=k_{2x}''\\ k_{1y}=k_{1y}'=k_{2y}''\end{cases}$$



$$k_{1y}=0$$

$$k_{1y}'=k_{2y}''=0$$



反射、折射的基本规律：

① 反射波、折射波的频率与入射波的**频率相等；**

② 入射线、反射线和折射线处于同一平面（入射面）内。

$$k_{1x} = k_{1x}' = k_{2x}''$$

$$k_{1y} = k_{1y}' = k_{2y}''$$

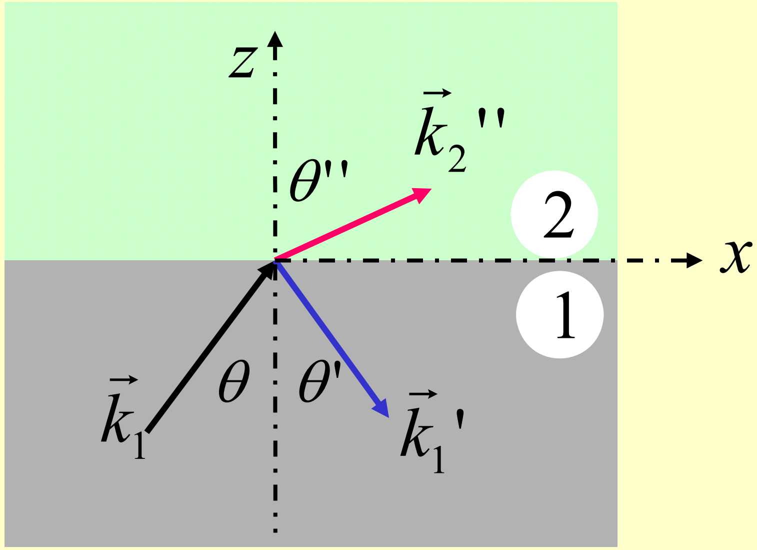
根据 $\quad k_{1x} = k_{1x}'$

$$k_1 = k_1' = n_1 k$$

得到

$$k_1 \sin\theta = k_1' \sin\theta'$$

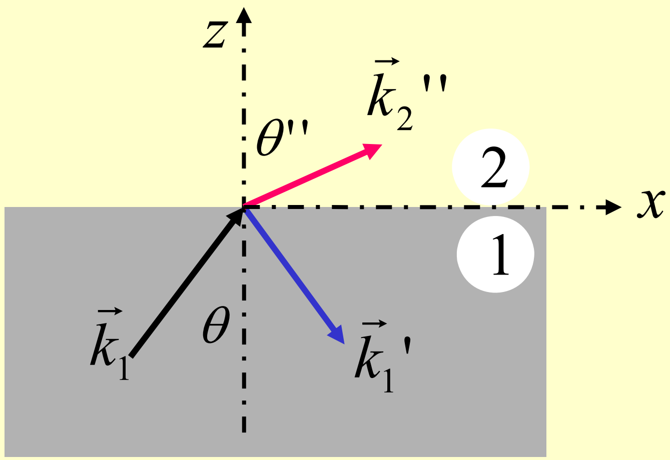
或者 $\quad \theta = \theta'$



③ 入射角等于反射角。

根据 $k_{1x} = k_{2x}''$

$$k_1 = n_1 k, \quad k_2'' = n_2 k$$



得到：$\boxed{n_1 \sin\theta = n_2 \sin\theta''}$

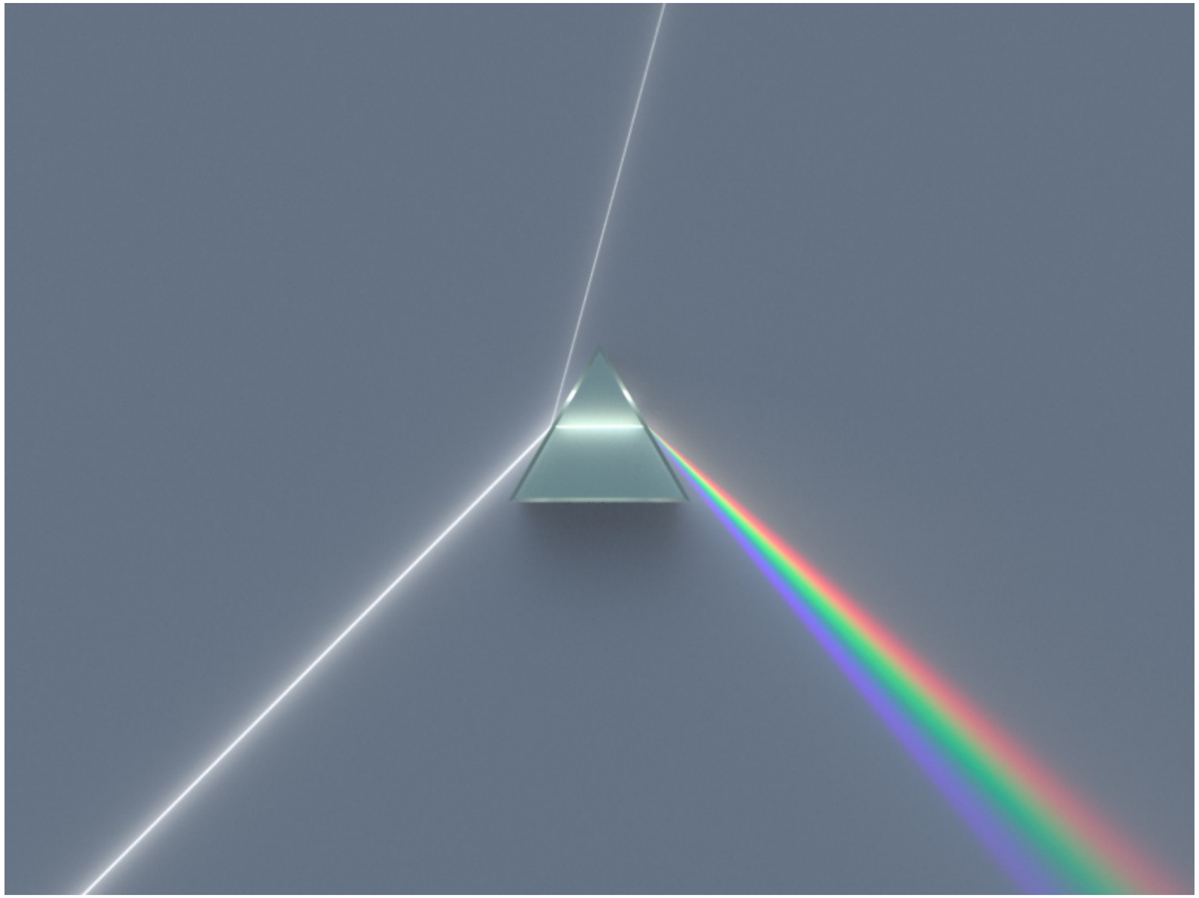
——光学中的折射定律

- 折射率反映了电磁波进入介质时传播方向的折射程度；
- 如果材料(一般是非磁性)的折射率是频率的函数

$$n(\omega) = \sqrt{\mu_r \varepsilon_r(\omega)} = \sqrt{\varepsilon_r(\omega)}$$ ——介质色散

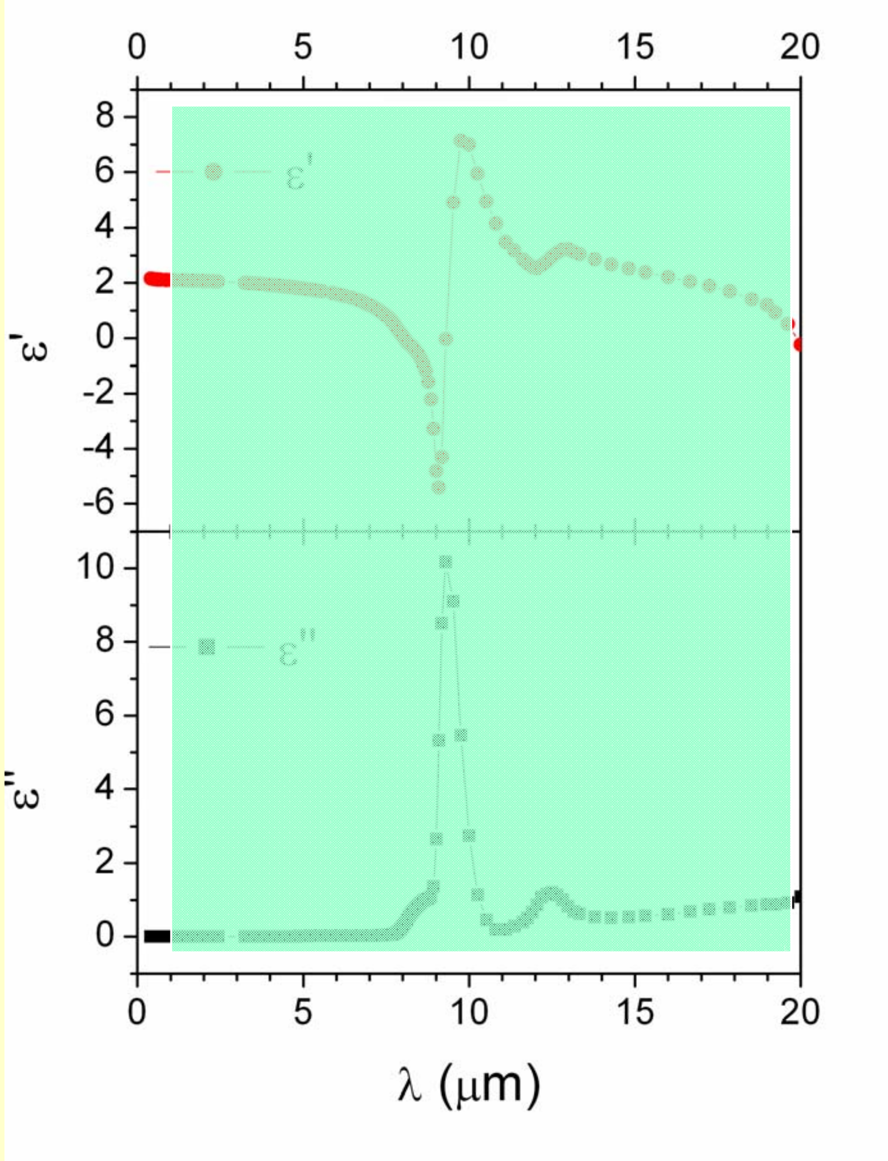
$$\Rightarrow \theta'' = \theta''(\omega)$$

生活中常见光的折射



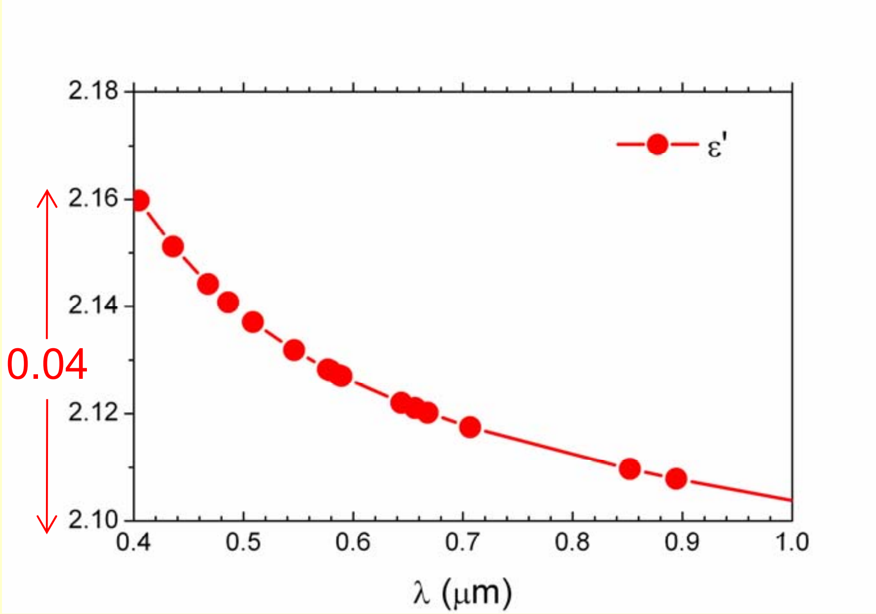
1672年，牛顿用三棱镜将太阳光分解成彩色光带，这是人们首次做的色散实验，说明物质的折射率和光的颜色(波长)有关。

玻璃的介电常数与光波长的关系



$$n = n' + in''$$ 折射率

$$\varepsilon = n^2 = \varepsilon' + i\varepsilon''$$



2、入射波、反射波和折射波的振幅关系

——菲涅耳（Fresnel）公式

$$\boxed{E_{1t} = E_{2t}}$$

$$\boxed{H_{1t} = H_{2t}}$$

$$\boxed{E_{1t} = E_{2t}}$$

$$\boxed{H_{1t} = H_{2t}}$$

$$\vec{n}\times\vec{E}_0\exp\left[\mathrm{i}\left(\vec{k}_1\cdot\vec{x}-\omega t\right)\right]\Big|_{z=0} + \vec{n}\times\vec{E}_0'\exp\left[\mathrm{i}\left(\vec{k}_1'\cdot\vec{x}-\omega' t\right)\right]\Big|_{z=0}$$

$$\equiv \vec{n}\times\vec{E}_0''\exp\left[\mathrm{i}\left(\vec{k}_2''\cdot\vec{x}-\omega'' t\right)\right]\Big|_{z=0}$$

$$\left.\begin{aligned}\vec{E}_{0t}+\vec{E}_{0t}' &= \vec{E}_{0t}''\\ \vec{H}_{0t}+\vec{H}_{0t}' &= \vec{H}_{0t}''\end{aligned}\right\}$$

$$\begin{cases} \vec{E}_{0t} + \vec{E}_{0t}' = \vec{E}_{0t}'' \\ \\ \vec{H}_{0t} + \vec{H}_{0t}' = \vec{H}_{0t}'' \end{cases}$$

$$\vec{B} = \mu \vec{H}$$

$$\left|\vec{B}\right| = \sqrt{\mu\varepsilon}\left|\vec{E}\right|$$

$$\boxed{Z = \sqrt{\frac{\mu}{\varepsilon}}}$$

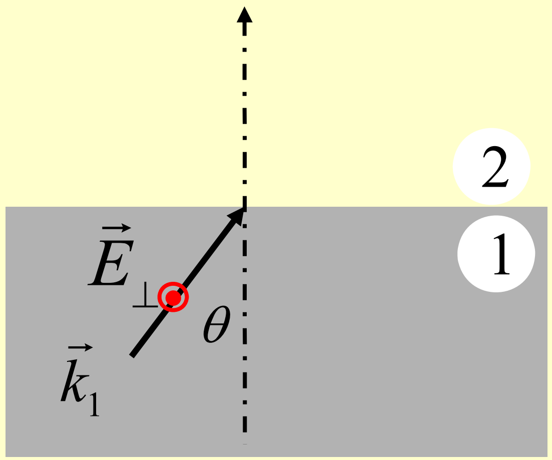
$$\boxed{\left|\vec{E}\right| = Z\left|\vec{H}\right|}$$

