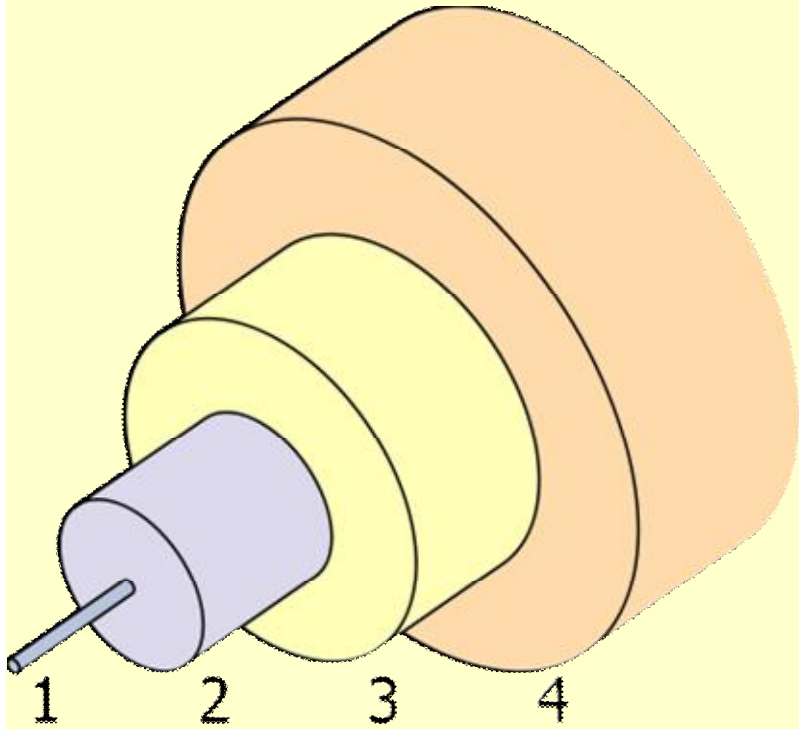


补充二：光波导

- 一. 采用(三层)透明电介质材料构成的平面光波导
- 二. 采用金属/电介质分界面构成的SPP波导

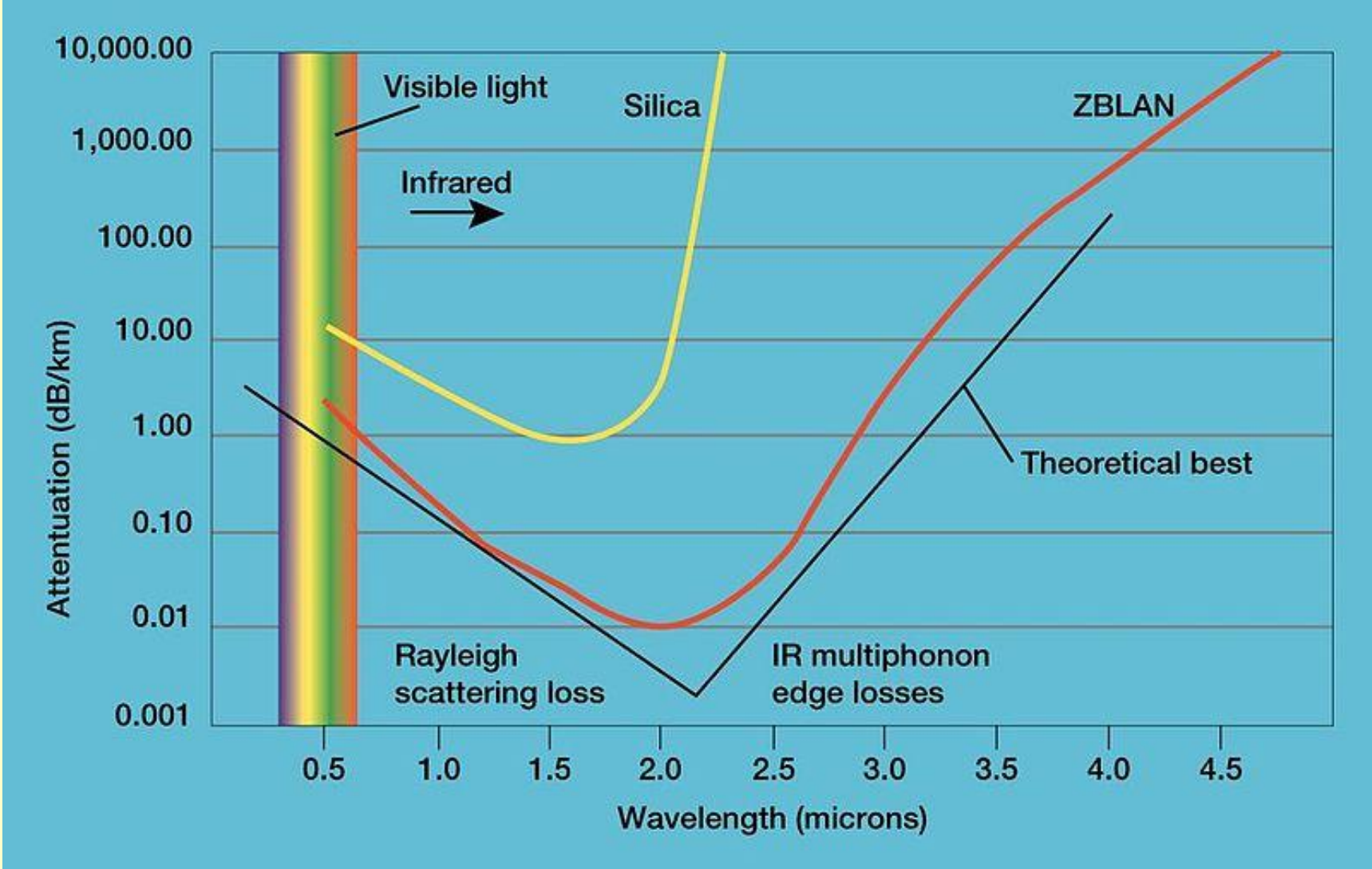
一、采用三层电介质材料构成的平面光波导

- **透明电介质**在光频范围内可以看作是无损耗的, 因此光在波导中的传播损耗是很低的
- 根据**全反射**的特性, 可以采用电介质制成工作在**光频段的光波导**
- 远距离光的传输是采用光纤, 而且波长是选在光在光纤中的传输损耗很小, 可以经过数百公里才需要进行信号放大



The structure of a typical single-mode fiber.

1. Core: 8 μm diameter
2. Cladding: 125 μm dia.
3. Buffer: 250 μm dia.
4. Jacket: 400 μm dia.



1、平面光波导



1) 介质平面波导由三种材料组成：

- ① 中间一层折射率为 n_1 的薄膜称为**导波层**，其厚度 d 一般为微米量级，可与波长相当；
- ② 导波层两侧是折射率分别为 n_2 和 n_3 的衬底和覆盖层；
- ③ 一般情况下， $n_1 > n_2 > n_3$ 。



2) 两个界面，对于在介质1中传播的光而言，存在两个全反射临界角：

$$\theta_{c(\text{cover})} = \arcsin(n_3/n_1)$$

$$\theta_{s(\text{substrate})} = \arcsin(n_2/n_1)$$

$$n_1 > n_2 > n_3 \quad \Rightarrow \quad \theta_c < \theta_s$$

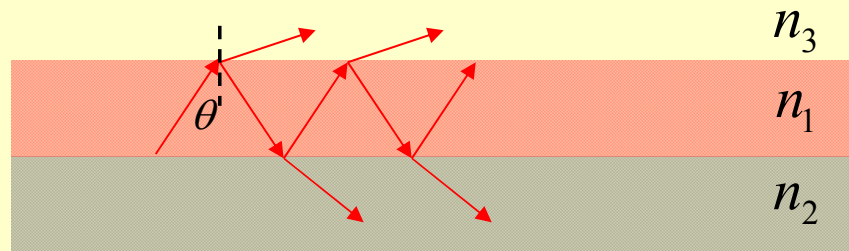
$$\theta_c = \arcsin(n_3/n_1)$$

$$\theta_s = \arcsin(n_2/n_1)$$

$$n_1 > n_2 > n_3$$

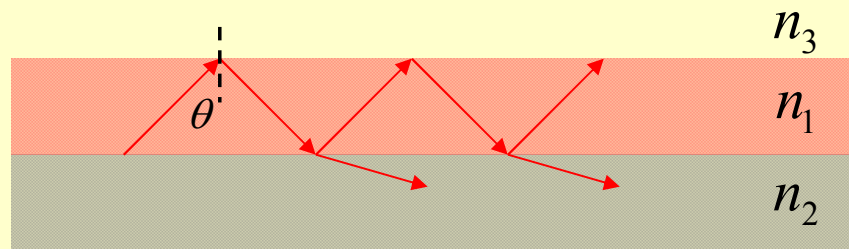
3) 分为三种情况:

$$\theta < \theta_c < \theta_s$$



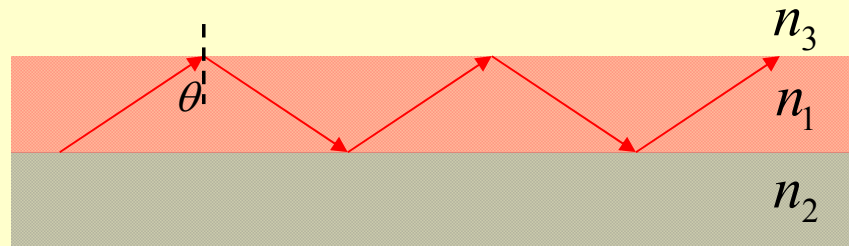
辐射模

$$\theta_c < \theta < \theta_s$$



辐射模

$$\theta_c < \theta_s < \theta$$



导模

$$k_2'' = n_{21} k_1$$

全反射的折射波矢 z 分量:

$$k_{2x}'' = k_{1x} = k_1 \sin \theta$$

$$\rightarrow k_{2z}'' = \sqrt{(k_2'')^2 - (k_{2x}'')^2}$$

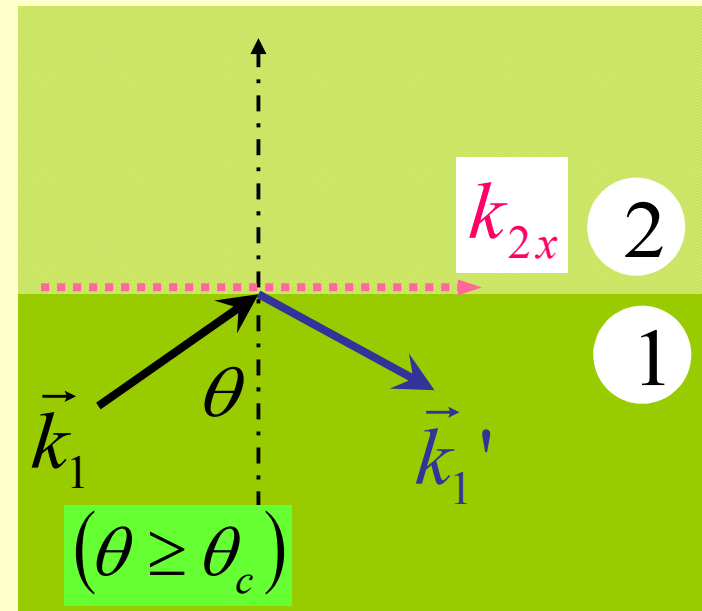
$$= \sqrt{k_1^2 n_{21}^2 - k_1^2 \sin^2 \theta}$$

$$= i k_1 \sqrt{\sin^2 \theta - n_{21}^2}$$

(一个纯虚数)

定义: $k_{2z}'' = i\kappa$

$$\kappa = k_1 \sqrt{\sin^2 \theta - n_{21}^2}$$

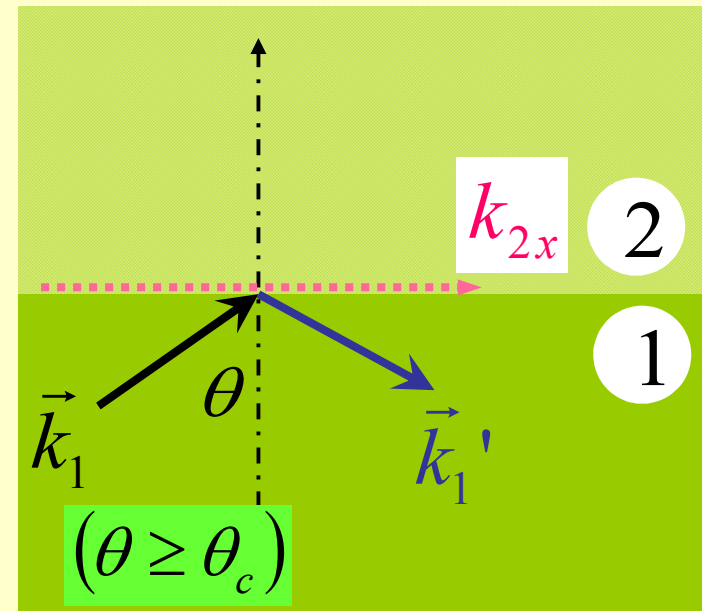


全反射的折射波矢 进入另一侧介质的波矢z 分量：

$$k_{2z}'' = \sqrt{(k_2'')^2 - (k_{2x}''')^2}$$

$$k_{2z}'' = i\kappa$$

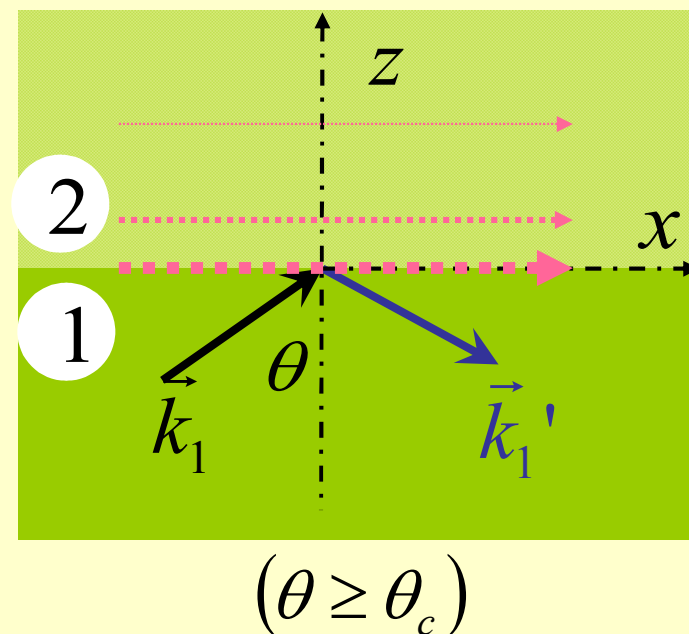
(一个纯虚数)



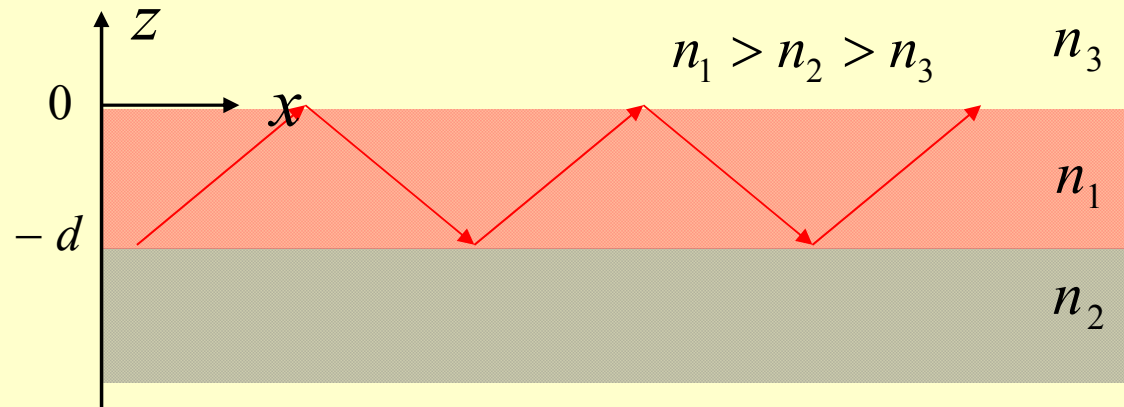
全反射时的折射波为表面波

$$\vec{E}''(\vec{x}, t) = \vec{E}_0'' e^{-ik_2 z} e^{i(k_2 x - \omega t)}$$

它表示沿 +x 方向传播、(从的 z=0 面开始) 振幅沿 +z 轴衰减的时谐波；



2、导模 ($\theta_c < \theta_s < \theta$)

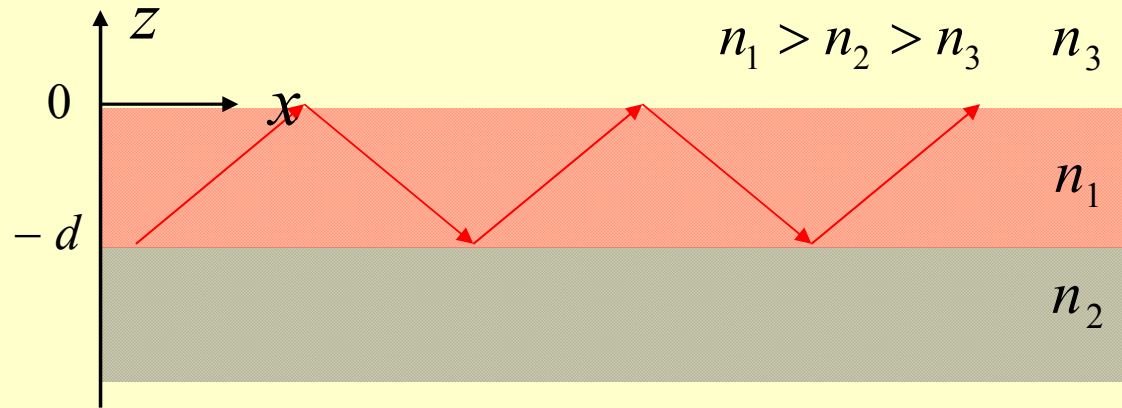


1) 根据导模性质，空间各处电磁场形式：

$$\vec{E}, \vec{H} \propto \left\{ \begin{array}{ll} e^{-k_{z3}z} e^{i(k_x x - \omega t)} & (z > 0) \quad k_{z3} = \sqrt{k_x^2 - n_3^2 k_0^2} \\ e^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & (z < -d) \quad k_{z2} = \sqrt{k_x^2 - n_2^2 k_0^2} \end{array} \right.$$

($z > 0$, 从 $z=0$ 位置开始，振幅沿 $+z$ 方向衰减的、沿 $+x$ 方向传播的表面波)

($z < -d$, 从 $z=-d$ 位置开始，振幅沿 $-z$ 方向衰减的、沿 $+x$ 方向传播的表面波)



根据导模性质，空间各处电磁场形式：

$$\vec{E}, \vec{H} \propto \begin{cases} e^{-k_{z3}z} e^{i(k_x x - \omega t)} & (z > 0) \\ [B e^{ik_{z1}(z+d)} + C e^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & (-d < z < 0) \\ e^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & (z < -d) \end{cases}$$

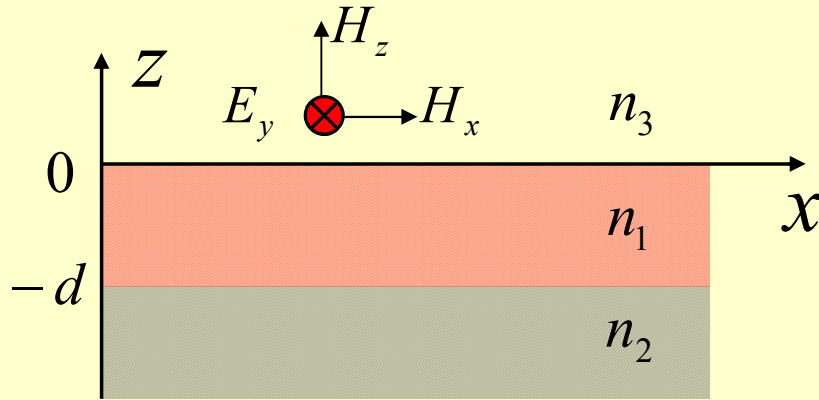
$$k_{z3} = \sqrt{k_x^2 - n_3^2 k_0^2}$$

$$k_{z1} = \sqrt{n_1^2 k_0^2 - k_x^2}$$

$$k_{z2} = \sqrt{k_x^2 - n_2^2 k_0^2}$$

2) TE模

$$E_y = \begin{cases} Ae^{-k_{z3}z} e^{i(k_x x - \omega t)} & (z > 0) \\ [Be^{ik_{z1}(z+d)} + Ce^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & (-d < z < 0) \\ De^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & (z < -d) \end{cases}$$



$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad H_x = \frac{i}{\mu_0 \omega} \frac{\partial E_y}{\partial z}$$

$$\rightarrow H_x = \begin{cases} \frac{i}{\mu_0 \omega} \cdot (-Ak_{z3}) e^{-k_{z3}z} e^{i(k_x x - \omega t)} & (z > 0) \\ \frac{i}{\mu_0 \omega} [iBk_{z1} e^{ik_{z1}(z+d)} - iCk_{z1} e^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & (-d < z < 0) \\ \frac{i}{\mu_0 \omega} \cdot Dk_{z2} e^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & (z < -d) \end{cases}$$

边界条件:

$$\vec{n}_{21} \cdot (\vec{D}_2 - \vec{D}_1) = 0$$

$$\vec{n}_{21} \times (\vec{E}_2 - \vec{E}_1) = 0 \quad \leftarrow$$

$$\vec{n}_{21} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

$$\vec{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = 0 \quad \leftarrow$$

$$\vec{n}_{21} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_f$$

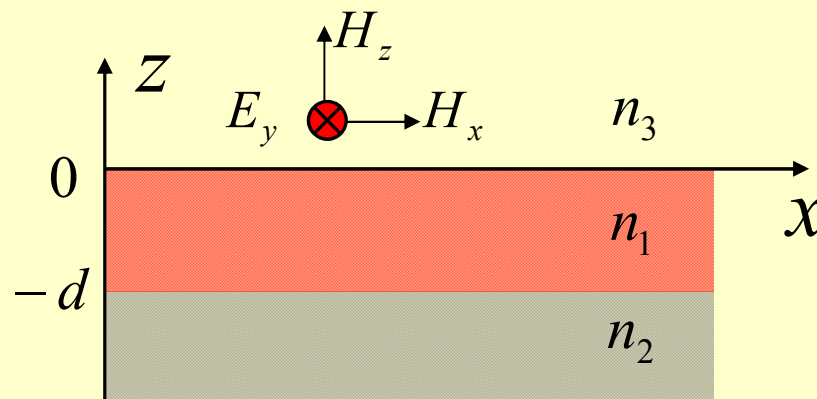
$$\vec{n}_{21} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

$$\vec{n}_{21} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}_f$$

$$E_{1t} = E_{2t}$$

$$H_{1t} = H_{2t}$$



$$E_y = \begin{cases} Ae^{-k_{z3}z} e^{i(k_x x - \omega t)} & z > 0 \\ [Be^{ik_{z1}(z+d)} + Ce^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & -d < z < 0 \\ De^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & z < -d \end{cases}$$

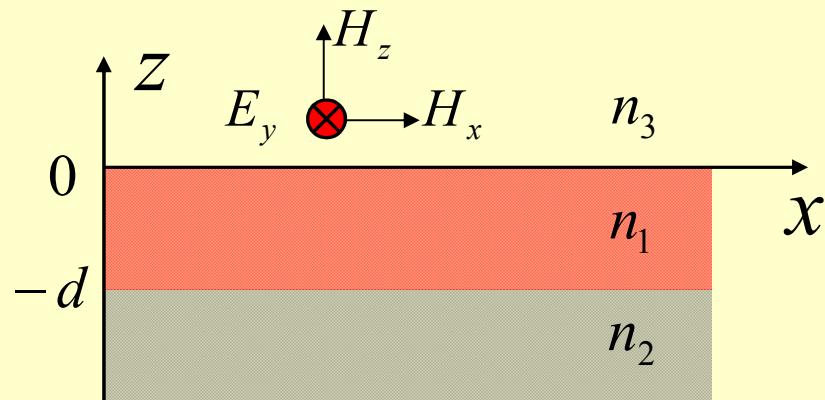
电场切向分量的边界连续条件 ($z=0$, $z=-d$) :

$$\begin{cases} A = Be^{ik_{z1}d} + C \\ B + Ce^{ik_{z1}d} = D \end{cases}$$

$$H_x = \begin{cases} \frac{i}{\mu_0 \omega} \cdot (-Ak_{z3}) e^{-k_{z3}z} e^{i(k_x x - \omega t)} & z > 0 \\ \frac{i}{\mu_0 \omega} [iBk_{z1} e^{ik_{z1}(z+d)} - iCk_{z1} e^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & -d < z < 0 \\ \frac{i}{\mu_0 \omega} \cdot Dk_{z2} e^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & z < -d \end{cases}$$

磁场切向分量的边界连续条件：

$$\begin{cases} -Ak_{z3} = iBk_{z1} e^{ik_{z1}d} - iCk_{z1} \\ iBk_{z1} - iCk_{z1} e^{ik_{z1}d} = Dk_{z2} \end{cases}$$