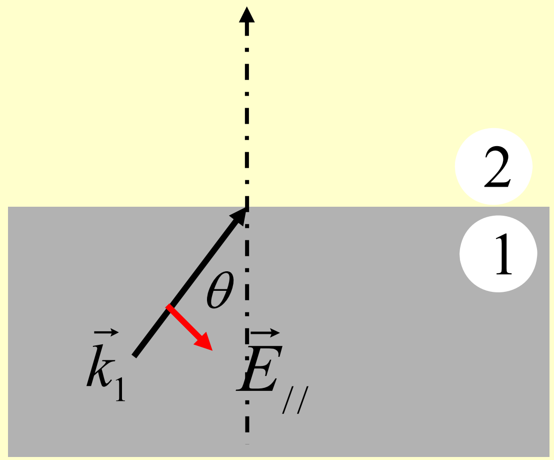
➢ 对于给定的波矢，电场存在两个独立的振动（偏振）方向；

➢ 为讨论方便，将电场矢量分解为

① 偏振方向垂直于入射面(**S-polarization**) $\left(\vec{E}_{\perp}\right)$

② 偏振方向处于入射面内(**P-polarization**) $\left(\vec{E}_{//}\right)$

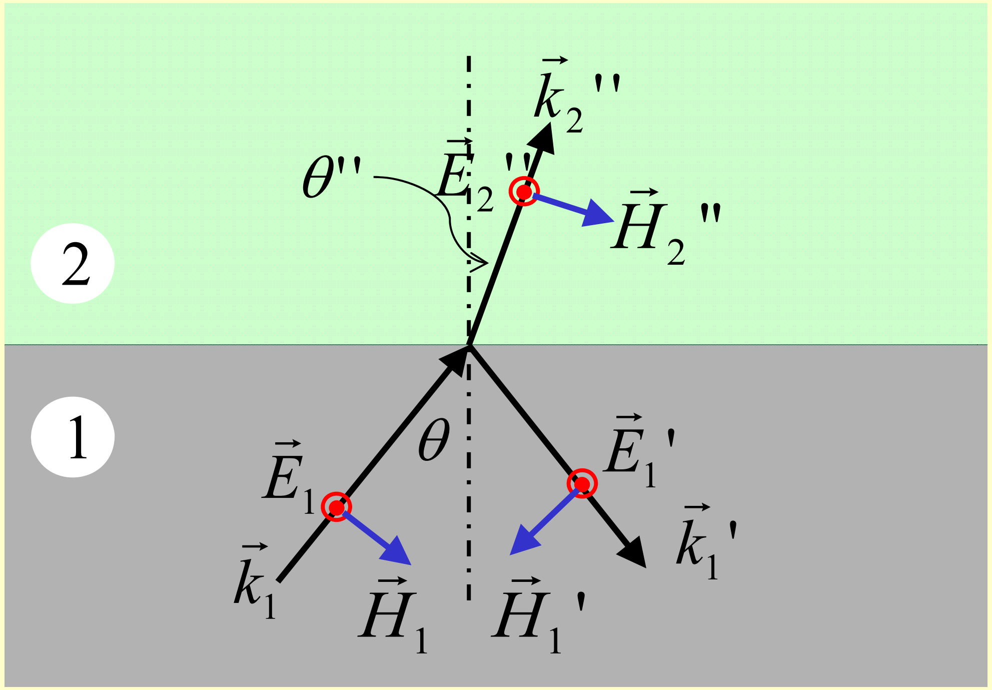


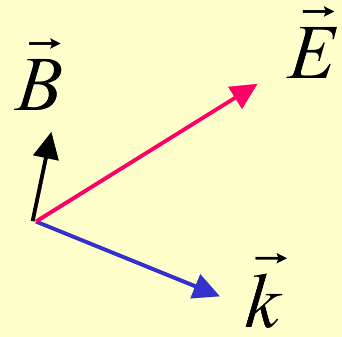


一般的情况下，$\vec{E} = \vec{E}_{//} + \vec{E}_{\perp}$

（1）偏振方向垂直于入射面（S-polarization）

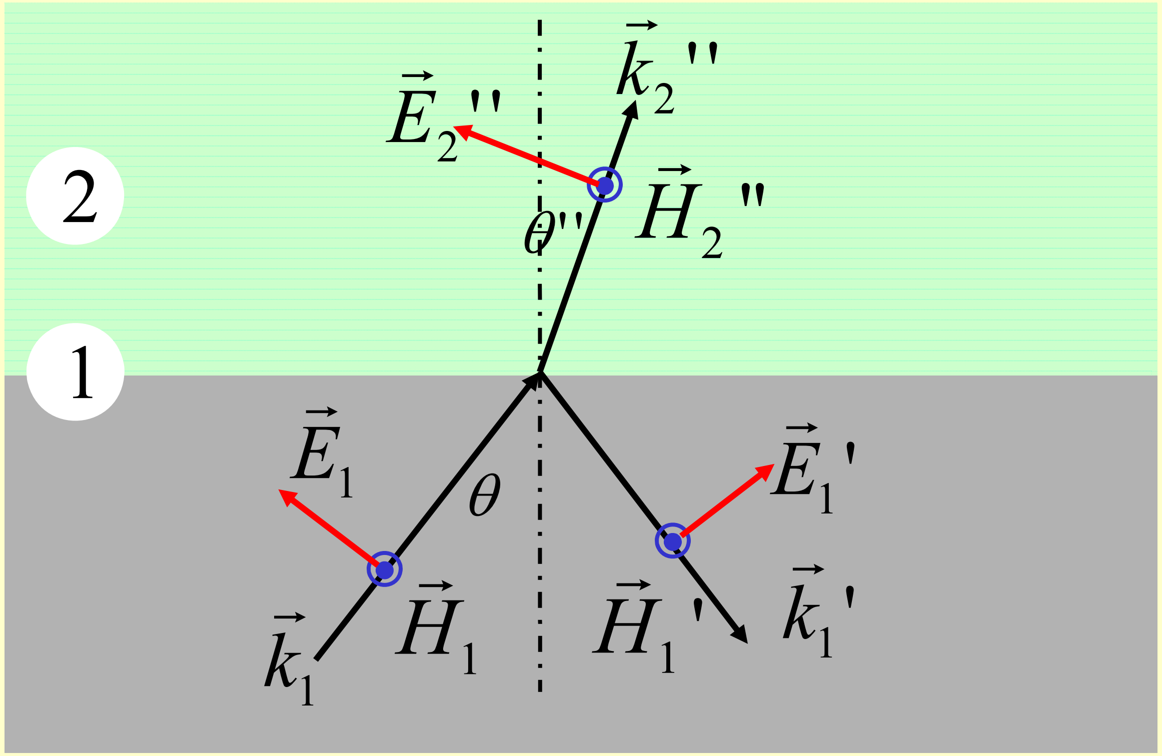
（选取垂直版面向外，作为电场的正方向）

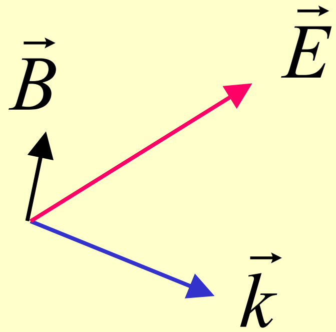




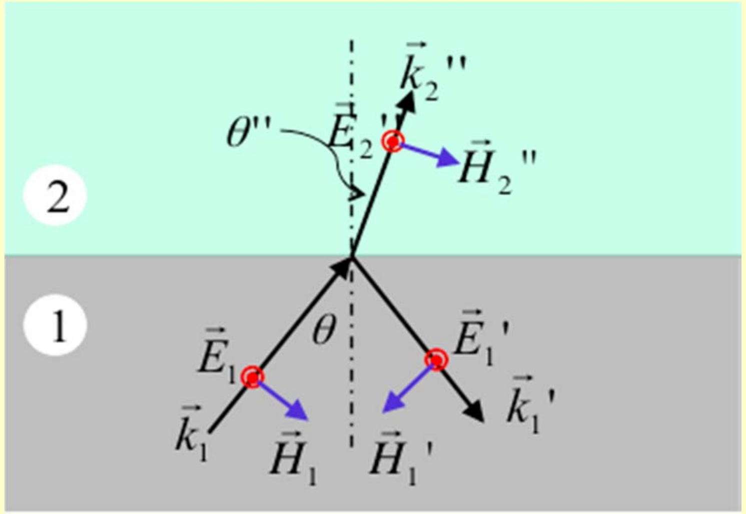
（2）偏振方向平行于入射面 (P-polarization)

（选取垂直版面向外，作为磁感应强度的正方向）





（1）偏振方向垂直于入射面（S-polarization）



$$\vec{E}_{0t} + \vec{E}_{0t}' = \vec{E}_{0t}''$$

$$\vec{H}_{0t} + \vec{H}_{0t}' = \vec{H}_{0t}''$$

$$\Rightarrow \quad E_0 + E_0' = E_0''$$

$$H_1 \cos\theta - H_1' \cos\theta = H_2'' \cos\theta''$$

$$\Rightarrow \quad \frac{1}{Z_1}\left[E_0 \cos\theta - E_0' \cos\theta\right] = \frac{1}{Z_2} E_0'' \cos\theta''$$

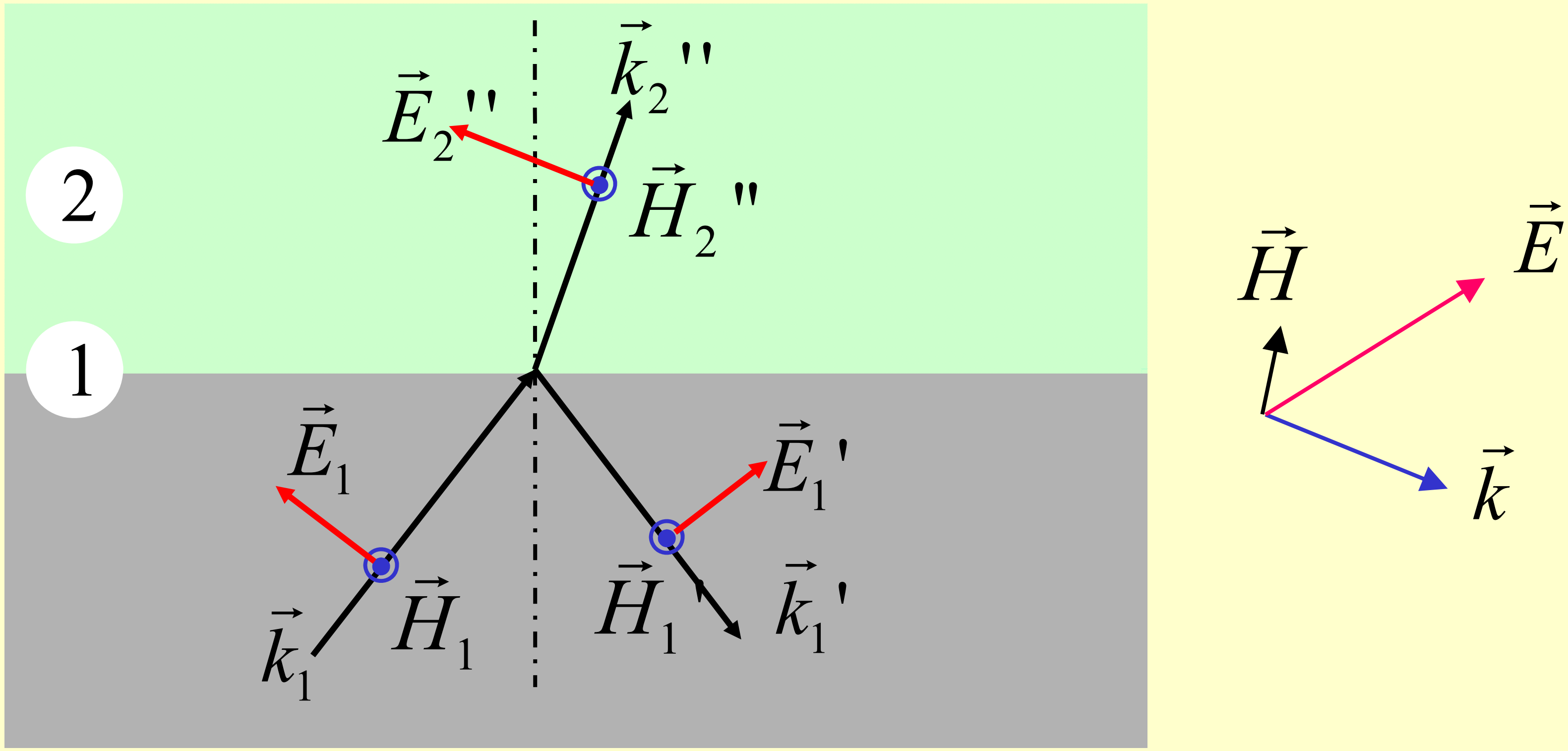
$$E_0 + E_0' = E_0''$$

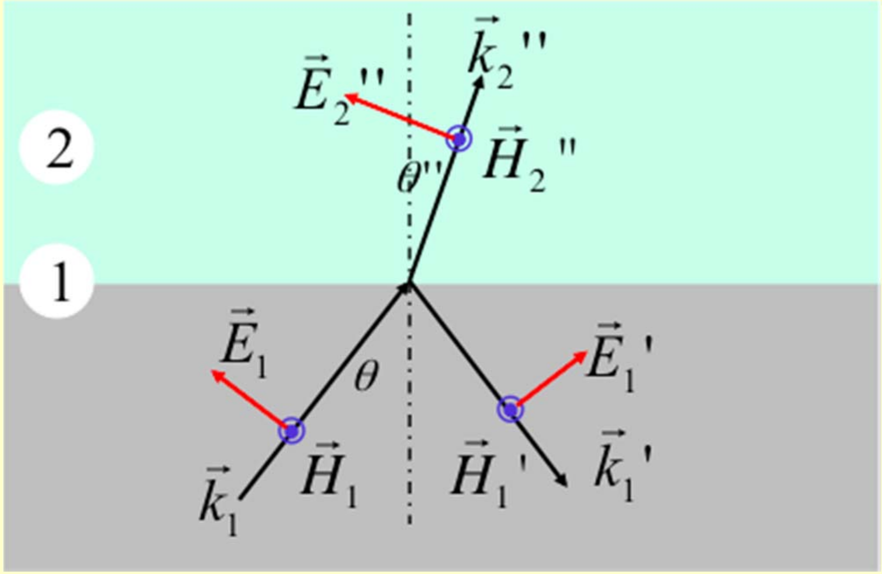
$$\frac{1}{Z_1}\left[E_0\cos\theta - E_0'\cos\theta\right] = \frac{1}{Z_2}E_0''\cos\theta''$$

S偏振入射下的反射波： $$\frac{E_0'}{E_0} = \frac{Z_2\cos\theta - Z_1\cos\theta''}{Z_2\cos\theta + Z_1\cos\theta''}$$

S偏振入射下的透射波： $$\frac{E_0''}{E_0} = \frac{2Z_2\cos\theta}{Z_2\cos\theta + Z_1\cos\theta''}$$

（2）偏振方向平行于入射面 (P-polarization)





$$\vec{E}_{0t} + \vec{E}_{0t}' = \vec{E}_{0t}''$$

$$\vec{H}_{0t} + \vec{H}_{0t}' = \vec{H}_{0t}''$$

$$\Rightarrow \quad H_0 + H_0' = H_0''$$

$$-E_0 \cos\theta + E_0' \cos\theta' = -E_0'' \cos\theta''$$

$$Z_1\left(-H_0 \cos\theta + H_0' \cos\theta'\right) = -Z_2 H_0'' \cos\theta''$$

$$\Rightarrow \quad \frac{1}{Z_2}\left(-H_0 \cos\theta + H_0' \cos\theta'\right) = -\frac{1}{Z_1} H_0'' \cos\theta''$$

求解得:

P偏振入射下的反射波

$$\frac{H_0'}{H_0} = \frac{-Z_1\cos\theta + Z_2\cos\theta''}{Z_1\cos\theta + Z_2\cos\theta''}$$

P偏振入射下的透射波

$$\frac{H_0''}{H_0} = \frac{2Z_2\cos\theta''}{Z_1\cos\theta + Z_2\cos\theta''}$$

归纳如下：

反射波

(s-polar.) $$\frac{E_0'}{E_0}=\frac{Z_2\cos\theta-Z_1\cos\theta''}{Z_2\cos\theta+Z_1\cos\theta''}$$

(p-polar.) $$\frac{H_0'}{H_0}=\frac{-Z_1\cos\theta+Z_2\cos\theta''}{Z_1\cos\theta+Z_2\cos\theta''}$$

折射波

$$\frac{E_0''}{E_0}=\frac{2Z_2\cos\theta}{Z_2\cos\theta+Z_1\cos\theta''}$$

$$\frac{H_0''}{H_0}=\frac{2Z_2\cos\theta''}{Z_1\cos\theta+Z_2\cos\theta''}$$

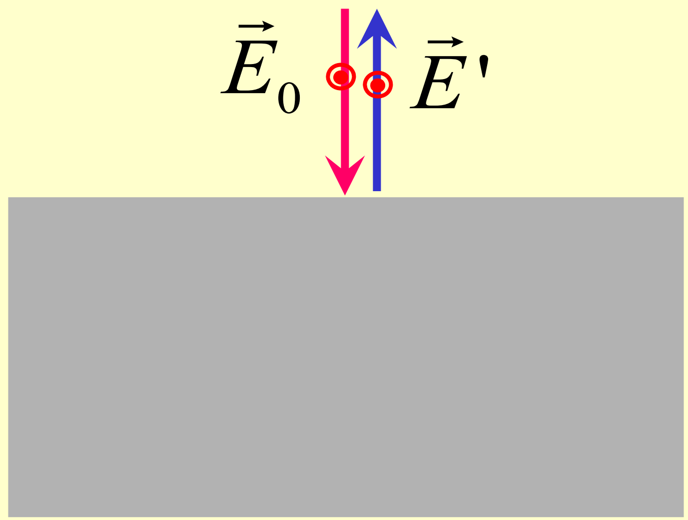
$$\frac{E_0'}{E_0}=\frac{Z_2\cos\theta-Z_1\cos\theta''}{Z_2\cos\theta+Z_1\cos\theta''}$$

(S polar.)

讨论：1）在垂直入射下：

$$\frac{E_0'}{E_0}=\frac{Z_2-Z_1}{Z_2+Z_1}$$

假设电磁波从真空入射到相对阻抗Zr的介质表面：



$$\frac{E_0'}{E_0}=\frac{Z_r-1}{Z_r+1}\quad\left(Z_r=\sqrt{\frac{\mu_r}{\varepsilon_r}}\right)$$

- 阻抗描述了电磁波入射到分界面时的振幅的比值（包括位相）关系；
- 在 $\varepsilon_r \rightarrow \infty$（理想导体）或 $\mu_r \rightarrow \infty$（理想磁导体），都能够产生强烈的反射。

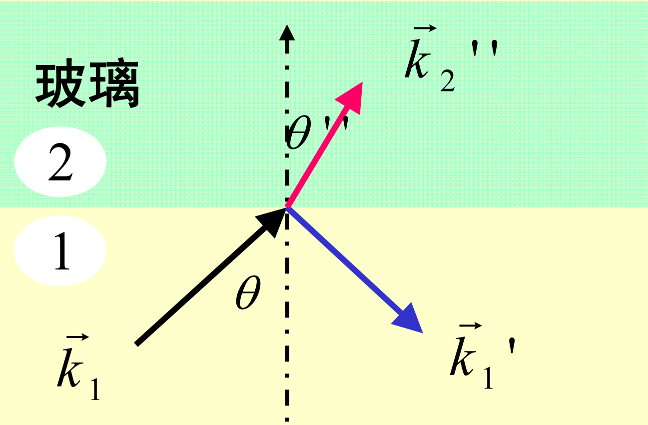
2）对于非磁性介质，μ$_1$＝μ$_2$＝μ$_0$的情况下，上述公式学过度到光学中的菲涅耳公式：

	反射波	折射波
（s-polar.）	$\dfrac{E_{0\perp}'}{E_{0\perp}}=\dfrac{\sin(\theta''-\theta)}{\sin(\theta''+\theta)}$	$\dfrac{E_{0\perp}''}{E_{0\perp}}=\dfrac{2\cos\theta\sin\theta''}{\sin(\theta''+\theta)}$
（p-polar.）	$\dfrac{E_{0//}'}{E_{0//}}=\dfrac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}$	$\dfrac{E_{0//}''}{E_{0//}}=\dfrac{2\cos\theta\sin\theta''}{\sin(\theta''+\theta)\cos(\theta-\theta'')}$

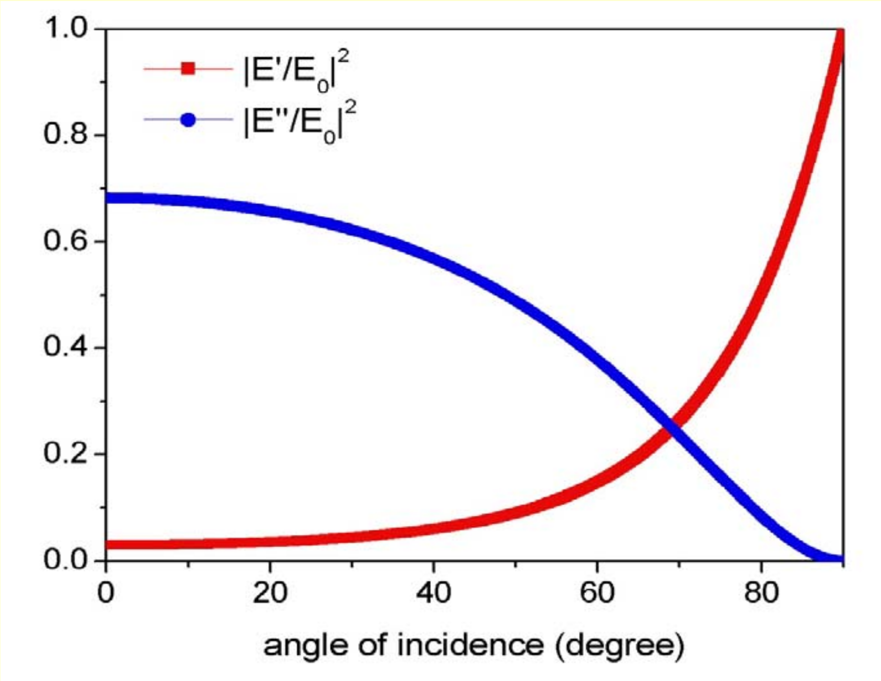
待继续

3）斜入射下情形：

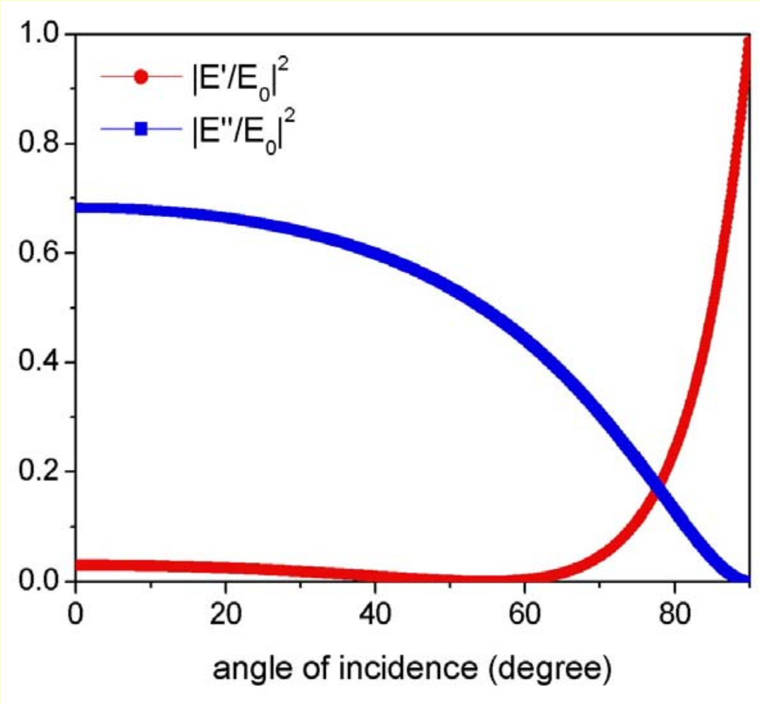
假设：$\left(n_1 = 1.00, \quad n_2 = 1.42\right)$



s－偏振入射：



p－偏振入射：



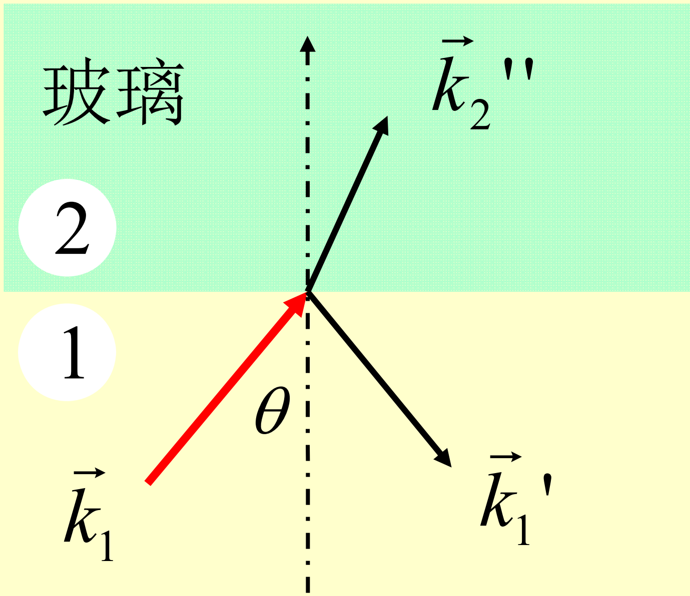
结论：

① 偏振方向垂直于入射面的光波反射、折射行为**不完全等同于**偏振平行于入射面的光波的反射和折射行为；

② 如果入射光为自然光（两种偏振的等量混合），经过反射（折射）后，由于两个偏振的反射（折射）强度不同，**反射光（折射光）变成了部分偏振光；**

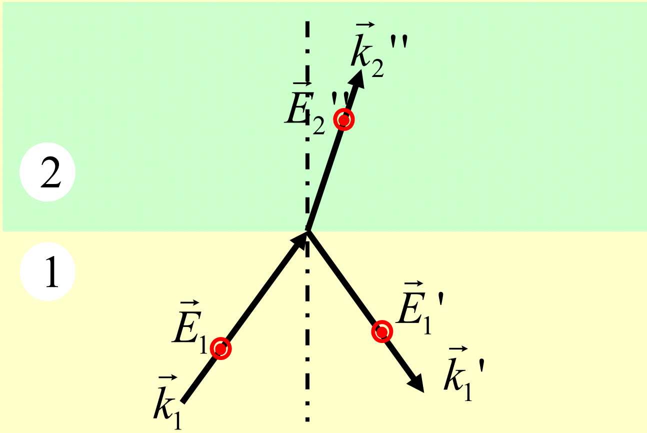
３、布儒斯特（Brewster）定律

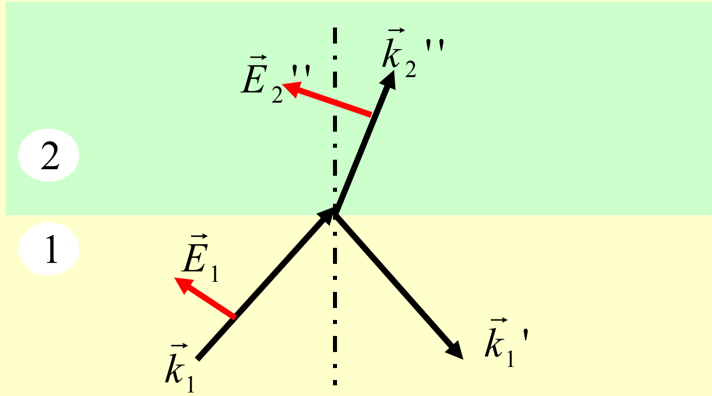
假设： $n_1 = 1.00,\ \ n_2 = 1.42$

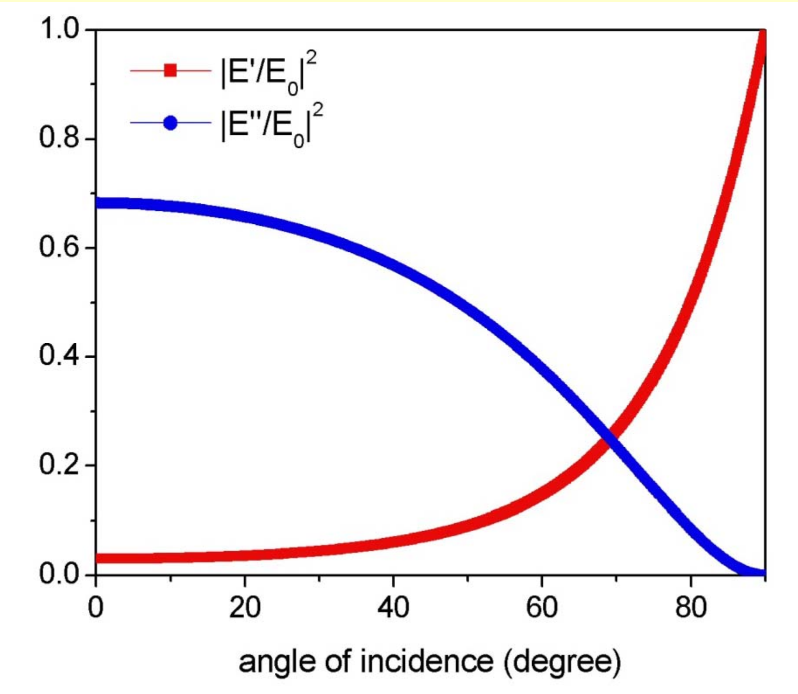


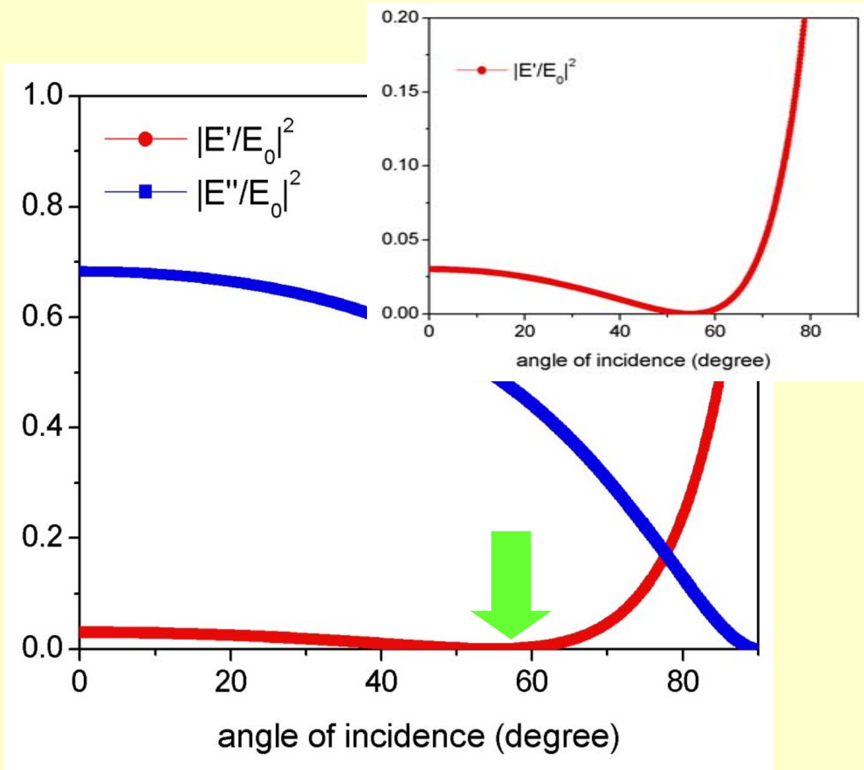
s-polarization

p-polarization

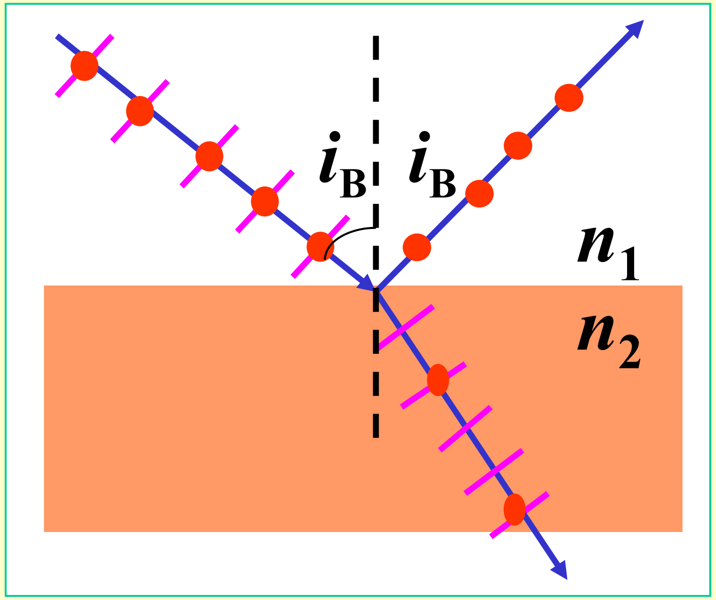


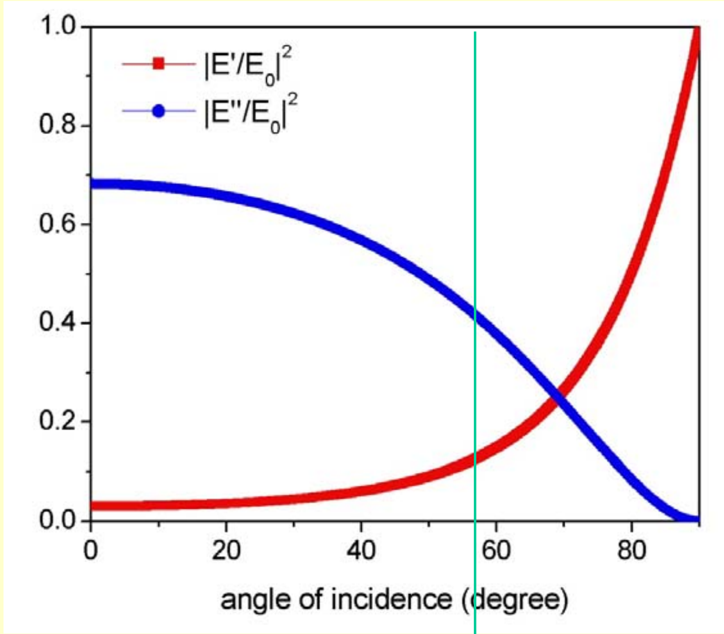


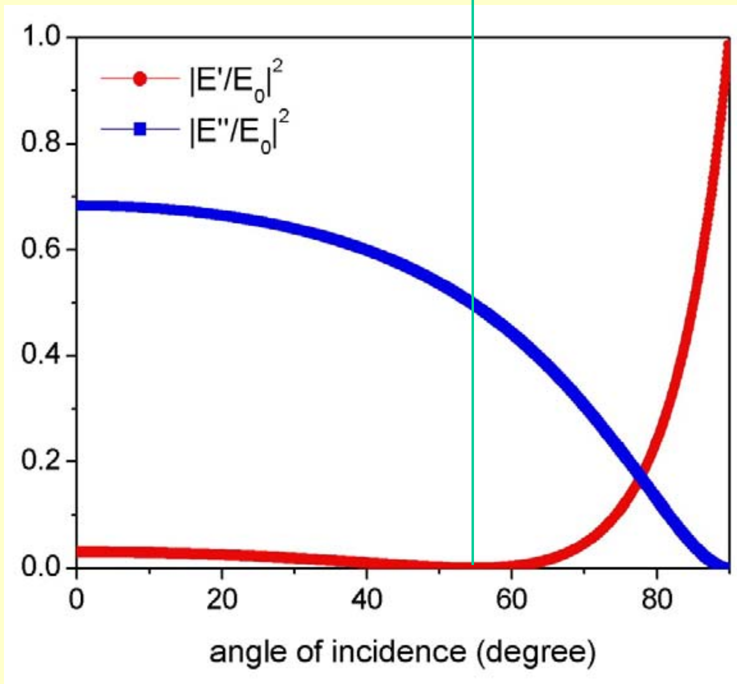




入射角度为布儒斯特角，含有两种偏振状态（P偏振的反射率为零！）

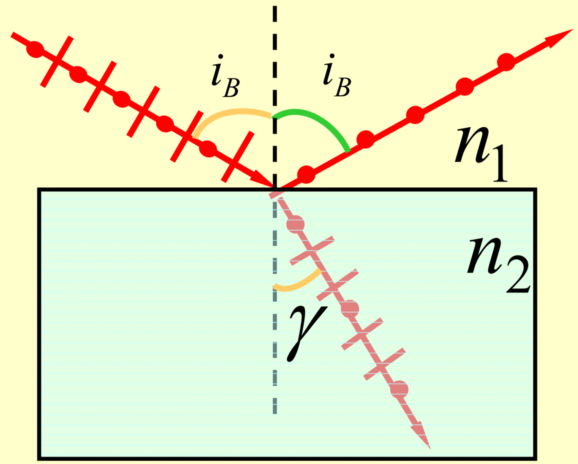




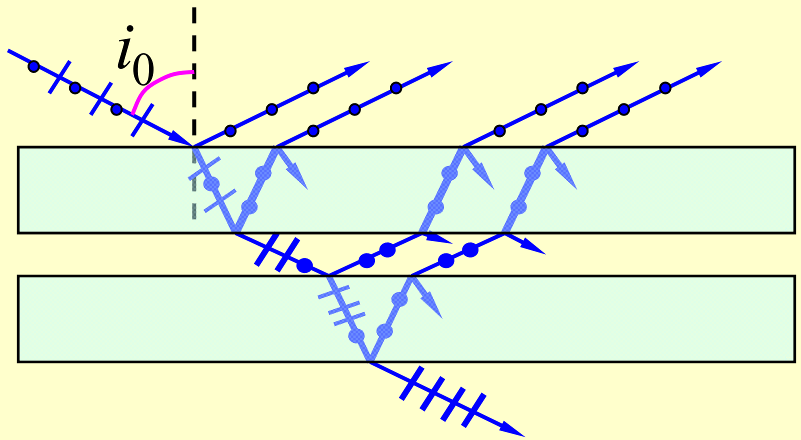


反射光：虽为纯的S偏振的线偏振，但反射率比较低，应用价值不大！

➢ 在入射角等于布儒斯特角时，在反射光中只有一种（S）偏振。



➢ 一般光学玻璃反射光的强度约占入射光强度的7.5%，大部分光将透过玻璃.