联立得到四个方程：

$$\begin{cases} A = Be^{ik_{z1}d} + C \\ B + Ce^{ik_{z1}d} = D \end{cases}$$

$$\begin{cases} -Ak_{z3} = iBk_{z1}e^{ik_{z1}d} - iCk_{z1} \\ iBk_{z1} - iCk_{z1}e^{ik_{z1}d} = Dk_{z2} \end{cases}$$

方程组有非零解的条件是系数矩阵行列式为零。



$$\tan(k_{z1}d) = \frac{k_{z1}(k_{z2} + k_{z3})}{k_{z1}^2 - k_{z2}k_{z3}}$$

平面波导的TE模式方程

$$\tan(k_{z1}d) = \frac{k_{z1}(k_{z2} + k_{z3})}{k_{z1}^2 - k_{z2}k_{z3}}$$

TE模式方程:

$$\tan(k_{z1}d) = \frac{k_{z1}(k_{z2} + k_{z3})}{k_{z1}^2 - k_{z2}k_{z3}} = \frac{\frac{k_{z2}}{k_{z1}} + \frac{k_{z3}}{k_{z1}}}{1 - \frac{k_{z2}}{k_{z1}} \cdot \frac{k_{z3}}{k_{z1}}}$$

$$k_{z3} = \sqrt{k_x^2 - n_3^2 k_0^2}$$

$$k_{z1} = \sqrt{n_1^2 k_0^2 - k_x^2}$$

$$k_{z2} = \sqrt{k_x^2 - n_2^2 k_0^2}$$

$$k_{z1}d = \arctan\left(\frac{k_{z2}}{k_{z1}}\right) + \arctan\left(\frac{k_{z3}}{k_{z1}}\right) + m\pi \quad (m \in \mathbb{Z})$$

$$k_0 = \omega / c$$

$$\text{Function}(\omega, k_x, m) = 0 \quad (m \in \mathbb{Z})$$

- 对于一定的厚度 d ，波导中存在的导模数目是有限的。
- d 越大，其所能承载的导模数目则越多。

$$k_{z1}d = \arctan\left(\frac{k_{z2}}{k_{z1}}\right) + \arctan\left(\frac{k_{z3}}{k_{z1}}\right) + m\pi \quad (m \in \mathbb{Z})$$

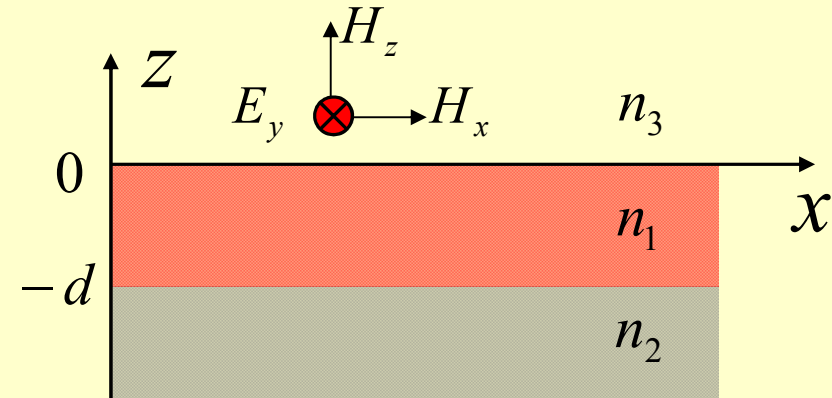
截止频率：

$$n_1 > n_2 > n_3$$

波导中存在导模的条件： $n_1 k_0 > k_x > n_2 k_0 (> n_3 k_0)$

导模截止条件：

$$k_x = n_2 k_0$$



⇒ 截止波长与厚度的关系：

$$d_{TE} = \frac{\lambda}{2\pi\sqrt{n_1^2 - n_2^2}} \left[\arctan\left(\frac{\sqrt{n_2^2 - n_3^2}}{\sqrt{n_1^2 - n_2^2}}\right) + m\pi \right]$$

$$k_{z3} = \sqrt{k_x^2 - n_3^2 k_0^2}$$

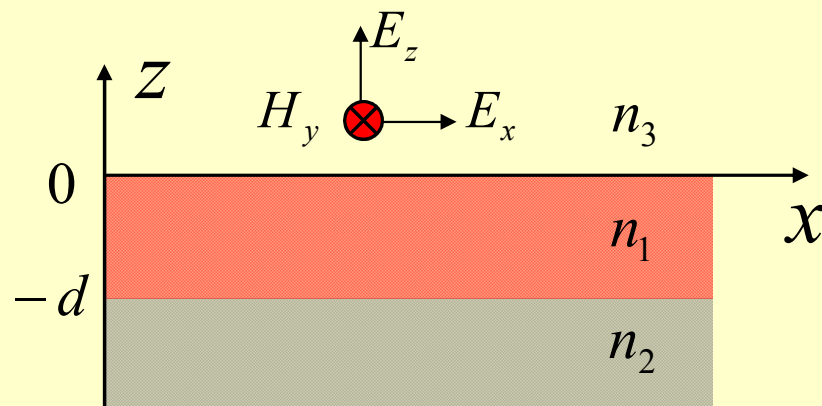
$$k_{z1} = \sqrt{n_1^2 k_0^2 - k_x^2}$$

$$k_{z2} = \sqrt{k_x^2 - n_2^2 k_0^2}$$

$$k_0 = \omega / c$$

(2) TM模

$$H_y = \begin{cases} Ae^{-k_{z3}z} e^{i(k_x x - \omega t)} & (z > 0) \\ [Be^{ik_{z1}(z+d)} + Ce^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & (-d < z < 0) \\ De^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & (z < -d) \end{cases}$$



$$\nabla \times \vec{H} = \varepsilon_0 \varepsilon \frac{\partial \vec{E}}{\partial t} \quad E_x = \frac{1}{i\varepsilon_0 \varepsilon \omega} \frac{\partial H_y}{\partial z}$$

$$E_x = \begin{cases} \frac{1}{i\varepsilon_0 \omega} \cdot \left(-\frac{Ak_{z3}}{n_3^2}\right) \exp[-k_{z3}z] e^{i(k_x x - \omega t)} & (z > 0) \\ \frac{1}{i\varepsilon_0 \omega} \left[\frac{iBk_{z1}}{n_1^2} e^{ik_{z1}(z+d)} - \frac{iCk_{z1}}{n_1^2} e^{-ik_{z1}z} \right] e^{i(k_x x - \omega t)} & (-d < z < 0) \\ \frac{1}{i\varepsilon_0 \omega} \cdot \frac{Dk_{z2}}{n_2^2} e^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & (z < -d) \end{cases}$$

$$H_y = \begin{cases} Ae^{-k_{z3}z} e^{i(k_x x - \omega t)} & z > 0 \\ [Be^{ik_{z1}(z+d)} + Ce^{-ik_{z1}z}] e^{i(k_x x - \omega t)} & -d < z < 0 \\ De^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & z < -d \end{cases}$$

$$E_x = \begin{cases} \frac{1}{i\epsilon_0\omega} \cdot \left(-\frac{Ak_{z3}}{n_3^2}\right) \exp[-k_{z3}z] e^{i(k_x x - \omega t)} & z > 0 \\ \frac{1}{i\epsilon_0\omega} \left[\frac{iBk_{z1}}{n_1^2} e^{ik_{z1}(z+d)} - \frac{iCk_{z1}}{n_1^2} e^{-ik_{z1}z} \right] e^{i(k_x x - \omega t)} & -d < z < 0 \\ \frac{1}{i\epsilon_0\omega} \cdot \frac{Dk_{z2}}{n_2^2} e^{k_{z2}(z+d)} e^{i(k_x x - \omega t)} & z < -d \end{cases}$$

利用电场、磁场强度的切向分量连续的边界条件：

$$\begin{cases} A = Be^{ik_{z1}d} + C \\ B + Ce^{ik_{z1}d} = D \\ -\frac{Ak_{z3}}{n_3^2} = \frac{iBk_{z1}}{n_1^2} e^{ik_{z1}d} - \frac{iCk_{z1}}{n_1^2} \\ \frac{iBk_{z1}}{n_1^2} - \frac{iCk_{z1}}{n_1^2} e^{ik_{z1}d} = \frac{Dk_{z2}}{n_2^2} \end{cases}$$

A、B、C、D存在非零解的条件是系数行列式为零。

$$\begin{cases} A = Be^{ik_{z1}d} + C \\ B + Ce^{ik_{z1}d} = D \\ -\frac{Ak_{z3}}{n_3^2} = \frac{iBk_{z1}}{n_1^2} e^{ik_{z1}d} - \frac{iCk_{z1}}{n_1^2} \\ \frac{iBk_{z1}}{n_1^2} - \frac{iCk_{z1}}{n_1^2} e^{ik_{z1}d} = \frac{Dk_{z2}}{n_2^2} \end{cases}$$



$$\tan(k_{z1}d) = \frac{\frac{k_{z1}}{n_1^2} \left(\frac{k_{z2}}{n_2^2} + \frac{k_{z3}}{n_3^2} \right)}{\left(\frac{k_{z1}}{n_1^2} \right)^2 - \frac{k_{z2}}{n_2^2} \frac{k_{z3}}{n_3^2}}$$

平面波导的TM模式方程

$$\tan(k_{z1}d) = \frac{\frac{k_{z1}}{n_1^2} \left(\frac{k_{z2}}{n_2^2} + \frac{k_{z3}}{n_3^2} \right)}{\left(\frac{k_{z1}}{n_1^2} \right)^2 - \frac{k_{z2}}{n_2^2} \frac{k_{z3}}{n_3^2}} = \frac{\frac{n_1^2}{n_2^2} \frac{k_{z2}}{k_{z1}} + \frac{n_1^2}{n_3^2} \frac{k_{z3}}{k_{z1}}}{1 - \left(\frac{n_1^2}{n_2^2} \frac{k_{z2}}{k_{z1}} \right) \cdot \left(\frac{n_1^2}{n_3^2} \frac{k_{z3}}{k_{z1}} \right)}$$

TM模式方程

$$k_{z1}d = \arctan\left(\frac{n_1^2}{n_2^2} \frac{k_{z2}}{k_{z1}}\right) + \arctan\left(\frac{n_1^2}{n_3^2} \frac{k_{z3}}{k_{z1}}\right) + m\pi \quad (m \in Z)$$