垂直振动S成分一次次被反射掉，折射光：近似线偏振光(P偏振)

p-polarization:

$$\frac{E_{0//}'}{E_{0//}}=\frac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}$$

在 $\underline{\theta+\theta''=90^{\circ}}$ 的特殊情况下， $E_{0//}'=0$ 。

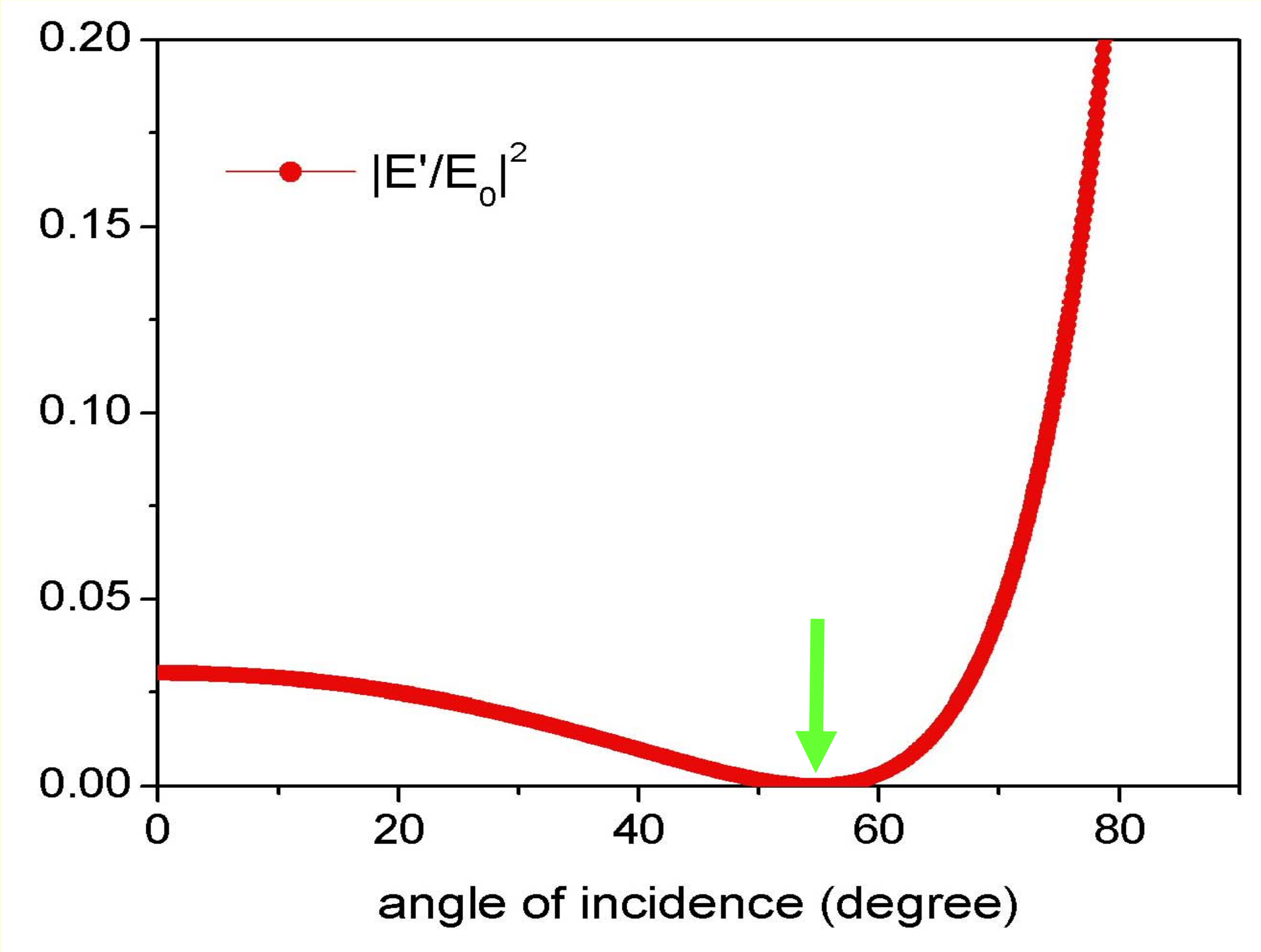
布儒斯特角：

$$n_1\sin\theta=n_2\sin\theta''$$

$$n_1\sin\theta_B=n_2\sin(90^{\circ}-\theta_B)=n_2\cos\theta_B$$

$$\boxed{\tan\theta_B=\frac{n_2}{n_1},\quad \theta_B=\arctan\frac{n_2}{n_1}}$$

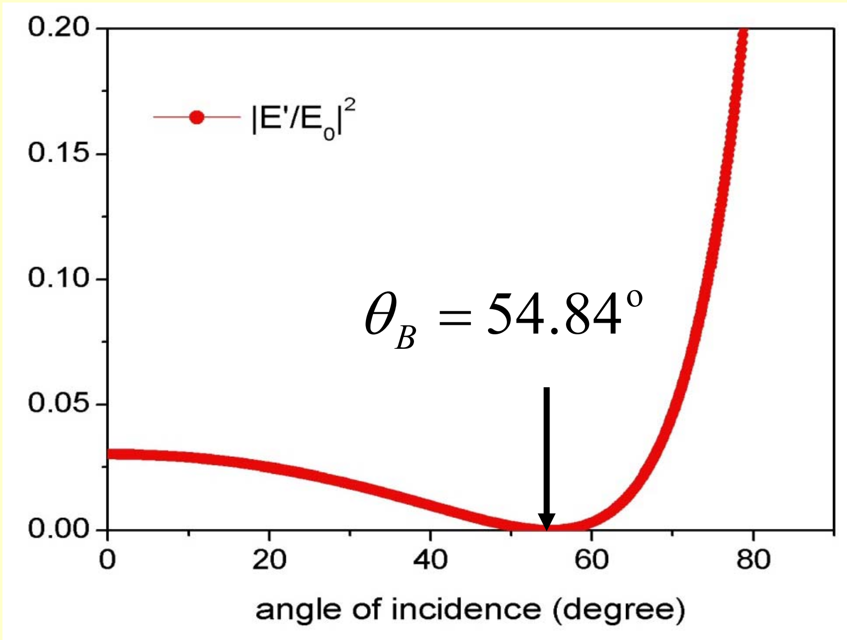
p-polarization



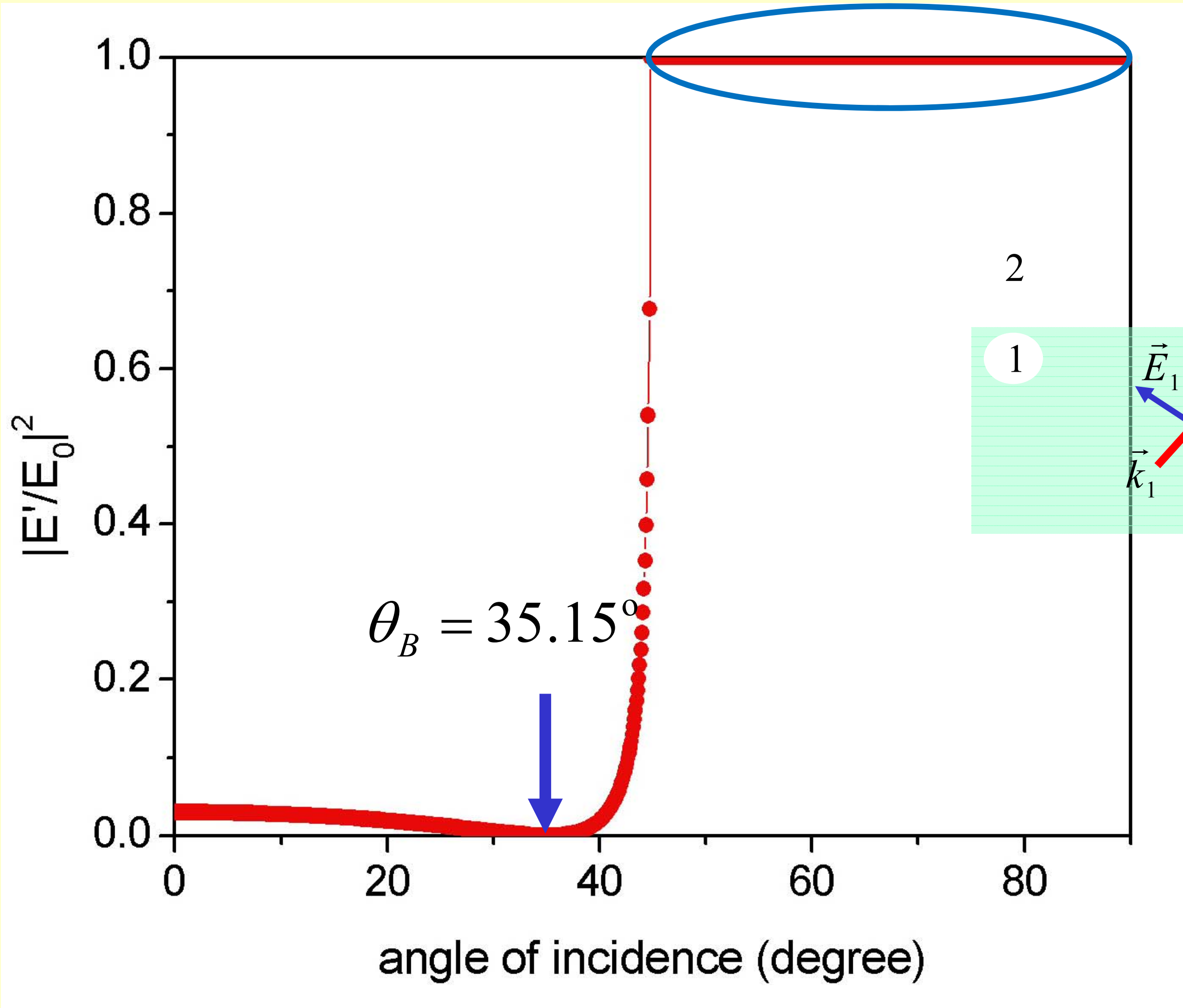
$$\tan\theta_B = \frac{n_2}{n_1}, \quad \theta_B = \arctan\frac{n_2}{n_1}$$

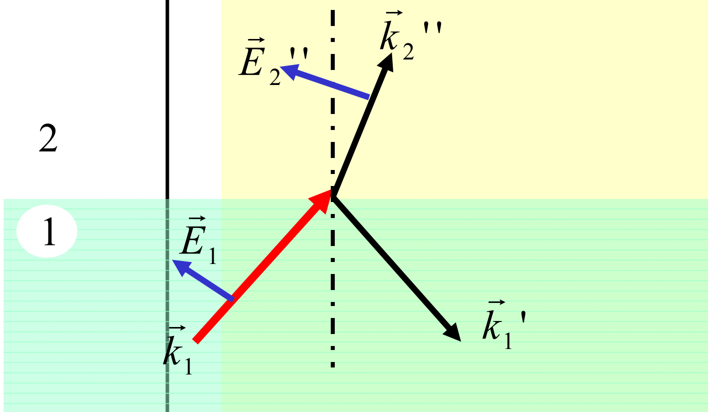
假设： $n_1 = 1.00,\ n_2 = 1.42$

$$\theta_B = \arctan\frac{n_2}{n_1} = \arctan 1.42 = 54.84^{\circ}$$

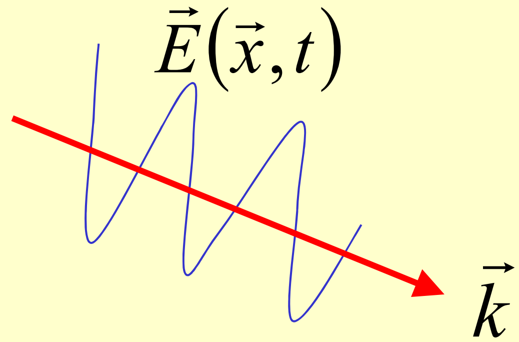


如果：$n_1 = 1.42,\ n_2 = 1.00 \qquad \theta_B = \arctan\frac{1}{1.42} = 35.15^{\circ}$





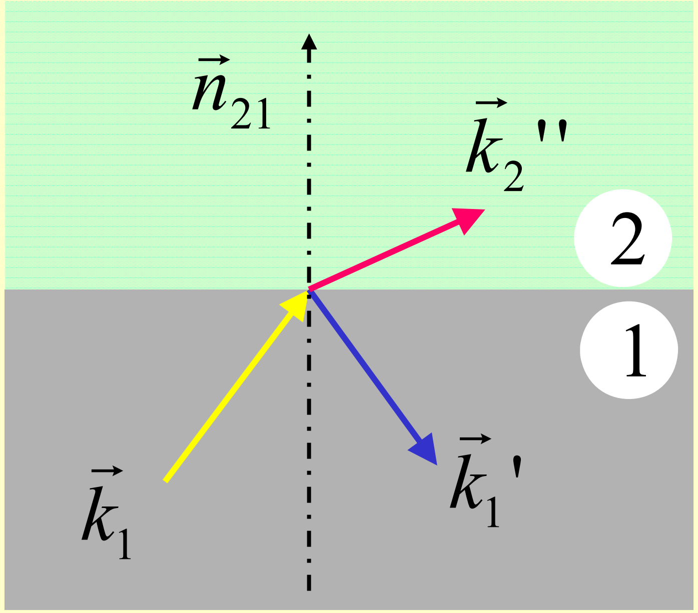
4、介质分界面的折射系数、反射系数



平面电磁波的平均能流密度：

$$\boxed{\langle\vec{S}\rangle=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\vec{e}_k}$$

$$\langle\vec{S}\rangle=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\vec{e}_k=\frac{1}{2}Z^{-1}E_0^2\vec{e}_k$$



1）反射波的平均能流密度

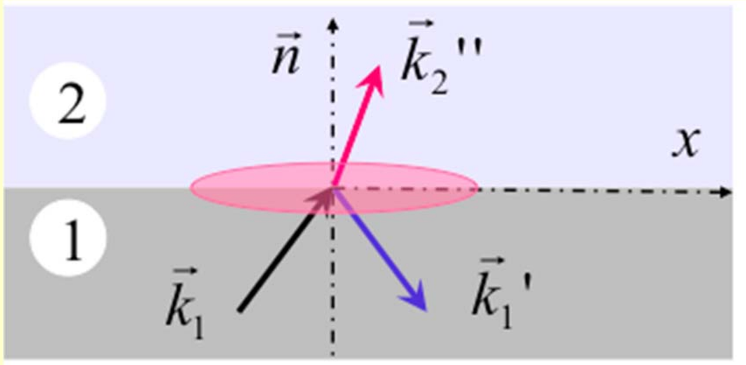
$$\langle\vec{S}'\rangle=\frac{1}{2}Z_1^{-1}|E_0'|^2\vec{e}_{k'}$$

2）折射波的平均能流密度

$$\langle\vec{S}''\rangle=\frac{1}{2}Z_2^{-1}|E_0''|^2\vec{e}_{k''}$$

3）反射系数：

$$R=\frac{\langle\vec{S}'\rangle\cdot\vec{n}}{\langle\vec{S}\rangle\cdot\vec{n}}=\frac{|E_0'|^2\vec{e}_{k'}\cdot\vec{n}}{|E_0|^2\vec{e}_k\cdot\vec{n}}=\frac{|E_0'|^2}{|E_0|^2}$$



4）折射系数：

$$T=\frac{\langle\vec{S}''\rangle\cdot\vec{n}}{\langle\vec{S}\rangle\cdot\vec{n}}=\frac{Z_2^{-1}|E_0''|^2\,\vec{e}_{k''}\cdot\vec{n}}{Z_1^{-1}|E_0|^2\,\vec{e}_k\cdot\vec{n}}=\frac{Z_2^{-1}|E_0''|^2\cos\theta''}{Z_1^{-1}|E_0|^2\cos\theta}$$

注意：

定义反射/透射率的时候，不是直接对入（反、透）射波的能流的比值，而是考虑这些能流对界面的通量。

Insect orientation to polarized moonlight

- Sunlight scatters when it strikes tiny particles in the atmosphere, giving rise to celestial polarization patterns
- Many creatures use the Sun's light-polarization pattern to orientate themselves

brief communicat

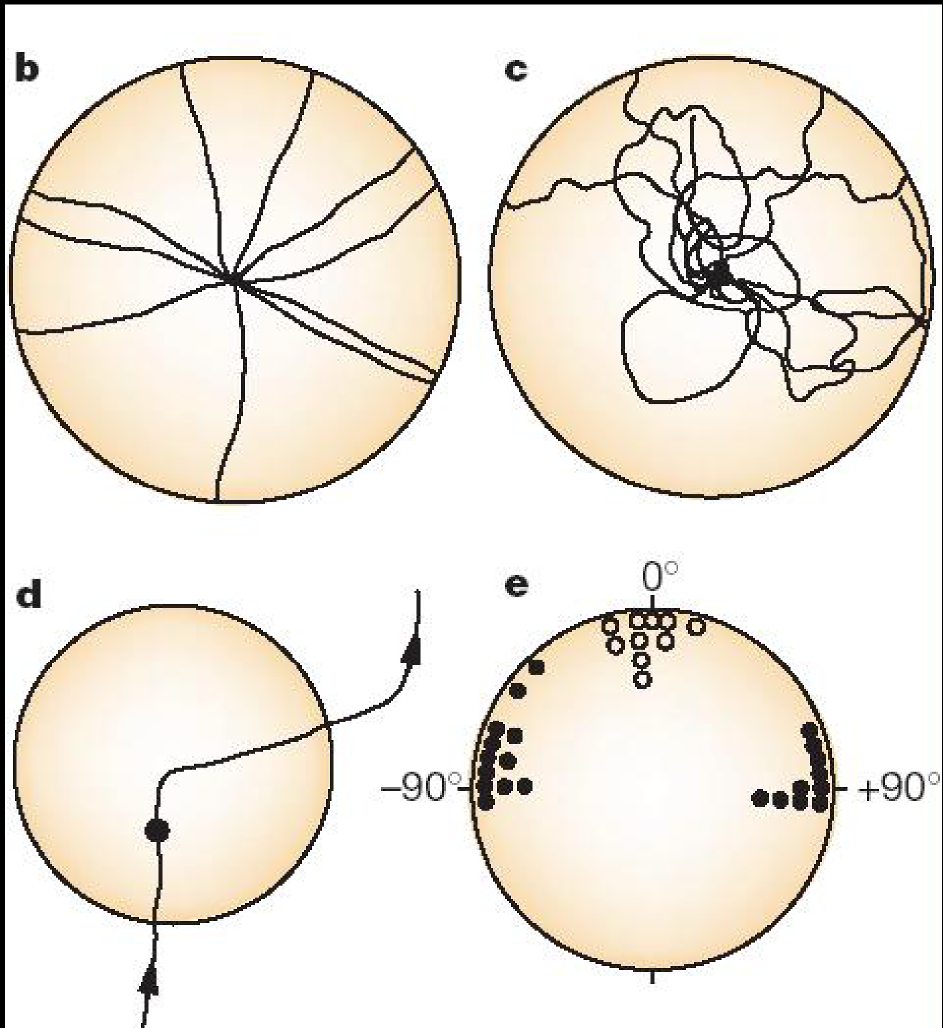
Insect orientation to polarized moonlight

An African dung beetle uses the moonlit sky to make a swift exit after finding food.

The beetle is rolling a ball of dung.

M. Dacke *et al.* *Nature* **424** (2003) 33

Paths taken by beetles (n = 10) moving dung balls outwards from the centre of an arena (diameter, 3 m).



b) On a moonlit night

c) On moonless nights

d) Change in direction (turn to the right by +70) taken by a beetle when a perpendicularly polarizing filter is placed over the beetle at the point indicated by the dot; the beetle resumes its direction of travel on exposure to the open sky.

brief communications

Polarized light as a butterfly mating signal

This optical feature of some iridescent wings catches a suitor's eye in the deep forest.

A. Sweeney *et al.* *Nature* **424** (2003) 31

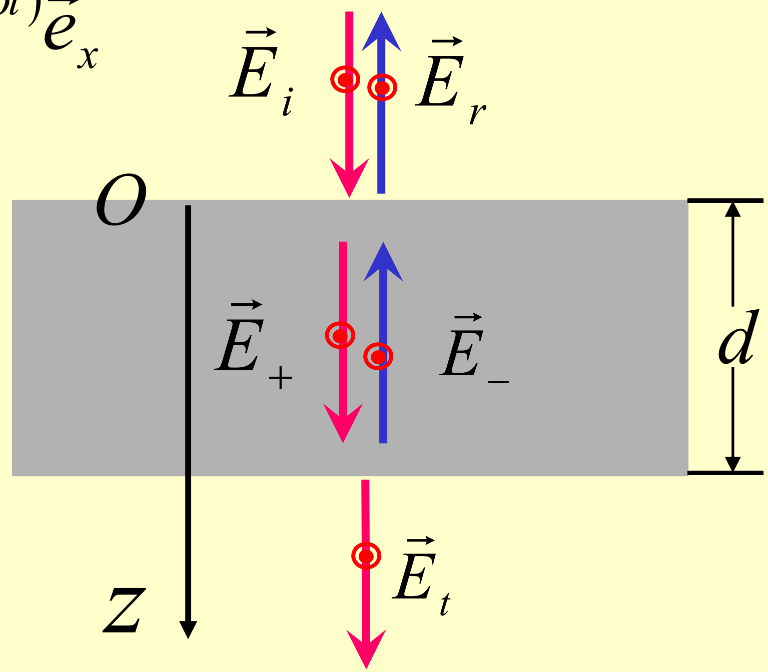
思考题： 单色平面电磁波垂直入射到一介质膜上，在入射区，存在入射和反射波；在介质膜内，存在前向和反向波；在透射区存在透射波，分别表示为

$$\vec{E}_i = E_{i0}\mathrm{e}^{\mathrm{i}(kz-\omega t)}\vec{e}_x,\quad \vec{E}_r = E_{r0}\mathrm{e}^{\mathrm{i}(-kz-\omega t)}\vec{e}_x$$

$$\vec{E}_+ = E_{+0}\mathrm{e}^{\mathrm{i}(k_m z-\omega t)}\vec{e}_x$$

$$\vec{E}_- = E_{-0}\mathrm{e}^{\mathrm{i}(-k_m z-\omega t)}\vec{e}_x$$

$$\vec{E}_t = E_{t0}\mathrm{e}^{\mathrm{i}(kz-\omega t)}\vec{e}_x$$



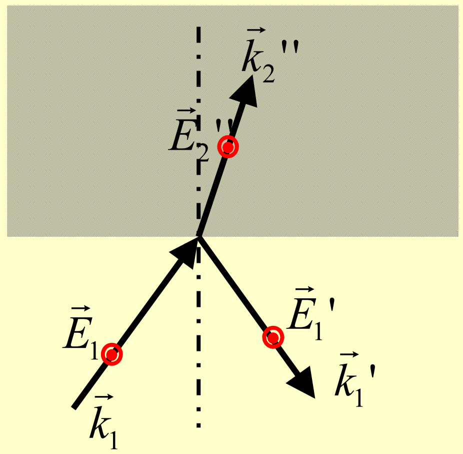
根据介质分界面处电磁场边界条件，写出上述电场振幅之间的关系。

５、(半无限大绝缘介质分界面上) 全反射现象

反射波的电场与入射波电场之比：

s 偏振 $\left(\vec{E}_{\perp}\right)$

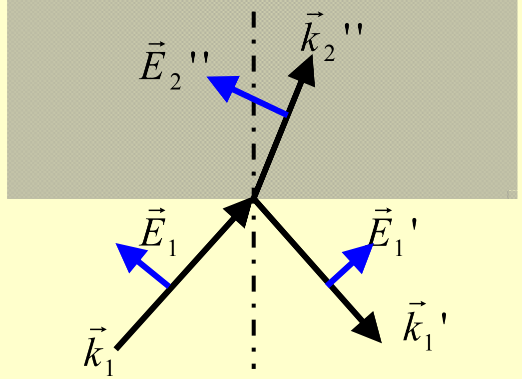
$$\frac{E_{0\perp}'}{E_{0\perp}}=\frac{\sqrt{\varepsilon_1}\cos\theta-\sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta+\sqrt{\varepsilon_2}\cos\theta''}$$



电场沿着相同方向振动，因此比值即代表了方向

p 偏振 $\left(\vec{E}_{//}\right)$

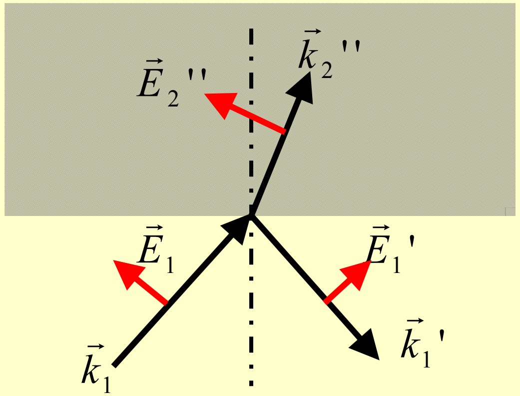
$$\frac{E_{0//}'}{E_{0//}}=\frac{\sqrt{\varepsilon_2}\cos\theta-\sqrt{\varepsilon_1}\cos\theta''}{\sqrt{\varepsilon_2}\cos\theta+\sqrt{\varepsilon_1}\cos\theta''}$$



一般情况下电场振动方向并不相同，除非在垂直入射情形！

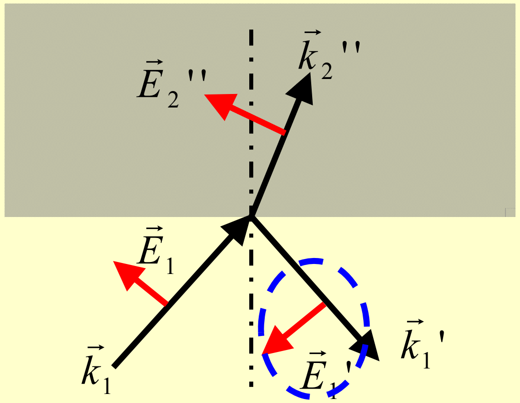
p 偏振：

$$\frac{E_{0//}'}{E_{0//}}=\frac{\sqrt{\varepsilon_2}\cos\theta-\sqrt{\varepsilon_1}\cos\theta''}{\sqrt{\varepsilon_2}\cos\theta+\sqrt{\varepsilon_1}\cos\theta''}$$

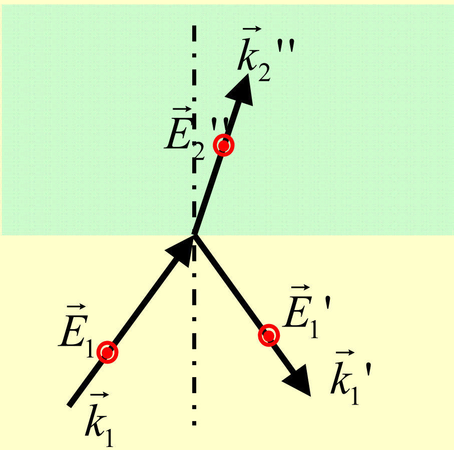


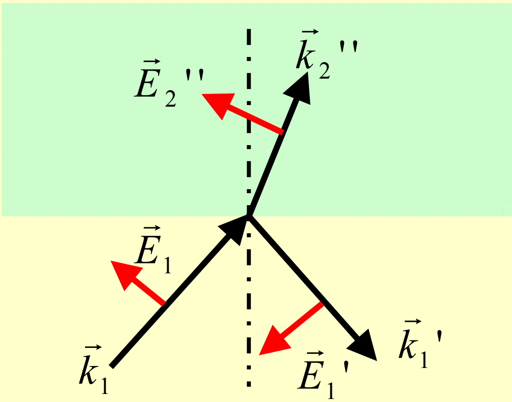
但按照图示，在垂直入射下，
比值为正，表示的是方向相反！

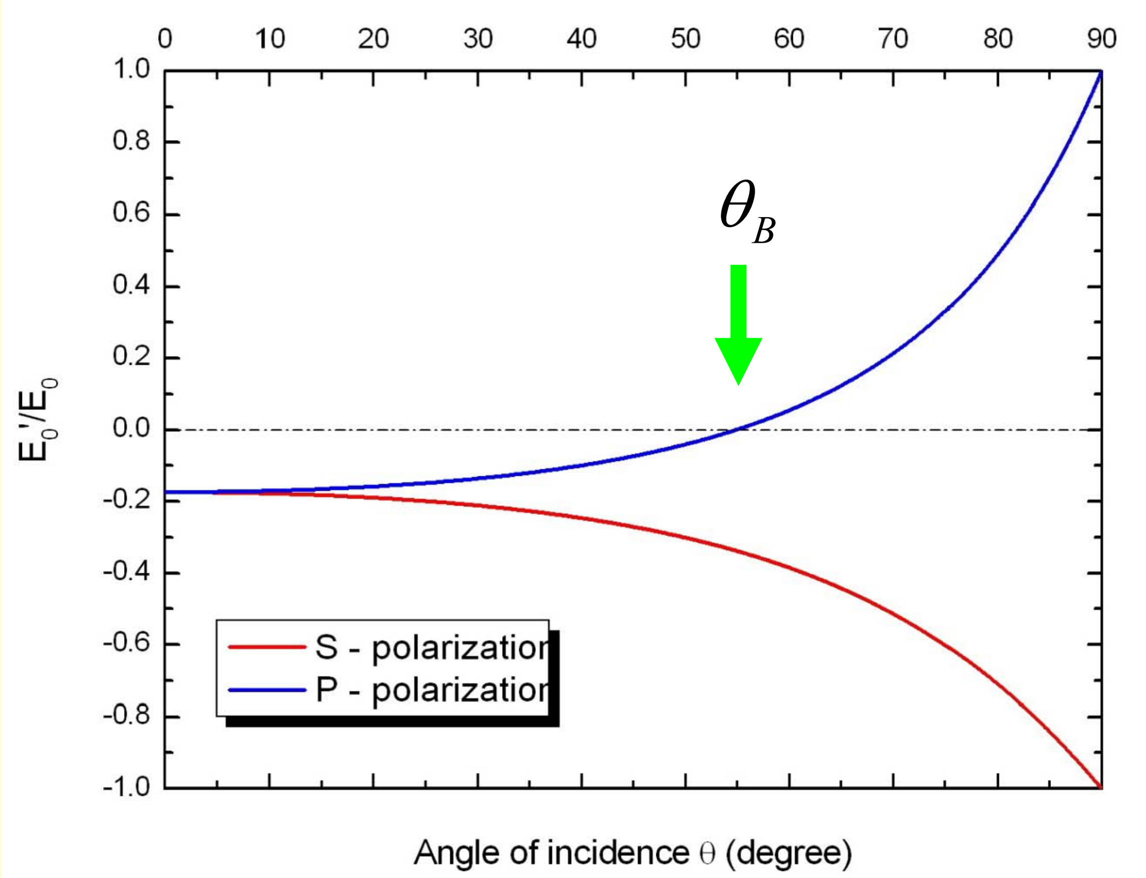
$$\frac{E_{0//}'}{E_{0//}}=-\frac{\sqrt{\varepsilon_2}\cos\theta-\sqrt{\varepsilon_1}\cos\theta''}{\sqrt{\varepsilon_2}\cos\theta+\sqrt{\varepsilon_1}\cos\theta''}$$



1）假设：$n_1 = 1.00,\ n_2 = 1.42\ \ (n_1 < n_2)$



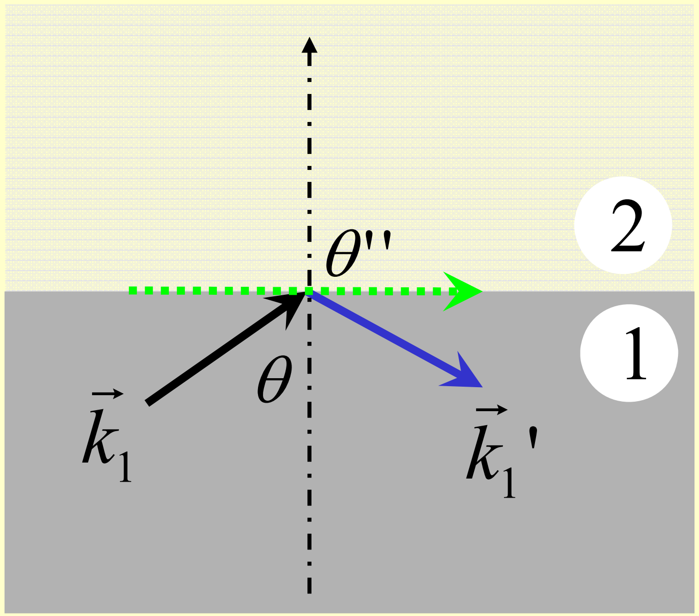




- **此时**，**对于s 偏振，反射波的电场与入射波的电场始终反向——电磁波反射过程中的半波损失。**

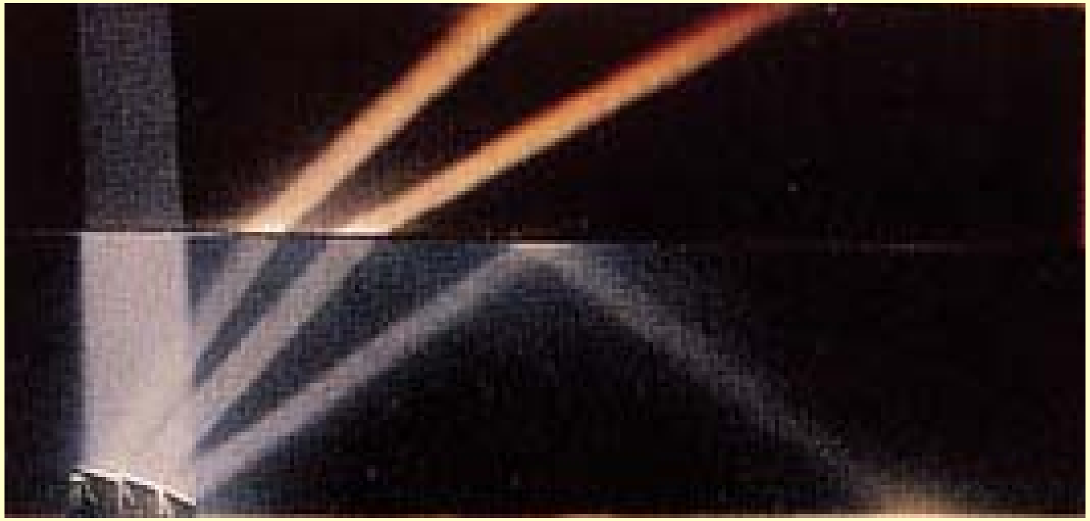
$n_1 \sin\theta = n_2 \sin\theta''$

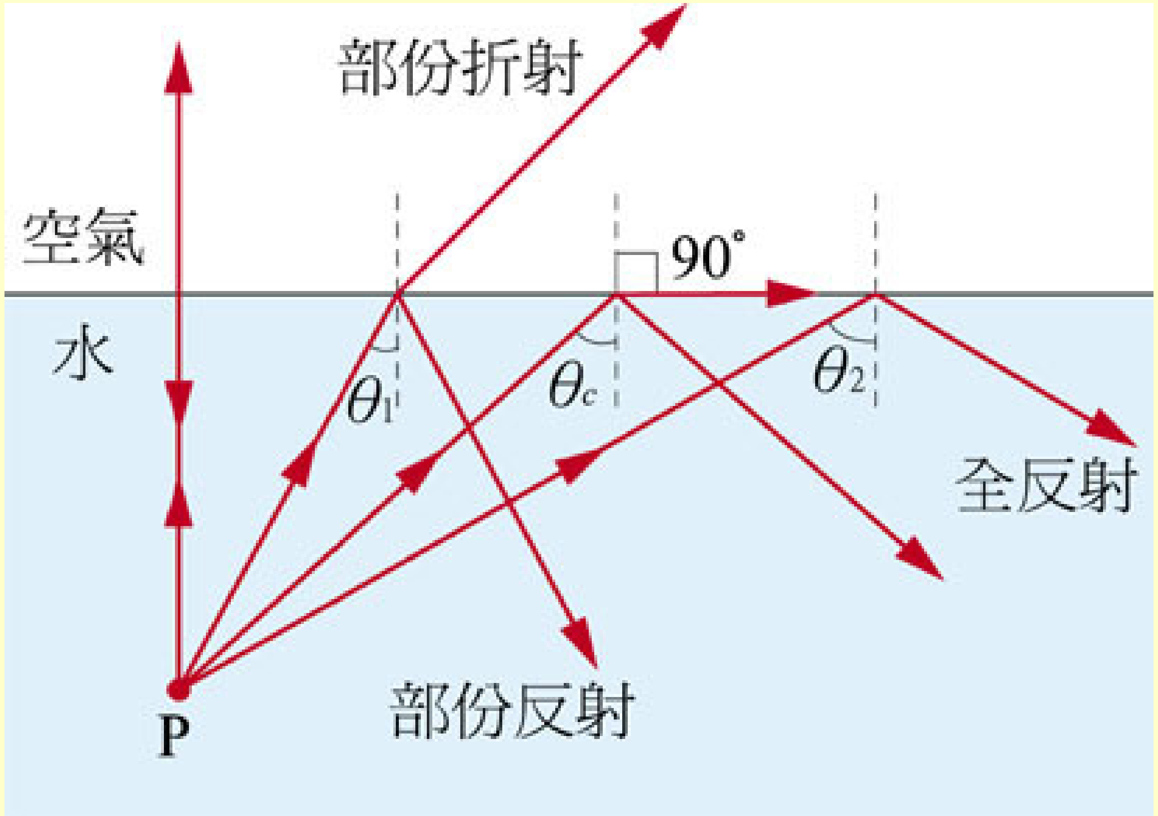
2）假设：电磁波从折射率高的介质（光子能量低）入射到折射率低（光子能量高）的介质分界面（会发生全反射现象）