导模截止条件： $k_x = n_2 k_0$

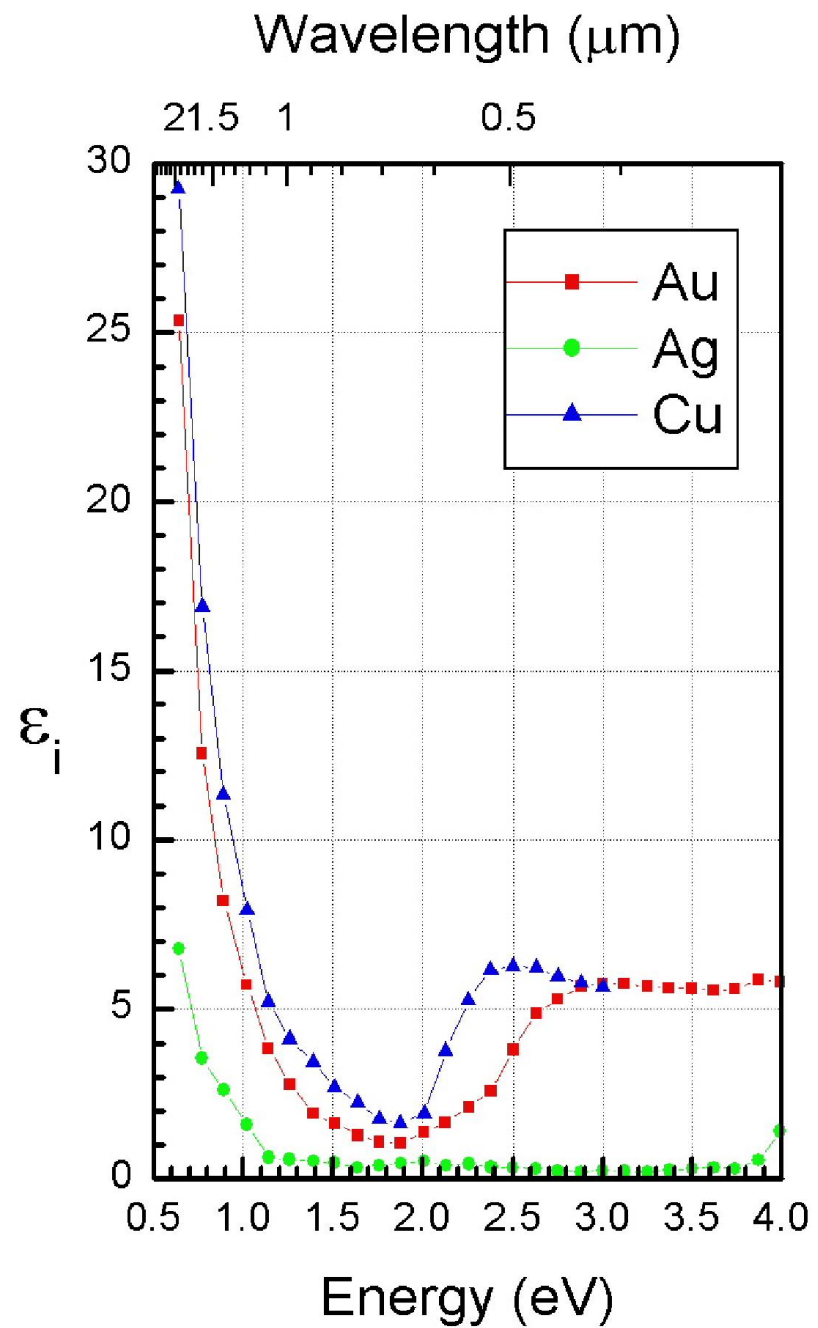
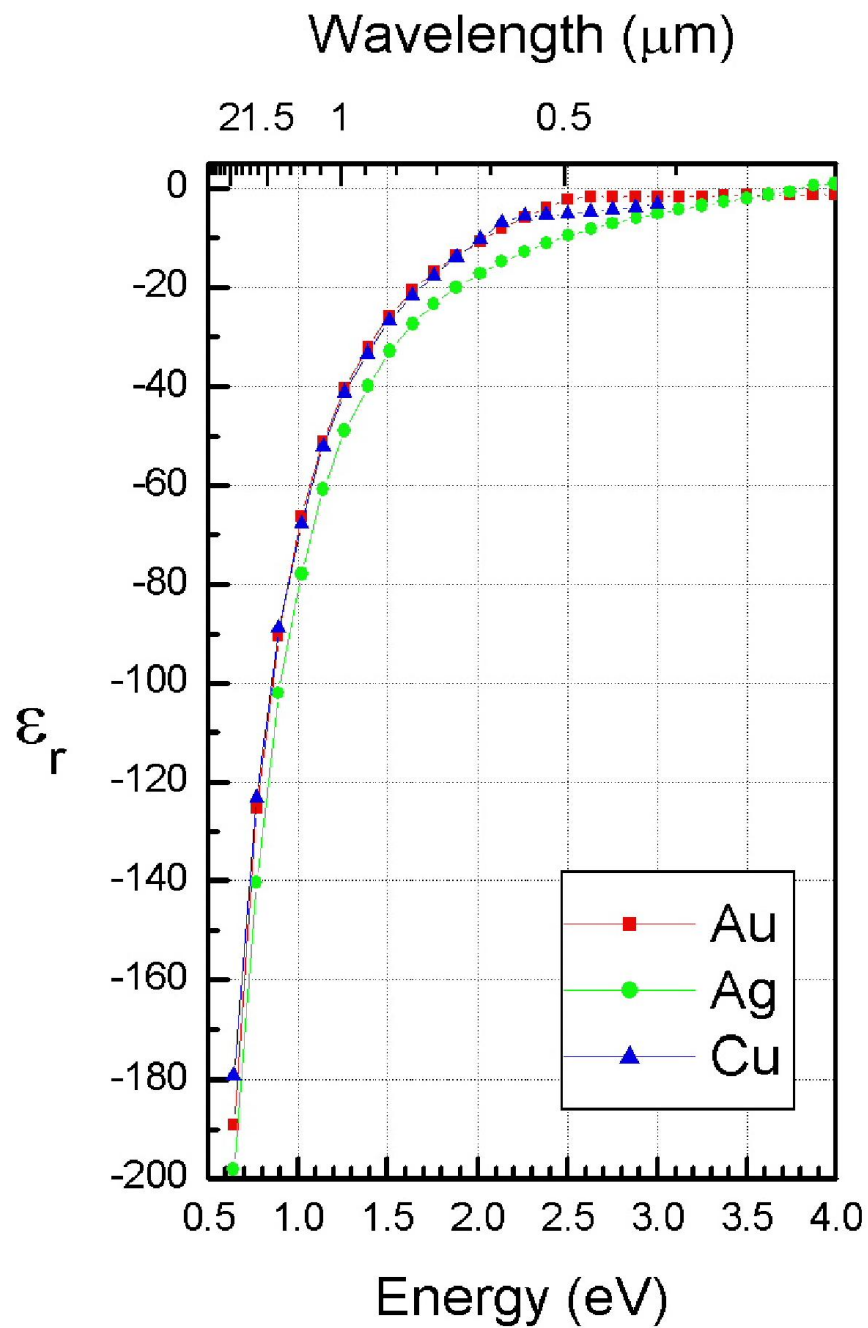
截止波长与厚度的关系：

$$d_{TM} = \frac{\lambda}{2\pi\sqrt{n_1^2 - n_2^2}} \left[\arctan\left(\frac{n_1^2}{n_3^2} \frac{\sqrt{n_2^2 - n_3^2}}{\sqrt{n_1^2 - n_2^2}}\right) + m\pi \right]$$

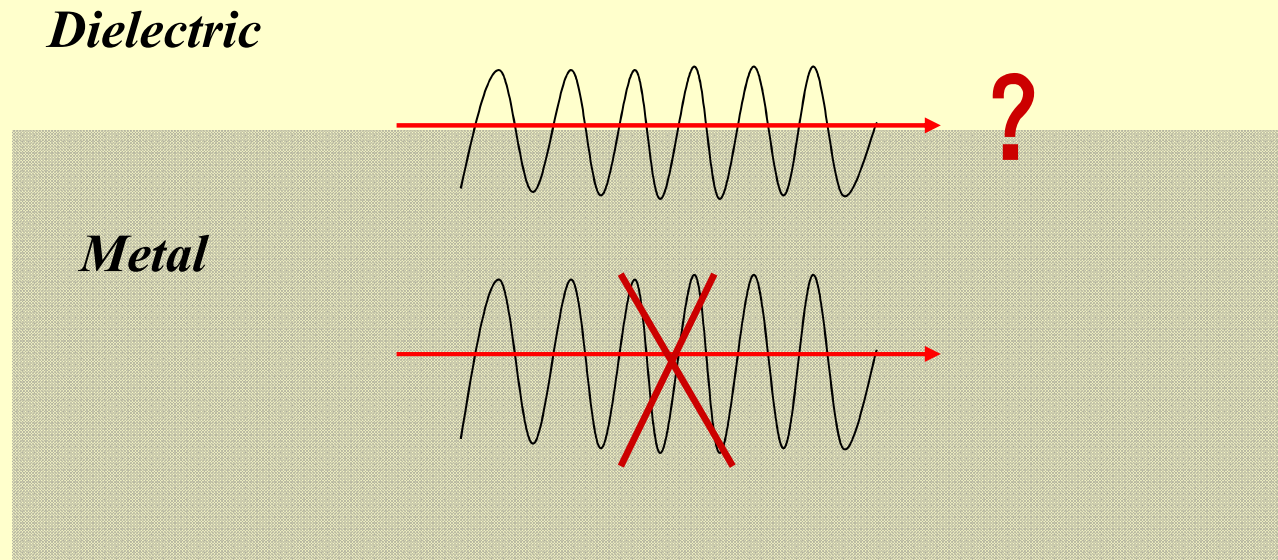
二、采用贵金属/电介质构成的SPP波导

- 在微波范围内，金属可以看作是理想的导体；
- 在光频范围内，金属不能够再堪看成是理想导体；
- 在光频范围内，金属的介电性质由基于自由电子气模型的Drude公式描述；
- 对于贵金属而言，在可见光/近红外区的损耗相对较小，但损耗仍远高于绝缘的透明介质！
- 这里的波导是SPP波导，传播的模式是SPP,全名是Surface plasmon（中文译成表面等离子激元）
- SP是光与金属中的自由电子耦合之后，形成的一种集体振荡模式。所以它具有光的特性，又受到电子电荷分布控制！

- 对于SPP波导，在平整的金属 / 介质分界面上，只支持TM模式的波导；
- SPP波导也是在可见 / 近红外波段（思考）。

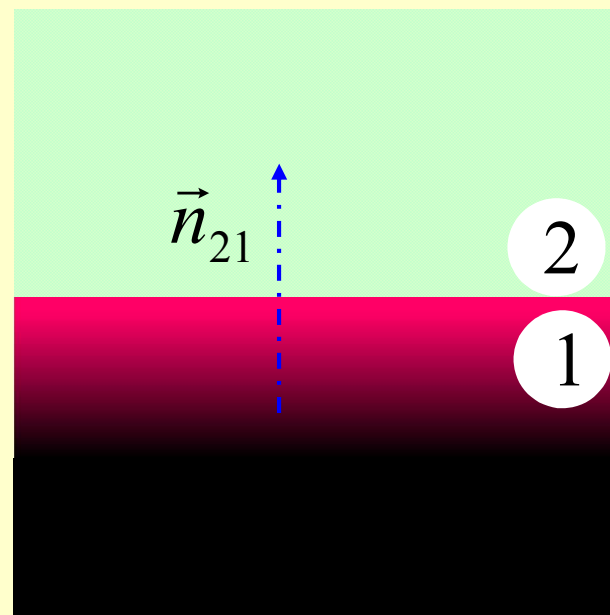


问题的提出：在金属/介质分界面上，电磁波可以沿着界面传播吗？即金属-介质表面是否存在着本征传播的表面模？



对于一般频率 ($\omega < \omega_p$) 的电磁波，不可以在金属内部传播。

- ✧ 对于导体，当附近的介质中存在电磁场时，其表面以下的穿透深度内存在一定的电流分布；
除非是理想导体，否则面电流密度可看成为零。



$$\begin{array}{l}
 \vec{n}_{21} \cdot (\vec{D}_2 - \vec{D}_1) = 0 \\
 \vec{n}_{21} \times (\vec{E}_2 - \vec{E}_1) = 0 \\
 \vec{n}_{21} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \\
 \vec{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = 0
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l}
 \vec{n}_{21} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_f \\
 \vec{n}_{21} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \\
 \vec{n}_{21} \times (\vec{E}_2 - \vec{E}_1) = 0 \\
 \vec{n}_{21} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}_f
 \end{array}$$

1、TM模

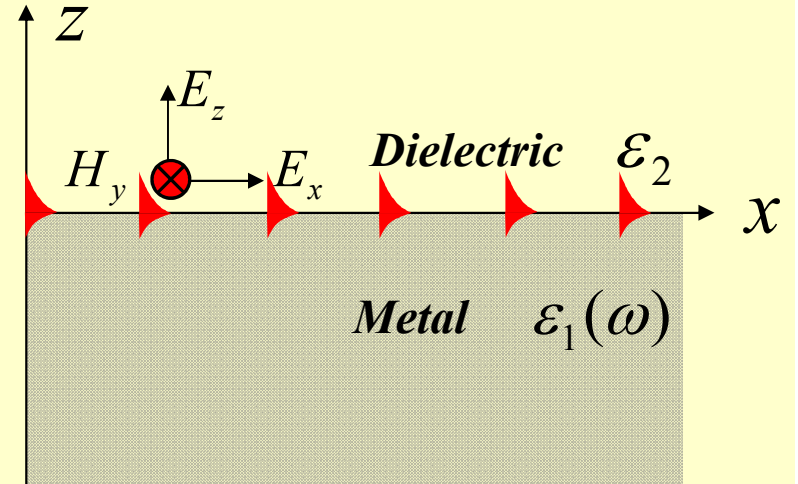
$$\vec{H} = \begin{cases} (0, A, 0) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ (0, B, 0) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$



$$\nabla \times \vec{H} = \epsilon_0 \epsilon \frac{\partial \vec{E}}{\partial t}$$



$$\vec{E} = \begin{cases} \frac{A}{\omega \epsilon_0 \epsilon_2} (k_{z2}, 0, -k_x) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ \frac{B}{\omega \epsilon_0 \epsilon_1} (k_{z1}, 0, -k_x) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$

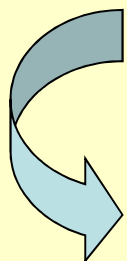


利用连续性边界条件：

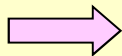
$$\vec{H} = \begin{cases} (0, A, 0) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ (0, B, 0) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$

$$\begin{cases} H_{y1} = H_{y2} \\ E_{x1} = E_{x2} \end{cases}$$

$$\vec{E} = \begin{cases} \frac{A}{\omega \epsilon_0 \epsilon_2} (k_{z2}, 0, -k_x) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ \frac{B}{\omega \epsilon_0 \epsilon_1} (k_{z1}, 0, -k_x) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$



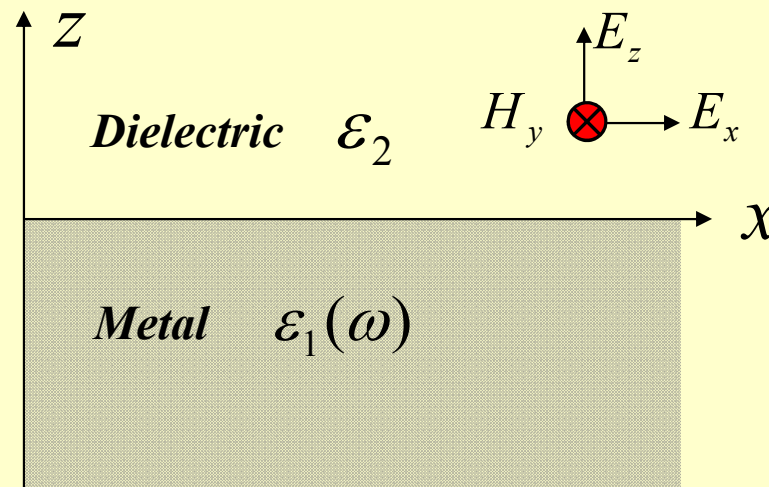
$$\begin{cases} A = B \\ \frac{A k_{z2}}{\epsilon_2} = \frac{B k_{z1}}{\epsilon_1} \end{cases}$$



$$\frac{k_{z2}}{\epsilon_2} = \frac{k_{z1}}{\epsilon_1}$$

同时

$$\epsilon_i \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2 \quad (i = 1, 2)$$



$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2}$$

(自行验证)

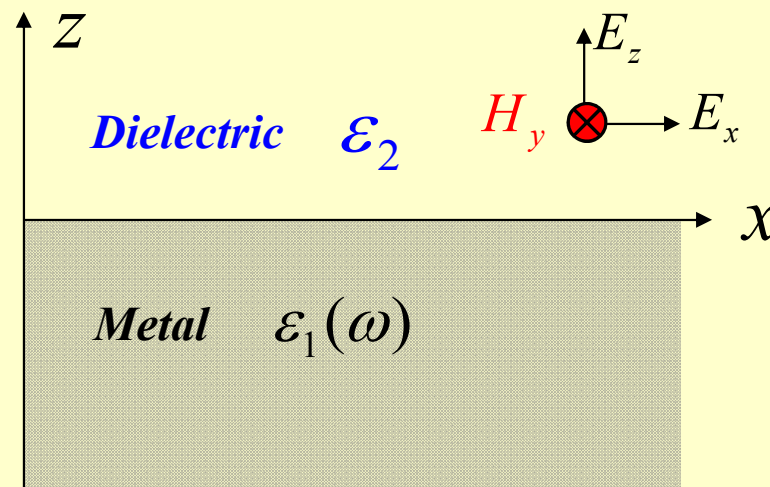
可见光区，金属介电常数—Drude模型：

$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2}$$

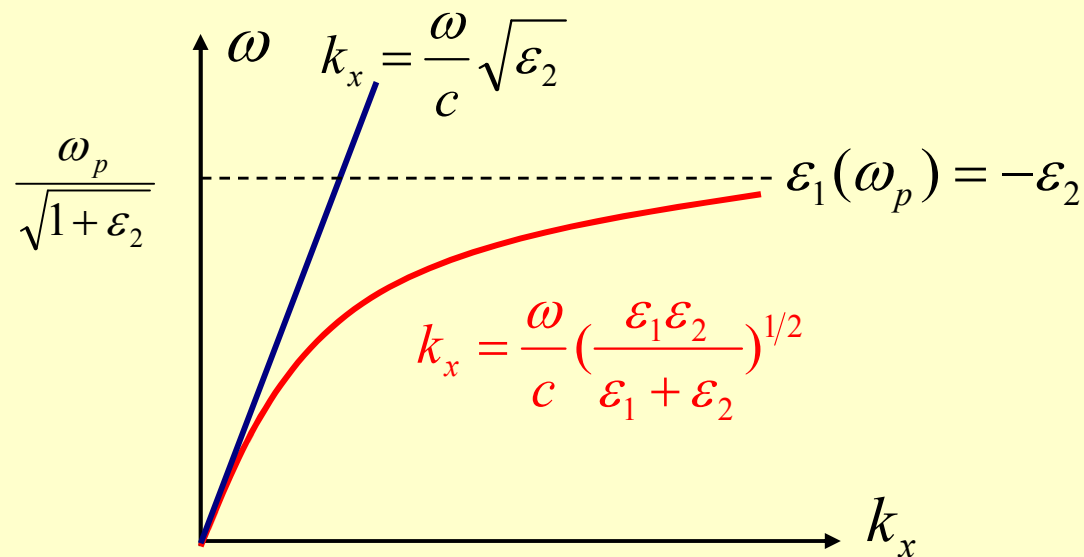
$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\omega_\tau}$$

A、暂忽略金属吸收 ($\omega_\tau=0$) :

$$\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$



$\omega \sim k_x$ 色散关系曲线图：

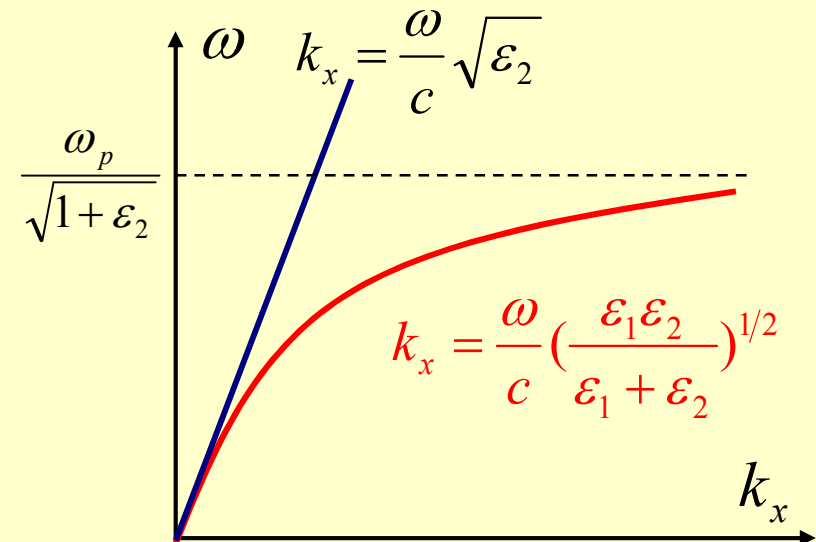


$$\vec{E}, \vec{H} \propto \begin{cases} \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$

$$\left. \begin{aligned} \varepsilon_i \left(\frac{\omega}{c}\right)^2 &= k_x^2 + k_{zi}^2 \quad (i=1,2) \\ 0 < \omega < \frac{\omega_p}{\sqrt{1+\varepsilon_2}}, \quad k_x > \frac{\omega}{c} \sqrt{\varepsilon_2} \end{aligned} \right\}$$

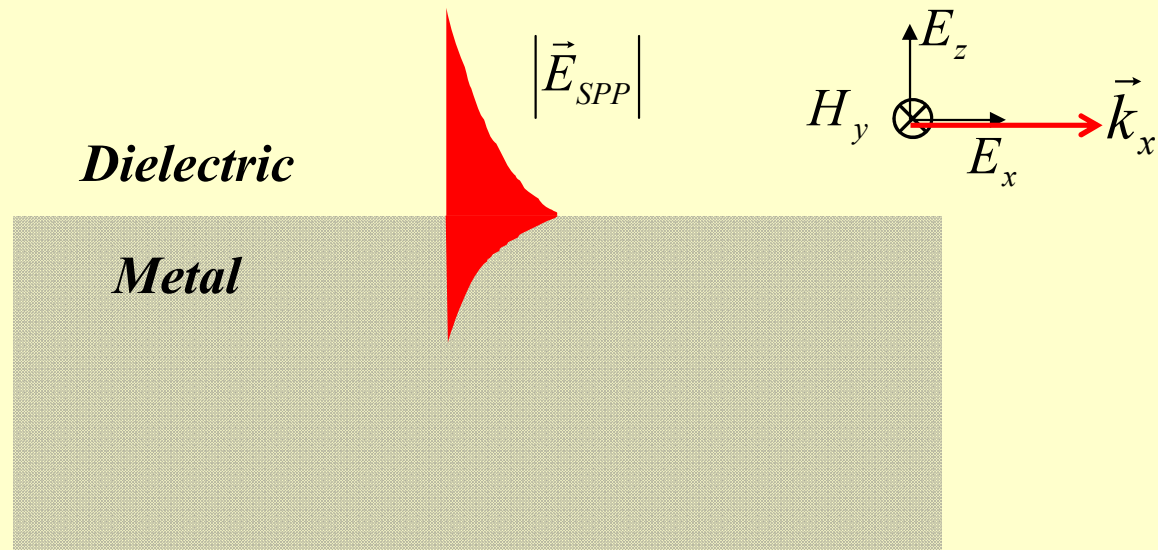
→ $k_{zi}^2 < 0$, k_{zi} 为虚数

- 电磁场沿着金属-介质界面传播，形成表面波；
- 在界面处为最大值，在金属和介质内部沿z方向衰减，具有高度的局域特点。



$\omega \sim k_x$ 色散关系曲线图

例如： Ag: $\lambda = 600 \text{ nm}$, $z_2 = 390 \text{ nm}$, $z_1 = 24 \text{ nm}$



具有这种性质的电磁场称为表面等离子激元
(Surface Plasmon Polaritons, SPP, SP)

B、实际的金属存在损耗

$$\epsilon_1 = \epsilon_1' + i\epsilon_1'' \quad (\epsilon_1'' \ll |\epsilon_1'|)$$

$$k_x = k_x' + ik_x''$$

$$\begin{cases} k_x' = \frac{\omega}{c} \left(\frac{\epsilon_1' \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2} \\ k_x'' = \frac{\omega}{c} \left(\frac{\epsilon_1' \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{3/2} \cdot \frac{\epsilon_1''}{2(\epsilon_1')^2} \end{cases}$$

$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2}$$

$$|\vec{E}_{SPP}| \propto \exp(ik_x x) = \exp(-k_x'' x) \cdot \exp(ik_x' x)$$

$$I_{SPP} = |\vec{E}_{SPP}|^2 \propto \exp(-2k_x'' x) \rightarrow \text{SPP损耗}$$

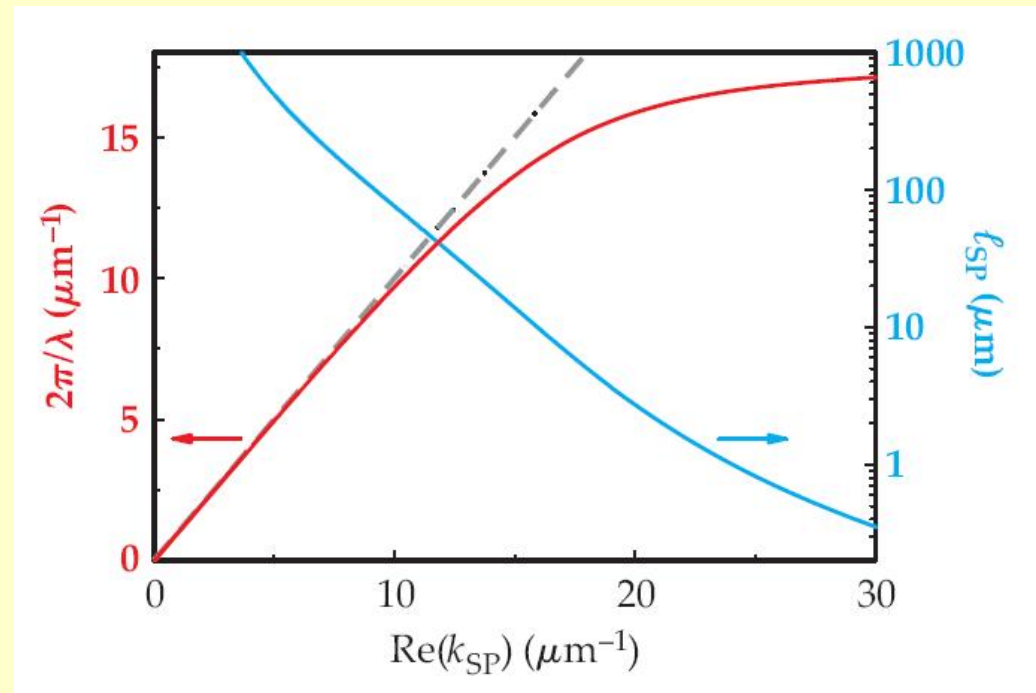
$$\text{SPP传播的距离: } L_{SPP} = (2k_x'')^{-1}$$

$$\text{例如: Ag} \quad \lambda = 514nm \quad L_{SPP} = 22\mu m$$

$$\lambda = 1060nm \quad L_{SPP} = 500\mu m$$

$$k_{sp} = \frac{2\pi}{\lambda} \sqrt{\frac{\epsilon_d \epsilon_m}{\epsilon_d + \epsilon_m}}$$

$$L_{spp} = \frac{2}{[\text{Re}(k_{sp})]^3} \frac{[2\pi \text{Re}(\epsilon_m)]^2}{\lambda^2 \text{Im}(\epsilon_m)}$$



对于SPP而言，最大的传播长度和最大的横向局域性是处在色散线的两端！

T. W. Ebbesen *et al.*, *Phys. Today* May 2008

2、是否存在TE模的SPP波?