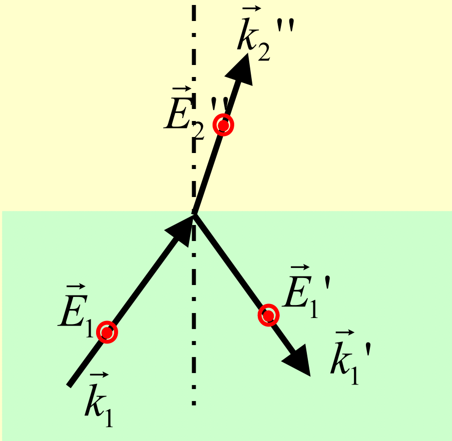
$(n_2 < n_1)$

假设：$n_1 = 1.42,\ n_2 = 1.00$

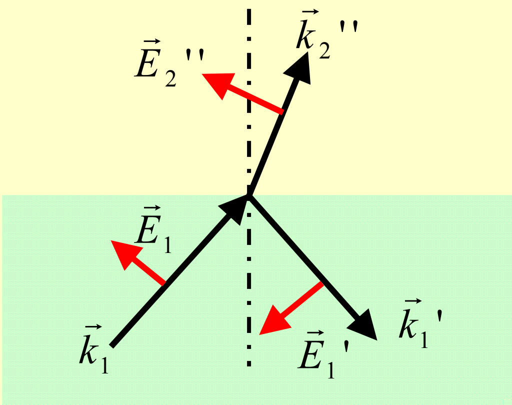



部份折射
空氣
90°
水
θ_1
θ_c
θ_2
全反射
部份反射
P

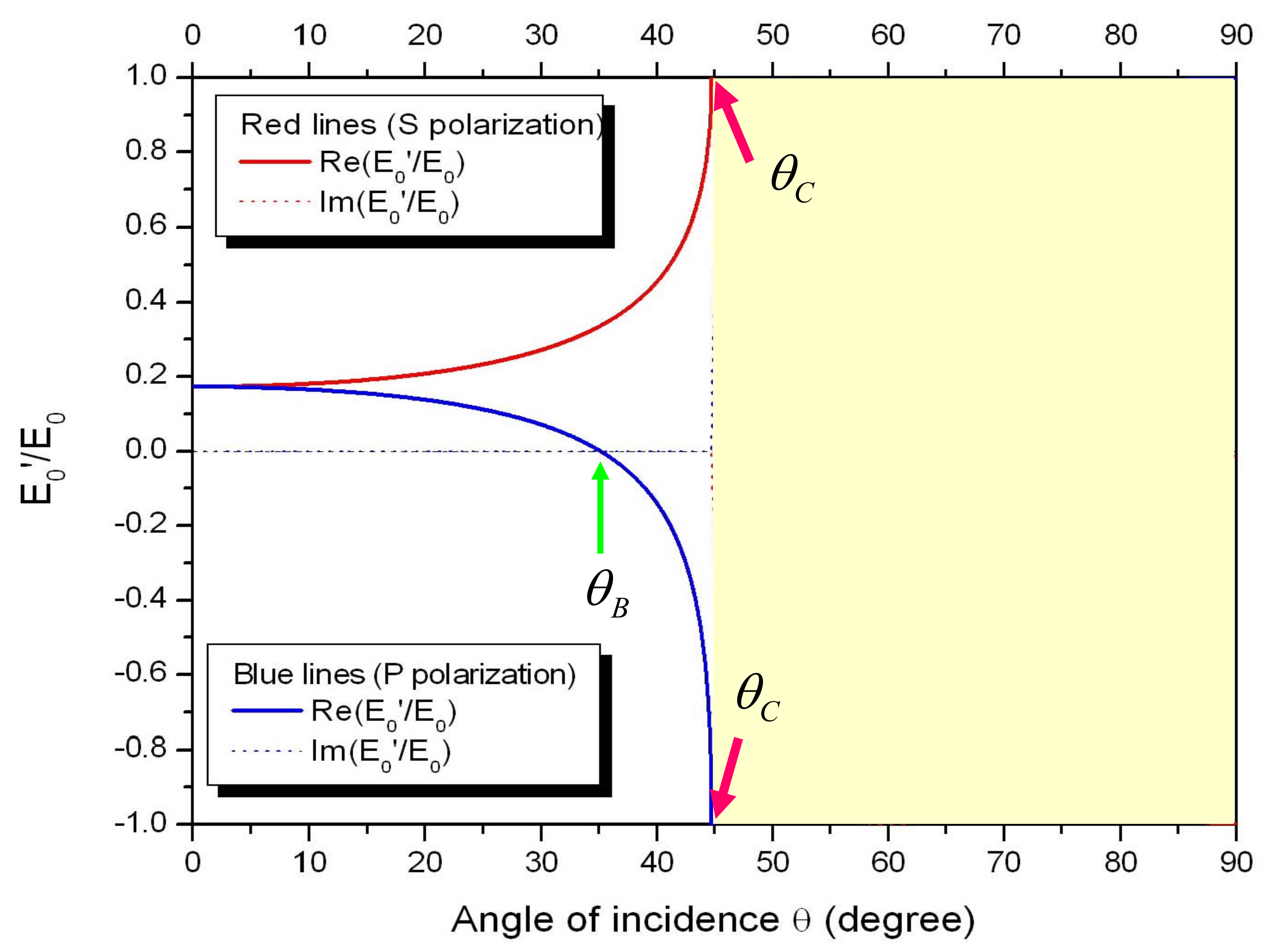
假设：$n_1 = 1.42,\ n_2 = 1.00$



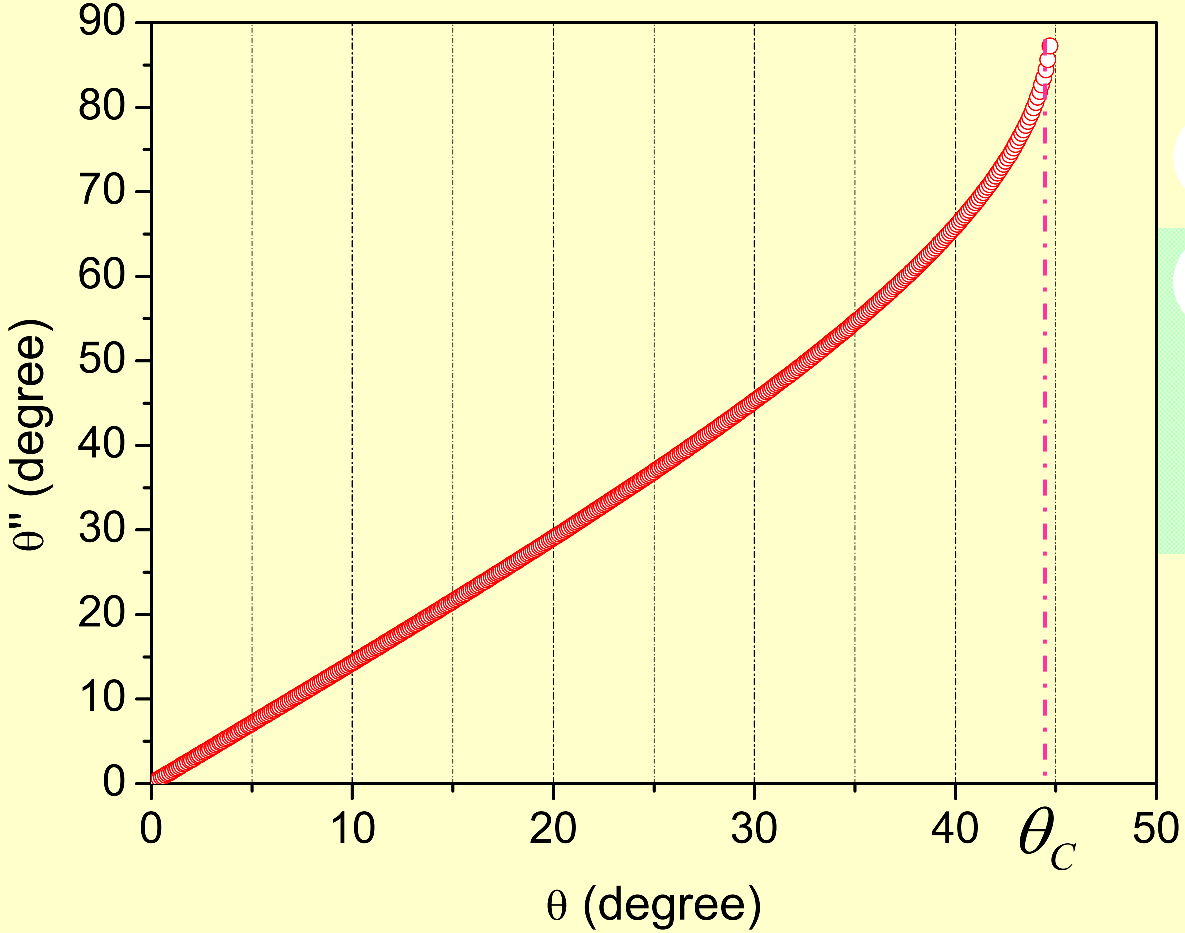
$$\frac{E_{0\perp}'}{E_{0\perp}} = \frac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''}$$

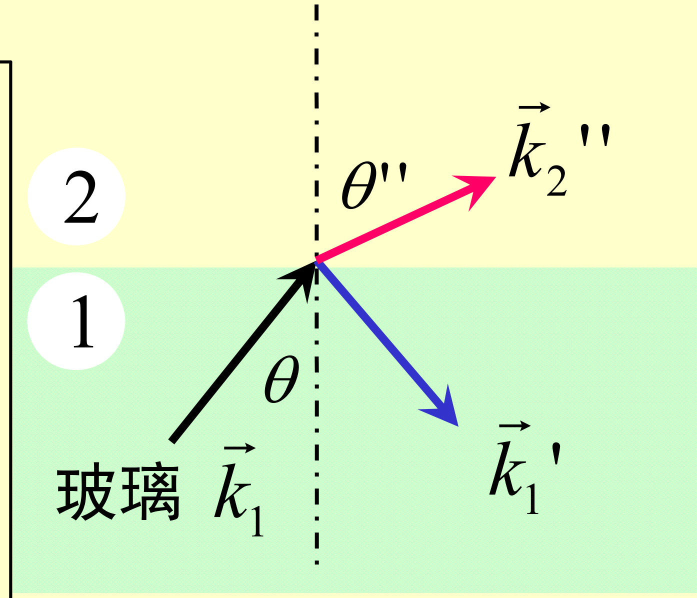


$$\frac{E_{0//}'}{E_{0//}} = -\frac{\sqrt{\varepsilon_2}\cos\theta - \sqrt{\varepsilon_1}\cos\theta''}{\sqrt{\varepsilon_2}\cos\theta + \sqrt{\varepsilon_1}\cos\theta''}$$



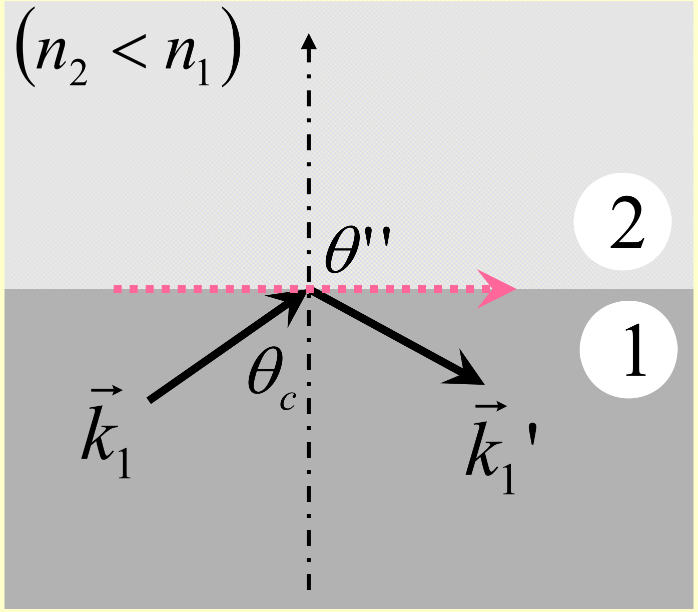
Red lines (S polarization)
Re(E0'/E0)
Im(E0'/E0)
Blue lines (P polarization)
Re(E0'/E0)
Im(E0'/E0)
θ_C
θ_B
θ_C
E0'/E0
Angle of incidence θ (degree)





发现：当 $\theta=\theta_c$ 时，$\theta''=90^{\circ}$

1）定义临界（入射）角:



$$\boxed{\sin\theta_c = n_{21}}$$

$$n_1 \sin\theta = n_2 \sin\theta''$$

① 当 $\theta > \theta_c$ 时，$\sin\theta'' > 1$

② 在 $\theta > \theta_c$ 时，θ”已失去它作为几何上的解释。

2）全反射情况下的反射波

$$n_2 \sin\theta'' = n_1 \sin\theta$$

当 $\theta > \theta_c$ 时，$\sin\theta'' > 1$

$$\sin\theta'' = \sin\theta / n_{21} \; (>1)$$

$$\cos\theta'' = \sqrt{1 - \sin^2\theta''}$$

$$= \mathrm{i}\sqrt{\left(\frac{\sin\theta}{n_{21}}\right)^2 - 1}$$

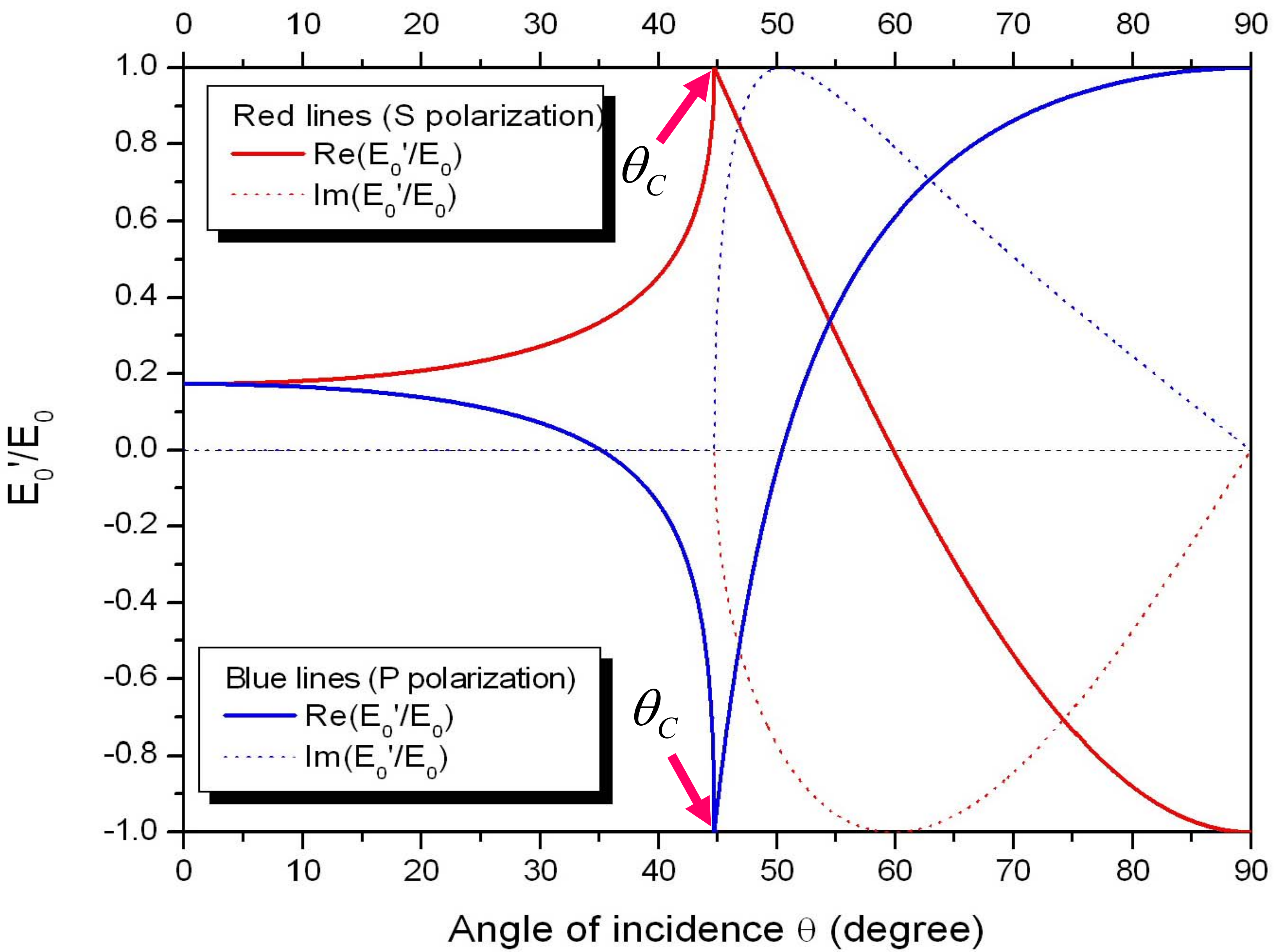
$$\frac{E_{0\perp}'}{E_{0\perp}} = \frac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''}$$

$$\frac{E_{0//}'}{E_{0//}} = -\frac{n_2\cos\theta - n_1\cos\theta''}{n_2\cos\theta + n_1\cos\theta''}$$

$$\frac{E_{0\perp}'}{E_{0\perp}} = \frac{\cos\theta - i\sqrt{\sin^2\theta - n_{21}^2}}{\cos\theta + i\sqrt{\sin^2\theta - n_{21}^2}}$$

$$\frac{E_{0//}'}{E_{0//}} = -\frac{n_{21}^2\cos\theta - i\sqrt{\sin^2\theta - n_{21}^2}}{n_{21}^2\cos\theta + i\sqrt{\sin^2\theta - n_{21}^2}}$$

Both complex beyond the critical angle !