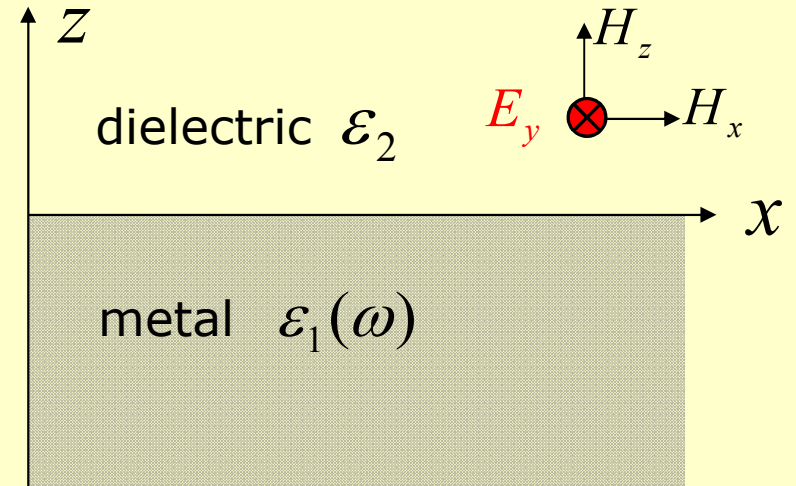
$$\vec{E} = \begin{cases} (0, A, 0) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ (0, B, 0) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$



$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (\mu \approx 1)$$



$$\vec{H} = \begin{cases} \frac{A}{\omega \mu_0} (-k_{z2}, 0, k_x) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ \frac{B}{\omega \mu_0} (-k_{z1}, 0, k_x) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$



$$\vec{E} = \begin{cases} (0, A, 0) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ (0, B, 0) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$

利用连续性边界条件：

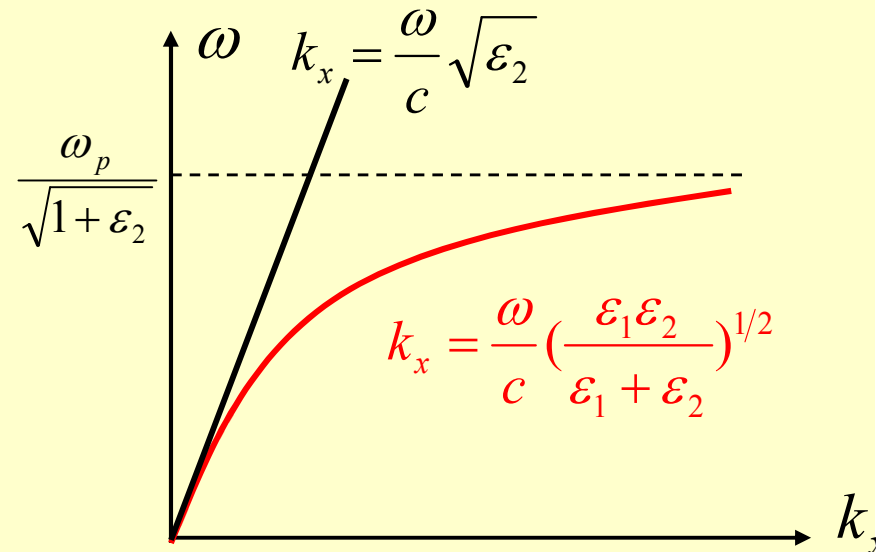
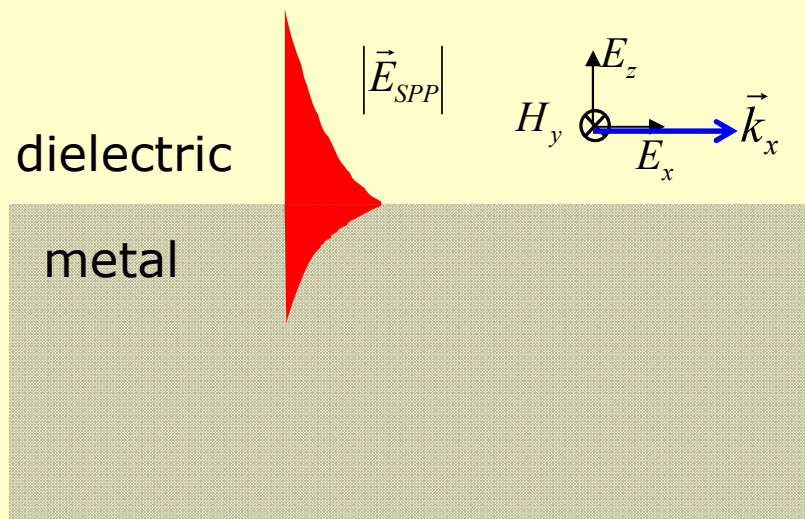
$$\vec{H} = \begin{cases} \frac{A}{\omega\mu_0} (-k_{z2}, 0, k_x) \exp[i(k_x x + k_{z2} z - \omega t)] & z > 0 \\ \frac{B}{\omega\mu_0} (-k_{z1}, 0, k_x) \exp[i(k_x x + k_{z1} z - \omega t)] & z < 0 \end{cases}$$

$$E_{x1} = E_{x2}$$

$$H_{y1} = H_{y2}$$

$$\begin{aligned} &\Rightarrow \left\{ \begin{array}{l} A = B \\ Ak_{z2} = Bk_{z1} \end{array} \right. \Rightarrow k_{z2} = k_{z1} \\ &\Rightarrow \varepsilon_i \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2, \quad k_{zi} = \left[\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2\right]^{1/2} \quad (i=1,2) \end{aligned} \quad \left. \vphantom{\begin{aligned} &\Rightarrow \left\{ \begin{array}{l} A = B \\ Ak_{z2} = Bk_{z1} \end{array} \right. \Rightarrow k_{z2} = k_{z1} \\ &\Rightarrow \varepsilon_i \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2, \quad k_{zi} = \left[\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2\right]^{1/2} \quad (i=1,2) \right\}$$

无解！



SPP主要特性：

- ① 沿着金属/介质分界面传播的TM模式（电场存在沿着波矢方向的分量）表面波；
- ② 电磁场具有很强的局域性（电场在表面局域增强）；
- ③ 由于金属存在损耗，使得SPP有限的传播距离；
- ④ 色散关系：
$$k_x = \frac{\omega}{c} \left(\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2}$$

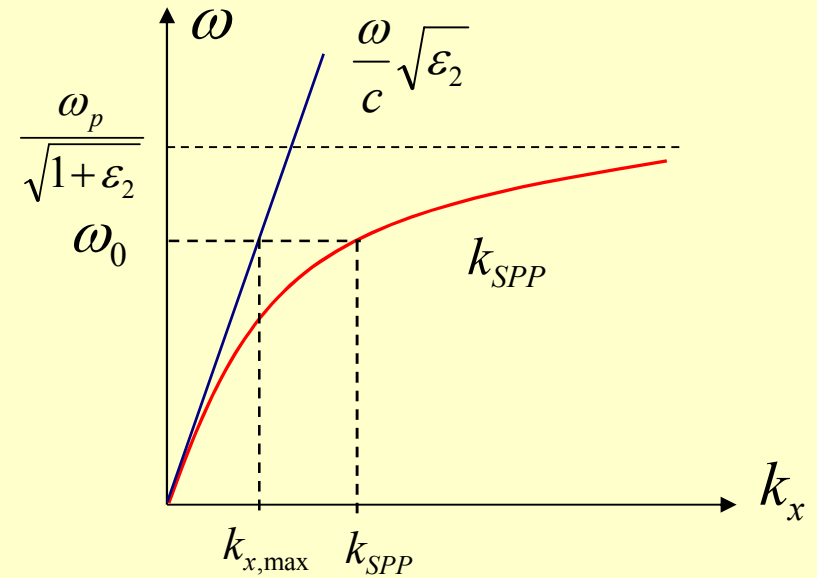
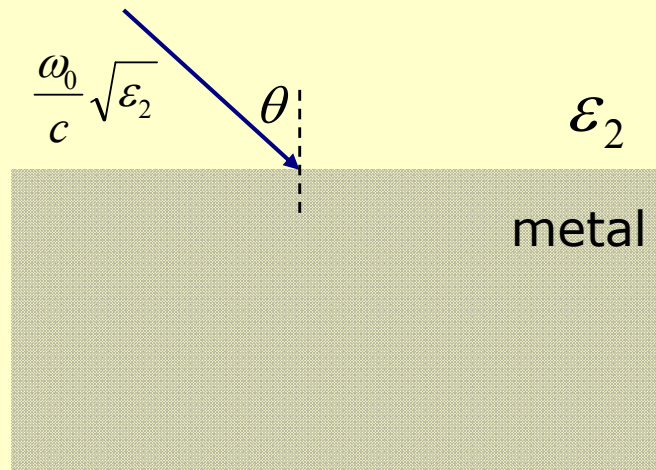
3、如何激发SPP ?

应满足条件：

- ① 频率相同
- ② 波矢相同
- ③ TM波 (P偏振)

关键问题：如何使得波矢匹配？

频率为 ω_0 的入射光



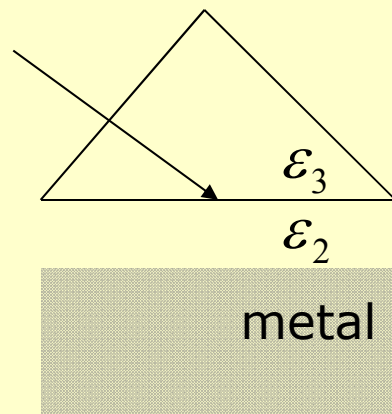
$$k_x = \frac{\omega_0}{c} \sqrt{\epsilon_2} \sin \theta,$$

$$k_{x,\max} = \frac{\omega_0}{c} \sqrt{\epsilon_2}$$

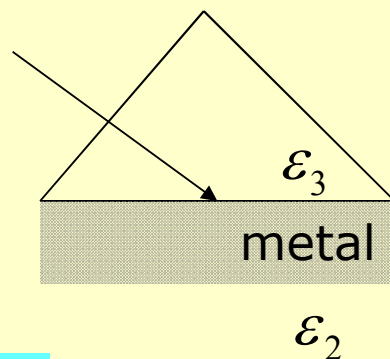
结论：平面光波直接照射金属表面不能激发SPP！

方法一：衰减全反射耦合 (attenuated total reflection, ATR)