Red lines (S polarization)
Re(E0'/E0)
Im(E0'/E0)
Blue lines (P polarization)
Re(E0'/E0)
Im(E0'/E0)
θ_C
θ_C
E0'/E0
Angle of incidence θ (degree)
0
10
20
30
40
50
60
70
80
90
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0

$$\frac{E_{0\perp}'}{E_{0\perp}} = \frac{\cos\theta - i\sqrt{\sin^2\theta - n_{21}^2}}{\cos\theta + i\sqrt{\sin^2\theta - n_{21}^2}}$$

$$\frac{E_{0//}'}{E_{0//}} = \frac{n_{21}^2\cos\theta - i\sqrt{\sin^2\theta - n_{21}^2}}{n_{21}^2\cos\theta + i\sqrt{\sin^2\theta - n_{21}^2}}$$

容易验证：当 $\theta > \theta_c$ 时,

$$\left|\frac{E_{0\perp}'}{E_{0\perp}}\right| = 1$$

$$\left|\frac{E_{0//}'}{E_{0//}}\right| = 1$$

在全反射条件下：

① 对于任意偏振, 都在同一个角度开始发生全反射；

② 反射波与入射波具有相同的振幅，但存在一定的位相差；

③ 反射波的平均能流密度等于入射波的平均能流密度。

3）全反射情况的折射波

全反射时的折射波成是沿着界面传播的表面波

$$k_{2z}'' = \sqrt{k_2^2 - k_{2x}^2} = \sqrt{k_2^2 - k_2^2 \sin^2\theta''} = k_2\sqrt{1-\sin^2\theta''}$$

$$k_{2z}'' = i\kappa$$

$$\vec{E}''(\vec{x},t) = \vec{E}_0''\exp\left[\mathrm{i}\left(\vec{k}_2''\cdot\vec{x} - \omega'' t\right)\right]$$

$$\vec{k}_2''\cdot\vec{x} = k_{2x}''x + k_{2z}''z = k_{2x}''x + i\kappa\cdot z$$

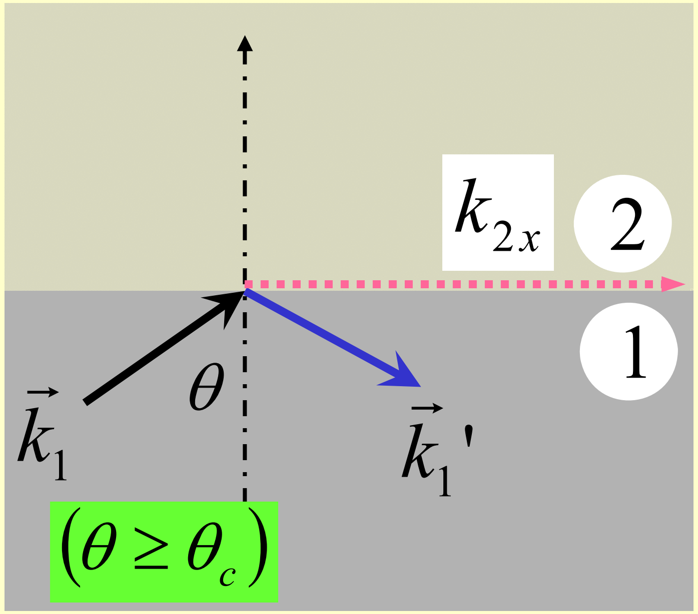
$$\vec{E}''(\vec{x},t) = \vec{E}_0''\exp(-\kappa z)\exp\left[\mathrm{i}\left(k_{2x}''x - \omega t\right)\right]$$

$$\vec{E}''(\vec{x},t) = \vec{E}_0''\mathrm{e}^{-\kappa z}\mathrm{e}^{\mathrm{i}\left(k_{2x}''x - \omega t\right)}$$

① 介质 2 中的折射波数:

$$k_2''/n_2 = k_1/n_1$$

$$\boxed{k_2'' = \frac{n_2}{n_1}k_1 = n_{21}k_1}$$



② 在全反射的情况下，波矢分量边值关系仍然成立

$$k_{2x}'' = k_{1x} = k_1 \sin\theta$$

$$k_2'' = n_{21} k_1$$

③ 全反射的折射波矢 z 分量：

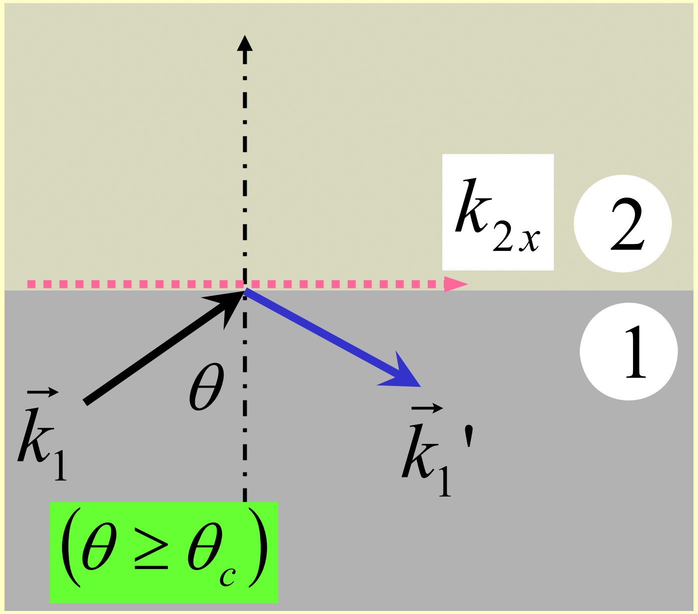
$$k_{2x}'' = k_{1x} = k_1 \sin\theta$$

$$\Rightarrow k_{2z}'' = \sqrt{(k_2'')^2 - (k_{2x}'')^2}$$

$$= \sqrt{k_1^2 n_{21}^2 - k_1^2 \sin^2\theta}$$

$$= \mathrm{i} k_1 \sqrt{\sin^2\theta - n_{21}^2}$$

（一个纯虚数）



定义： $k_{2z}'' = i\kappa$

$$\kappa = k_1 \sqrt{\sin^2\theta - n_{21}^2}$$

④　全反射时折射波—表面波

$$k_{2z}'' = i\kappa \qquad \kappa = k_1\sqrt{\sin^2\theta - n_{21}^2}$$

$$\vec{E}''(\vec{x},t) = \vec{E}_0''\exp\left[\mathrm{i}\left(\vec{k}_2''\cdot\vec{x} - \omega''t\right)\right]$$

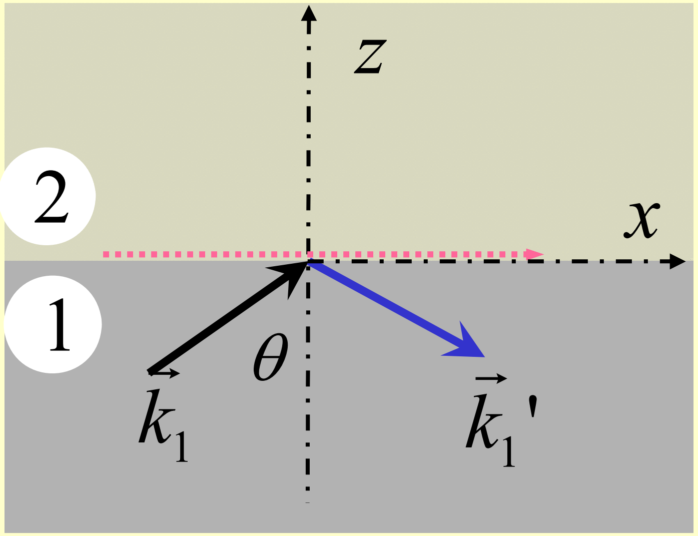
$$\vec{k}_2''\cdot\vec{x} = k_{2x}''x + k_{2z}''z = k_{2x}''x + i\kappa\cdot z$$

$$\vec{E}''(\vec{x},t) = \vec{E}_0''\exp(-\kappa z)\exp\left[\mathrm{i}\left(k_{2x}''x - \omega t\right)\right]$$

$$\vec{E}''(\vec{x},t) = \vec{E}_0''\mathrm{e}^{-\kappa z}\mathrm{e}^{\mathrm{i}\left(k_{2x}''x - \omega t\right)}$$

$$\vec{E}''(\vec{x},t)$$

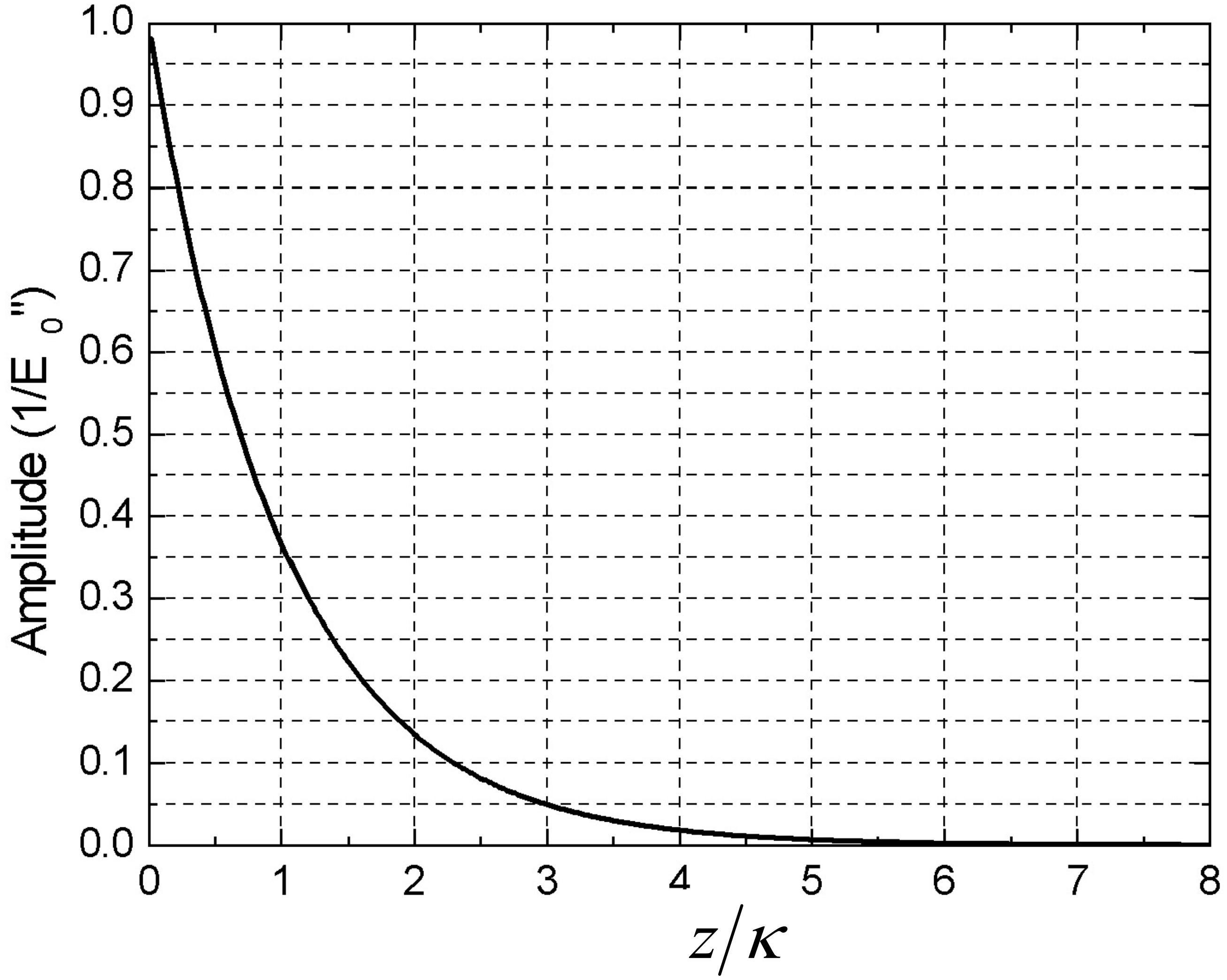
$$= \vec{E}_0'' e^{-\kappa z} e^{i(k_{2x}'' x - \omega t)}$$



$(\theta \geq \theta_c)$

- 它表示沿 x方向传播、振幅沿 z 轴衰减的时谐波；
- 这种时谐波只存在于界面附近一簿层内，该簿层的厚度在1~2个λ的线度内。
- 全反射时的折射波为表面波。

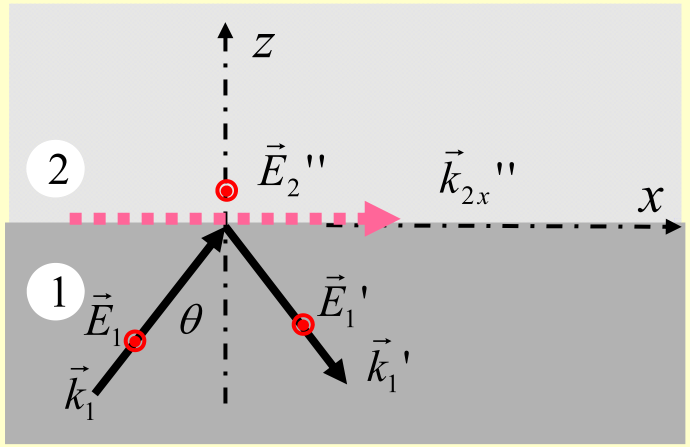
$$\vec{E}''(\vec{x},t) = \vec{E}_0''\exp(-\kappa z)\exp[\mathrm{i}(k_{2x}''x - \omega t)]$$