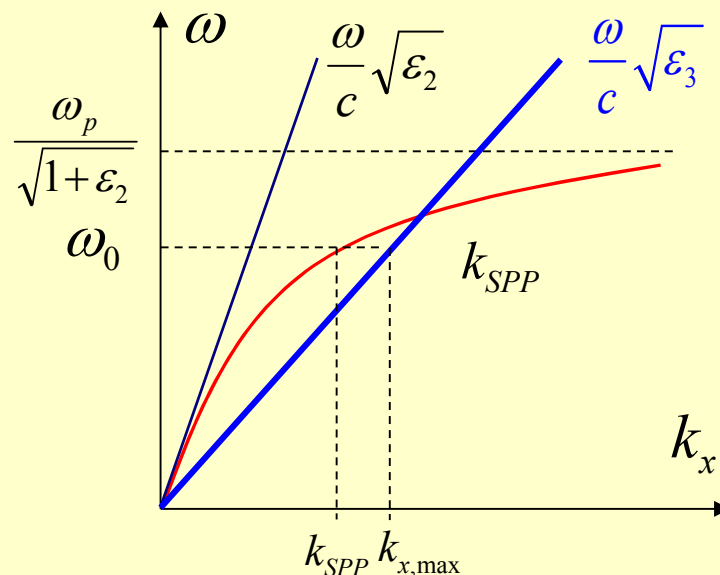
(1) Otto结构



(2) K-R结构

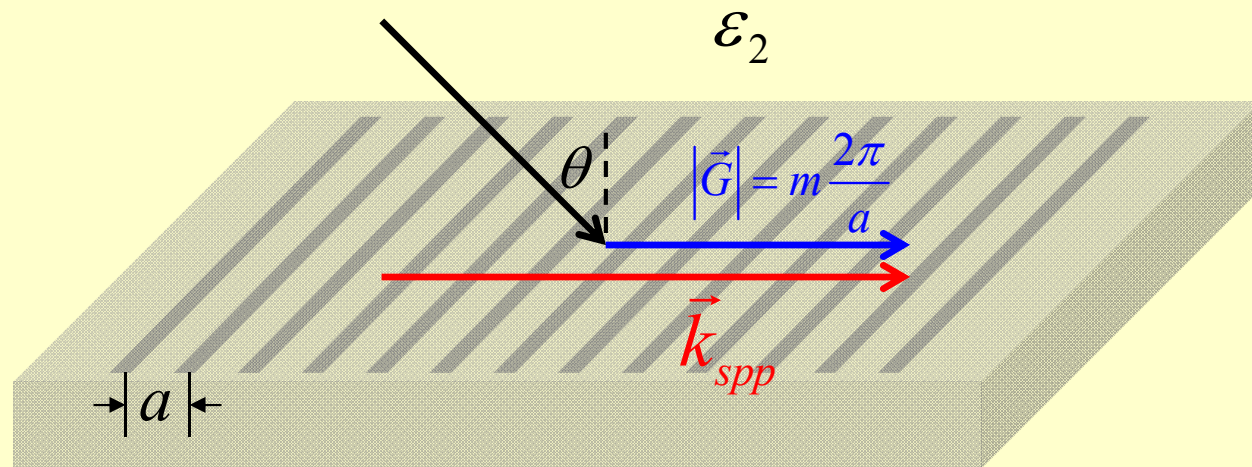


$$(\epsilon_3 > \epsilon_2)$$



由高介电常数的电介质提供较大的耦合波矢

方法二：光栅耦合



$$k_x = \frac{\omega}{c} \sqrt{\epsilon_2} \sin \theta + m \frac{2\pi}{a}$$

入射光与金属光栅相互作用，由光栅提供周期倒格矢。

5、国际上的最新进展

- 表面等离激元
 - SPP电路
 - 光学天线
 - Spaser (SPP amplification)
- 特异材料
 - 负折射材料
 - 超分辨成像
- 变换光学
 - 隐身

- **电子器件**：尺寸小、集成度高，带宽窄；
- **光学电路**：带宽大，集成度低；
- 利用**表面等离子激元**的物理特性和空间强烈的局域特性，采用亚波长结构而获得等离子激元器件，有可能把光子和电子融合在纳米尺度上，从而**实现在纳米尺度对光子的路由、操纵，甚至光器件的微型与集成**。

W.L.Barnes et al., *Nature* 424, 824 (2003).

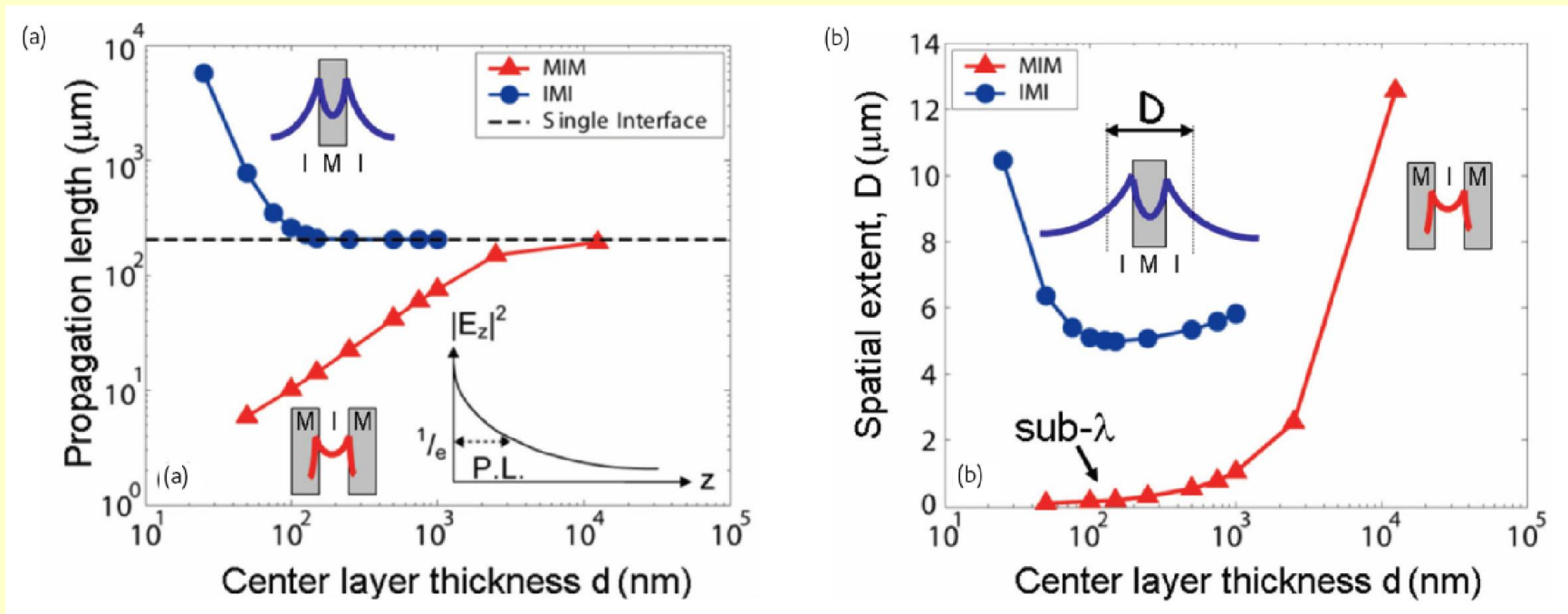
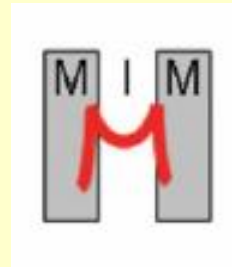
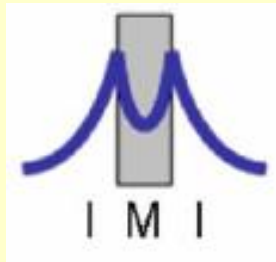
E.Ozbay, *Science* 311, 189 (2006).

M. L. Brongersma et al., *Mater. Today* 9, 20 (2006).

C.Genet et al., *Nature* 445, 39 (2007).

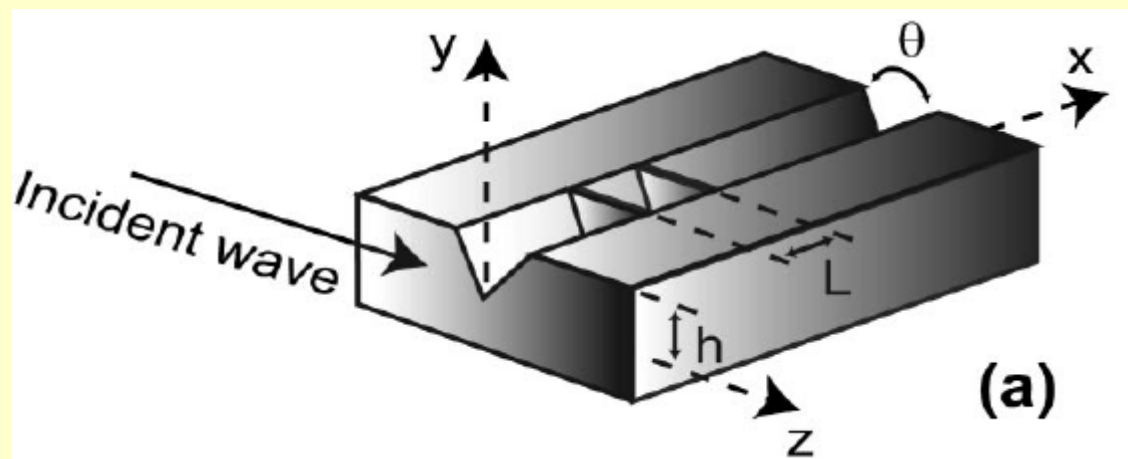
SPP波导:

➤介电/金属/介电、金属/介电/金属



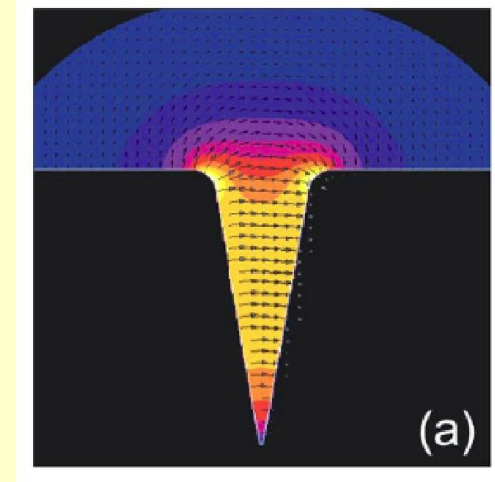
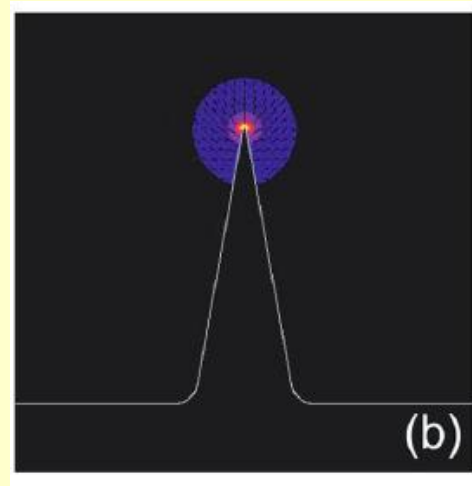
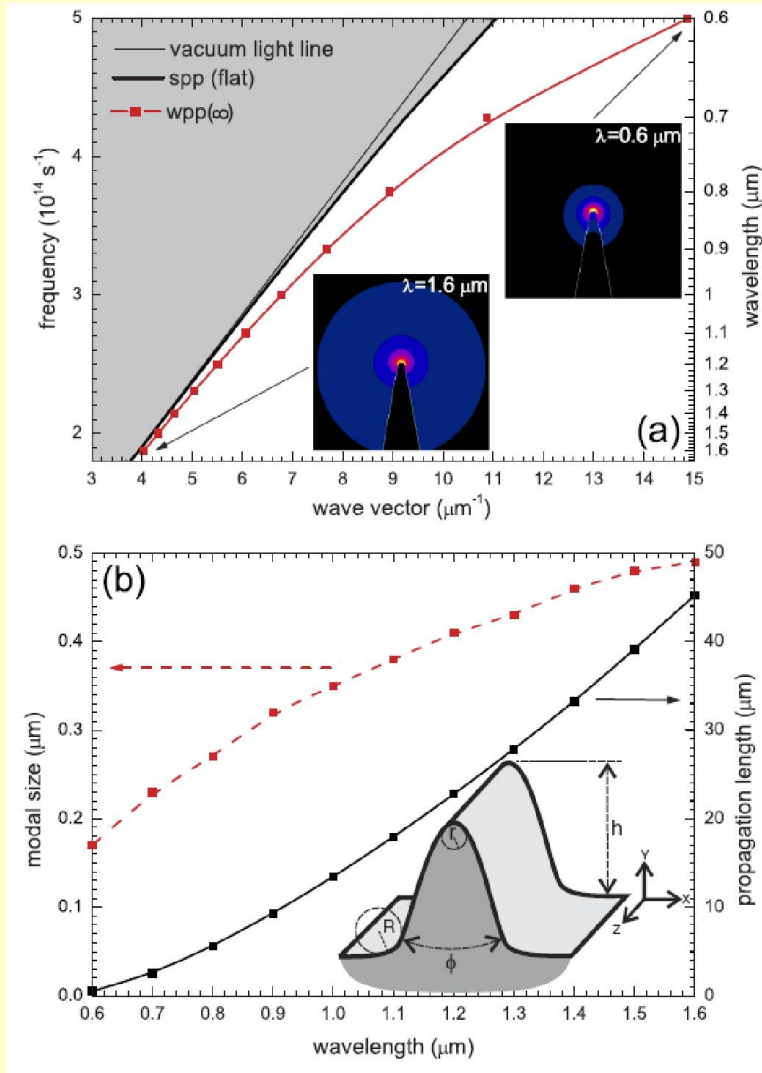
R. Zia et al., *Mater. Today*, 9, 20 (2006).