4）折射波的能流密度

$$\left(\vec{S} = \vec{E} \times \vec{H}\right)$$

假设入射波的电场垂直于入射面，即S偏振情形



介质 2 中的能流密度：

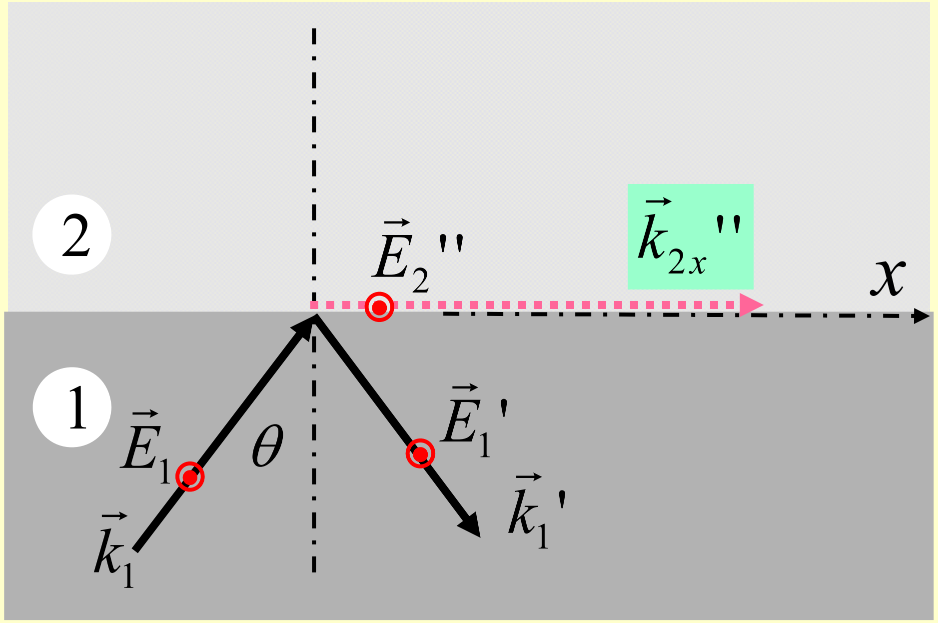
$$\boxed{\left(\vec{S} = \vec{E} \times \vec{H}\right)}$$

$$S_{2x} = E_2'' H_{2z} = \frac{1}{\mu_2} E_2'' B_{2z}''$$

须各取其实部

$$S_{2z} = -E_2'' H_{2x} = -\frac{1}{\mu_2} E_2'' B_{2x}''$$

对于S偏振情形：



$$\vec{B} = (1/\omega)\vec{k} \times \vec{E}$$

$$\vec{k}_2'' = \left(k_{2x}'', 0, k_{2z}''\right)$$

$$\vec{E}'' = \left(0, E'', 0\right)$$

折射波： $\vec{E}''(\vec{x},t) = \left(E_0'' \mathrm{e}^{-\kappa z}\right)\mathrm{e}^{\mathrm{i}\left(k_{2x}'' x - \omega t\right)}\,\vec{\mathrm{e}}_{\mathrm{y}}$

$$\vec{E}''(\vec{x},t)=\left(E_0''\mathrm{e}^{-\kappa z}\right)\mathrm{e}^{\mathrm{i}(k_{2x}''x-\omega t)}\,\vec{\mathrm{e}}_{\mathrm{y}}$$

$$\vec{k}_2''=\left(k_{2x}'',0,k_{2z}''\right)$$

$$\vec{B}=(1/\omega)\vec{k}\times\vec{E}$$

$$\vec{E}''=\left(0,E'',0\right)$$

$$\begin{cases} B_{2z}''=\dfrac{1}{\omega}k_{2x}''E_2'' \\ B_{2x}''=-\dfrac{1}{\omega}k_{2z}''E_2'' \end{cases}$$

$$k_2''=\omega\sqrt{\mu_2\varepsilon_2}$$

$$k_{2x}''=k_1\sin\theta$$

$$k_2''=n_{21}k_1$$

$$B_{2z}''=\frac{k_2''}{\omega}\frac{\sin\theta}{n_{21}}E_2''$$

$$=\sqrt{\mu_2\varepsilon_2}\frac{\sin\theta}{n_{21}}E_2''$$

$$B_{2z}'' = \sqrt{\mu_2 \varepsilon_2}\,\frac{\sin\theta}{n_{21}} E_2''$$

$$B_{2x}'' = -\frac{1}{\omega} k_{2z}'' E_2''$$

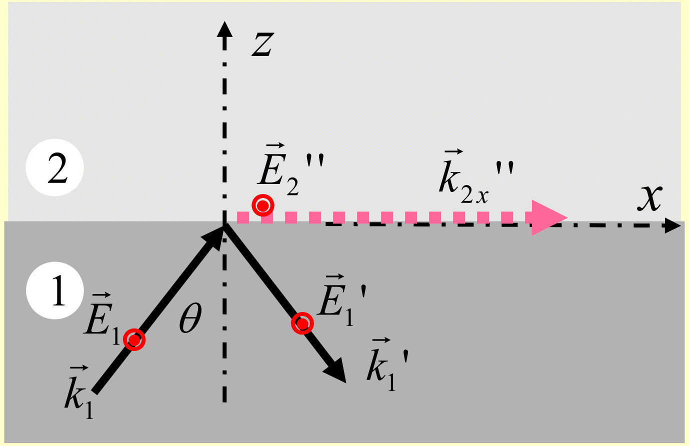
$$k_{2z}'' = \mathrm{i} k_1 \sqrt{\sin^2\theta - n_{21}^2}$$

$$B_{2x}'' = -\mathrm{i}\frac{1}{\omega} k_1 \sqrt{\sin^2\theta - n_{21}^2}\, E_2''$$

$$= -\mathrm{i}\frac{k_1 n_{21}}{\omega}\sqrt{\left(\frac{\sin^2\theta}{n_{21}^2} - 1\right)}\, E_2''$$

$$k_2''/n_2 = k_1/n_1$$

$$= -\mathrm{i}\frac{k_2''}{\omega}\sqrt{\left(\frac{\sin^2\theta}{n_{21}^2} - 1\right)}\, E_2''$$



$$\vec{E}''(\vec{x},t)=\left(E_0''\mathrm{e}^{-\kappa z}\right)\mathrm{e}^{\mathrm{i}\left(k_{2x}''x-\omega t\right)}\,\vec{\mathrm{e}}_y$$

$$B_{2x}''=-\mathrm{i}\frac{k_2''}{\omega}\sqrt{\frac{\sin^2\theta}{n_{21}^2}-1}\;E_2''$$

$$B_{2z}''=\sqrt{\mu_2\varepsilon_2}\,\frac{\sin\theta}{n_{21}}E_2''$$

介质 2 中的能流密度：$\left(\vec{S}=\vec{E}\times\vec{H}\right)$

$$S_{2x}=E_2''H_{2z}=\frac{1}{\mu_2}\underline{E_2''}\,\underline{B_{2z}''}$$

$$S_{2z}=-E_2''H_{2x}=-\frac{1}{\mu_2}\underline{E_2''}\,\underline{B_{2x}''}$$

$$B_{2z}'' = \sqrt{\mu_2 \varepsilon_2}\,\frac{\sin\theta}{n_{21}}\,E_2''$$

$$B_{2x}'' = -\mathrm{i}\,\frac{k_2''}{\omega}\sqrt{\frac{\sin^2\theta}{n_{21}^2} - 1}\;E_2''$$

$$\vec{E}''(\vec{x}, t) = \left(E_0''\,\mathrm{e}^{-\kappa z}\right)\mathrm{e}^{\mathrm{i}(k_{2x}''x - \omega t)}\;\vec{\mathrm{e}}_y$$

$$\mathrm{Re}(B_{2z}'') = \sqrt{\mu_2 \varepsilon_2}\,\frac{\sin\theta}{n_{21}}\,\mathrm{Re}(E_2'')$$

$$\mathrm{Re}(B_{2x}'') = \frac{k_2''}{\omega}\sqrt{\frac{\sin^2\theta}{n_{21}^2} - 1}\;\mathrm{Im}(E_2'')$$

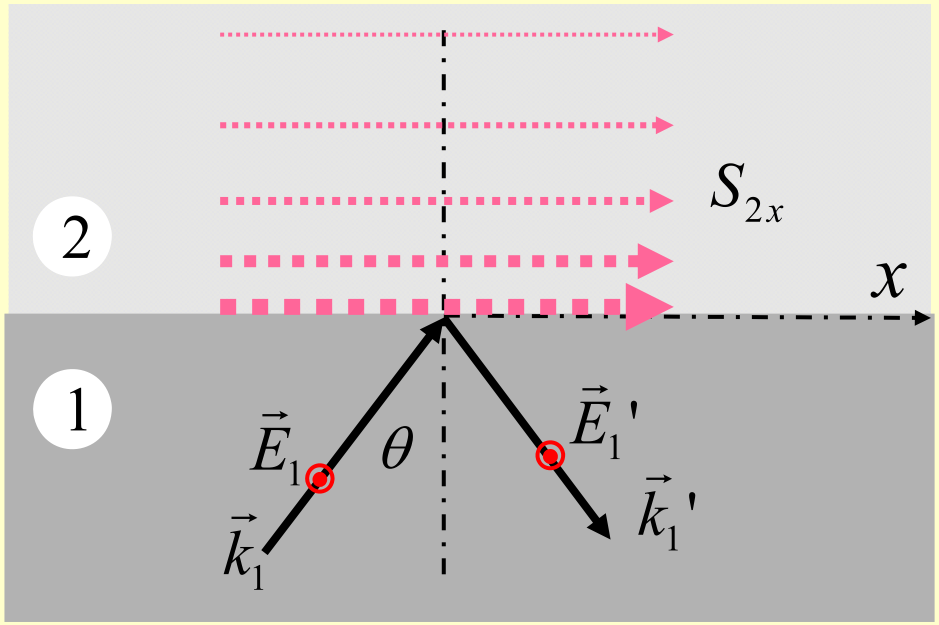
$$S_{2x} = \frac{1}{\mu_2}\mathrm{Re}(E_2'')\cdot\mathrm{Re}(B_{2z}'') = \frac{1}{\mu_2}\sqrt{\mu_2 \varepsilon_2}\,\frac{\sin\theta}{n_{21}}\left(\mathrm{Re}\,E_2''\right)^2$$

$$= \sqrt{\frac{\varepsilon_2}{\mu_2}}\,\frac{\sin\theta}{n_{21}}\left(E_0''\right)^2 \mathrm{e}^{-2\kappa z}\cos^2\left(k_{2x}''x - \omega t\right)$$

$$= \frac{1}{2}\sqrt{\frac{\varepsilon_2}{\mu_2}}\,\frac{\sin\theta}{n_{21}}\left(E_0''\right)^2 \mathrm{e}^{-2\kappa z}\left\{1 + \cos\left[2\left(k_{2x}''x - \omega t\right)\right]\right\}$$

$$S_{2\mathrm{x}} = \frac{1}{2}\sqrt{\frac{\varepsilon_2}{\mu_2}}\frac{\sin\theta}{n_{21}}\left(E_0''\right)^2 \mathrm{e}^{-2\kappa z}\left\{1+\cos\left[2\left(k_{2x}''x-\omega t\right)\right]\right\}$$

$$\left\langle S_{2x}\right\rangle = \frac{1}{2}\sqrt{\frac{\varepsilon_2}{\mu_2}}\frac{\sin\theta}{n_{21}}\left(E_0''\right)^2 \mathrm{e}^{-2\kappa z}$$



$$\vec{E}''(\vec{x},t)=\left(E_0''\,\mathrm{e}^{-\kappa z}\right)\mathrm{e}^{\mathrm{i}(k_{2x}''x-\omega t)}\,\vec{\mathrm{e}}_{\mathrm{y}}$$

$$\mathrm{Re}\left(B_{2x}''\right)=\frac{k_2''}{\omega}\sqrt{\frac{\sin^2\theta}{n_{21}^2}-1}\,\mathrm{Im}\left(E_2''\right)$$

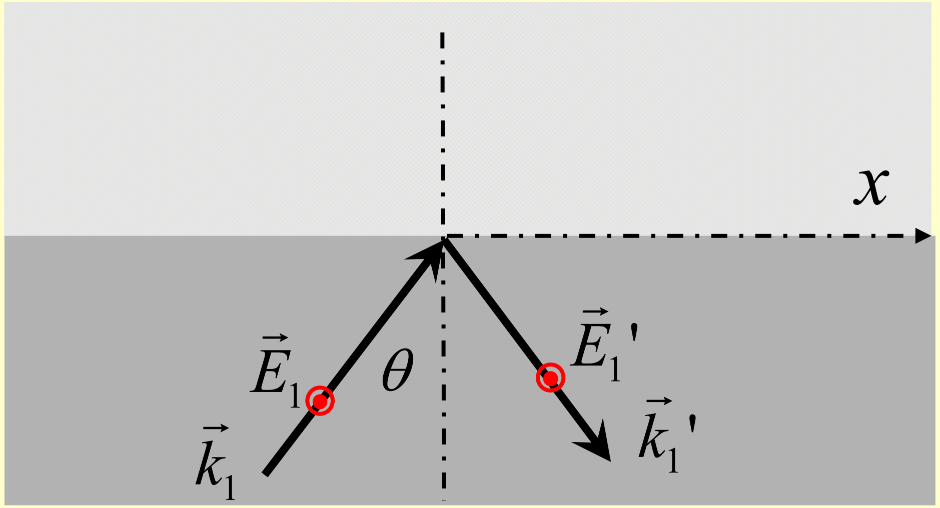
沿 z 轴的能流

$$S_{2z}=-\left(1/\mu_2\right)\mathrm{Re}\left(E_2''\right)\mathrm{Re}\left(B_{2x}''\right)$$

$$=-\frac{1}{\mu_2}\frac{k_2''}{\omega}\left(\frac{\sin^2\theta}{n_{21}^2}-1\right)^{1/2}\mathrm{e}^{-2\kappa z}\left(E_0''\right)^2$$

$$\cdot\sin\left(k_{2x}''x-\omega t\right)\cos\left(k_{2x}''x-\omega t\right)$$

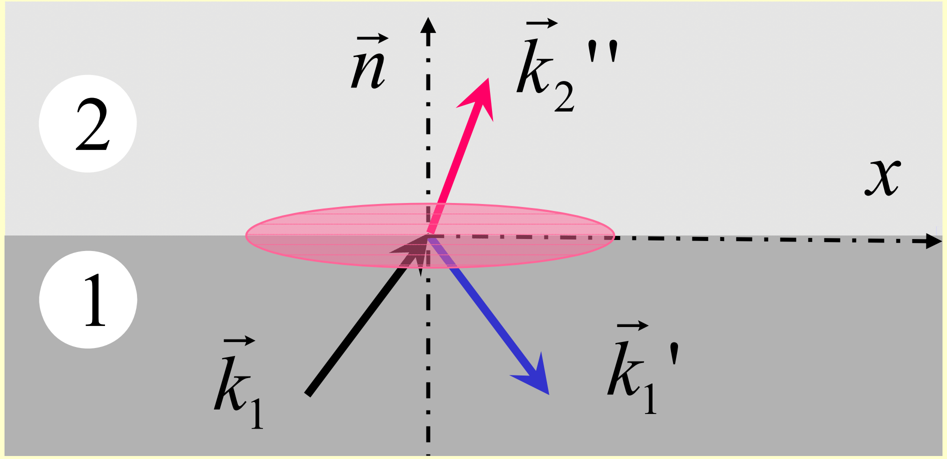
$$=-\frac{k_2''}{2\mu_2\omega}\left(E_0''\right)^2\mathrm{e}^{-2\kappa z}\left(\frac{\sin^2\theta}{n_{21}^2}-1\right)^{1/2}\cdot\sin\left[2\left(k_{2x}''x-\omega t\right)\right]$$



$$S_{2z} = -\frac{k_2''}{2\mu_2\omega}(E_0'')^2 e^{-2\kappa z}\left(\frac{\sin^2\theta}{n_{21}^2}-1\right)^{1/2}\cdot\sin[2(k_{2x}''x-\omega t)]$$

沿 z 轴能流在一个周期内的平均值：

$$\langle S_{2z}\rangle = 0$$



3）反射系数：

$$R=\frac{\langle \vec{S}'\rangle\cdot\vec{n}}{\langle \vec{S}\rangle\cdot\vec{n}}=\frac{|E_0'|^2\,\vec{e}_{k'}\cdot\vec{n}}{|E_0|^2\,\vec{e}_k\cdot\vec{n}}=\frac{|E_0'|^2}{|E_0|^2}$$

4）折射系数：

$$T=\frac{\langle \vec{S}''\rangle\cdot\vec{n}}{\langle \vec{S}\rangle\cdot\vec{n}}=\frac{n_2|E_0''|^2\,\vec{e}_{k''}\cdot\vec{n}}{n_1|E_0|^2\,\vec{e}_k\cdot\vec{n}}=\frac{n_2|E_0''|^2\cos\theta''}{n_1|E_0|^2\cos\theta}$$

在全反射条件下，

① 折射波介质 2 的存在沿着表面传播的折射波，因此存在能流；

② 平均能流密度存在沿分界面的分量；

③ 而沿面法线方向而透入第二介质的平均能流密度为零。

作业

第四章：习题3

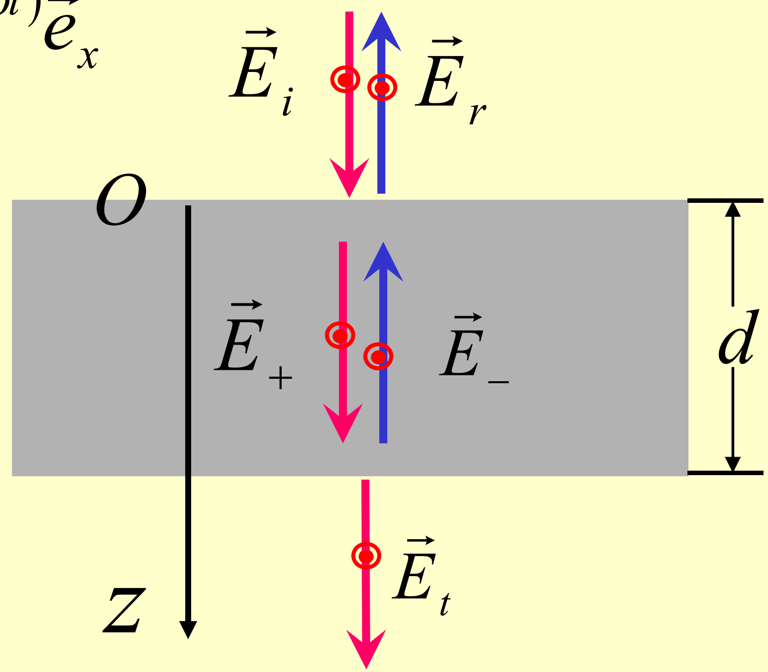
思考题：单色平面电磁波垂直入射到一介质膜上，在入射区，存在入射和反射波；在介质膜内，存在前向和反向波；在透射区存在透射波，分别表示为

$$\vec{E}_i = E_{i0}\mathrm{e}^{\mathrm{i}(kz-\omega t)}\vec{e}_x,\quad \vec{E}_r = E_{r0}\mathrm{e}^{\mathrm{i}(-kz-\omega t)}\vec{e}_x$$

$$\boxed{\vec{E}_+ = E_{+0}\mathrm{e}^{\mathrm{i}(k_m z-\omega t)}\vec{e}_x}$$

$$\boxed{\vec{E}_- = E_{-0}\mathrm{e}^{\mathrm{i}(-k_m z-\omega t)}\vec{e}_x}$$

$$\vec{E}_t = E_{t0}\mathrm{e}^{\mathrm{i}(kz-\omega t)}\vec{e}_x$$



试根据介质分界面处电磁场边界条件，写出上述电场振幅之间的关系。