➤ V型槽金属波导



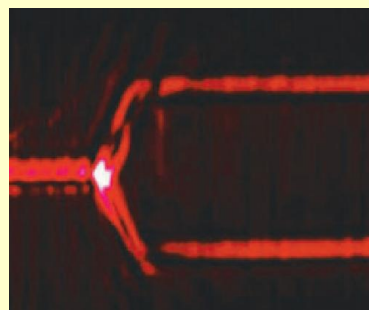
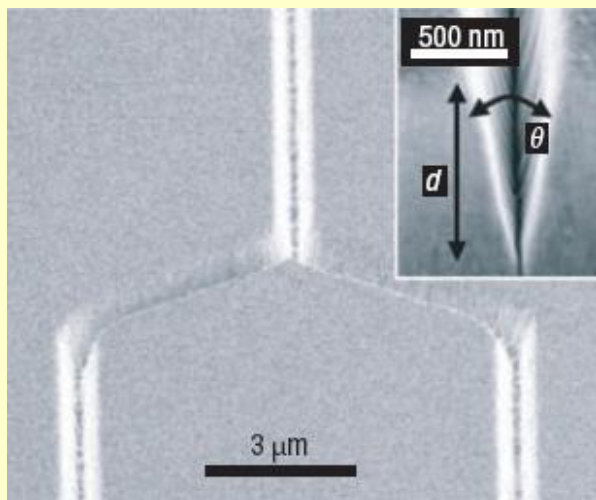
- SP沿着V型槽的底部传播，它具有亚波长的限制尺度，相对较小的传播损耗，单模工作，以及在拐弯处保持高的传输效率。
- **技术指标：**直线型的V型槽金属波导在通讯波段实现的传播距离可以到100微米，模式尺寸1.1微米；（槽宽0.6微米、槽深1.0微米）；

► 劈尖形金属波导 (理论)

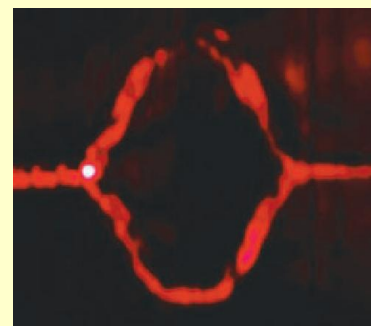


Esteban Moreno et al., PRL **100**, 023901 (2008)

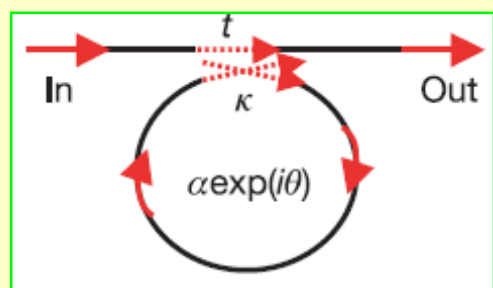
➤ 基于 V 型槽金属波导的相关SP器件



Y形波导分束器



Mach-Zehnder干涉器



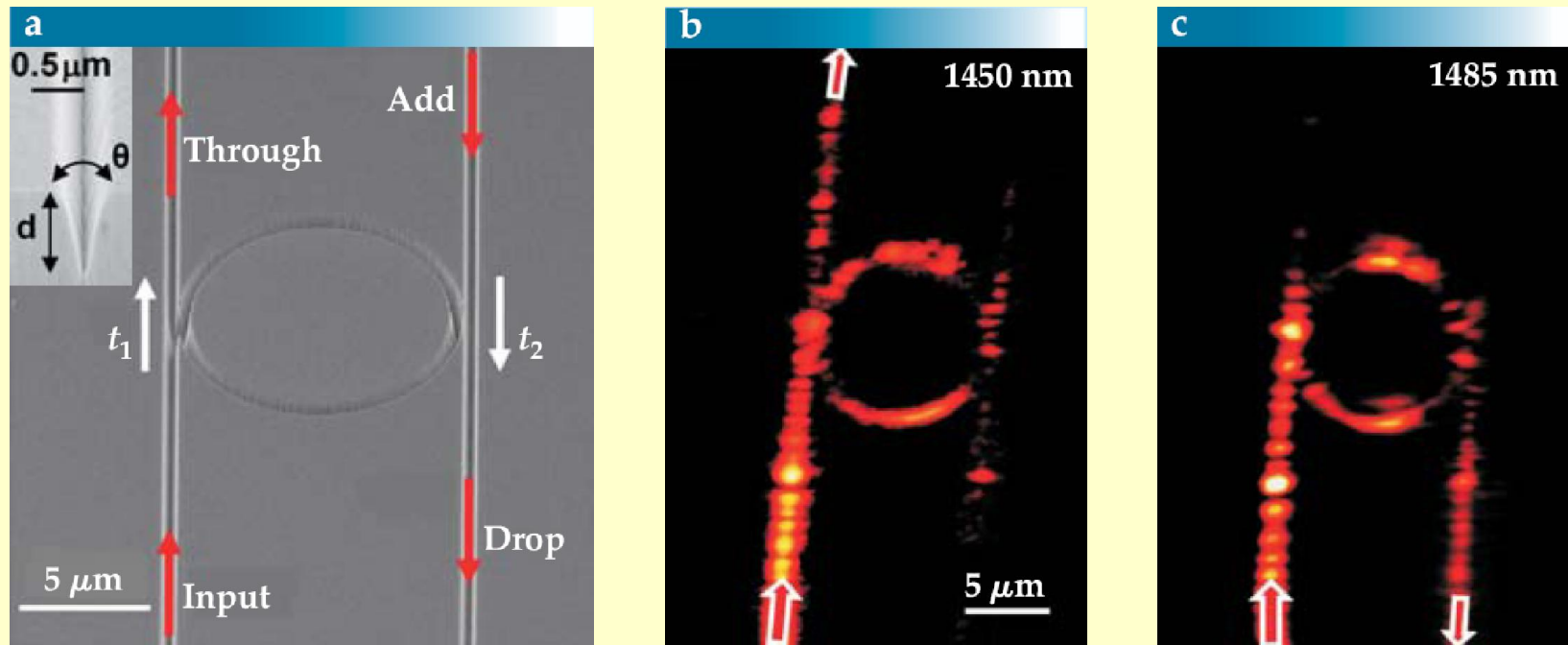
传输损耗小于3dB，尺寸在200平方微米，滤波带宽40纳米。实现了依赖于SPP波长的电磁能量传输。



波导环形共振腔

S. I. Bozhevolnyi *et al.*, *Nature*, 440, 508 (2006).

环形多通道SP波分复用器

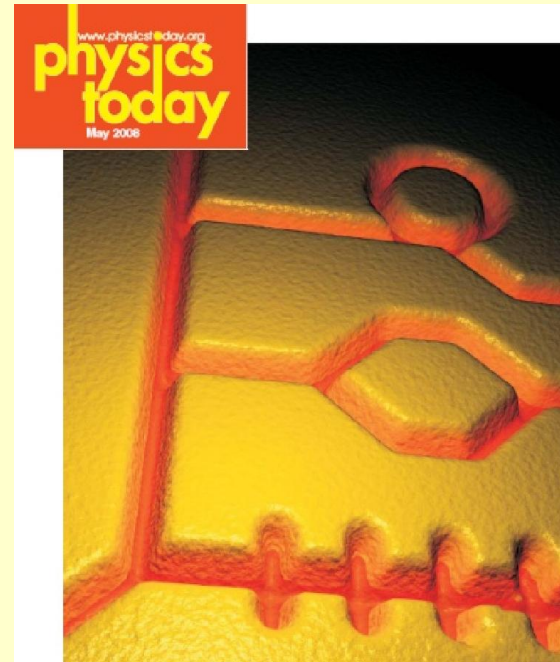


SP “上下话路器”：当1450 nm和1485 nm不同波长的SPP传播进输入端，经过环形波分复用器，会选择不同的波导通道出射。

S. I. Bozhevolnyi et al., Nano Lett. 7, 880 (2007)

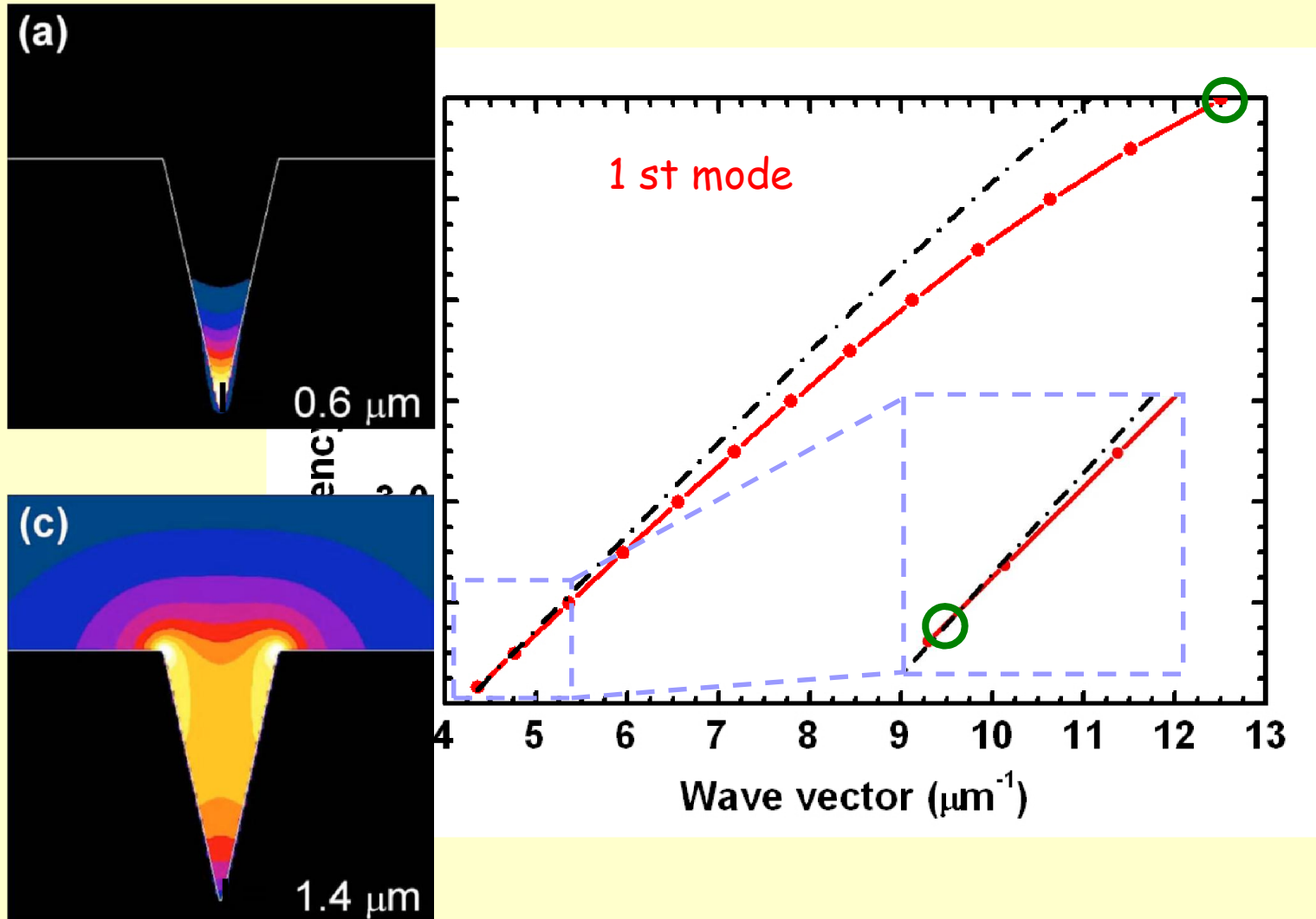
等离激元电路器件单元

- 波导器件
- 分束、干涉、环形共振腔
- 波分复用器
-

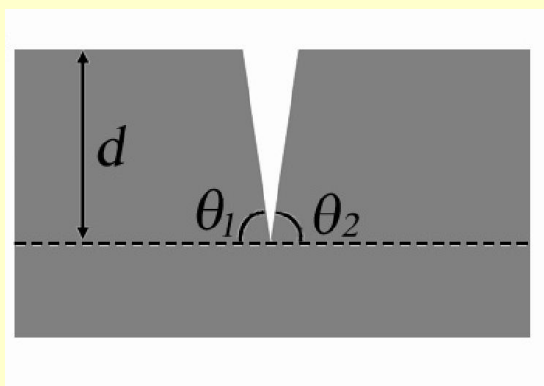


T. W. Ebbesen *et al.*, *Phys. Today* May 2008.

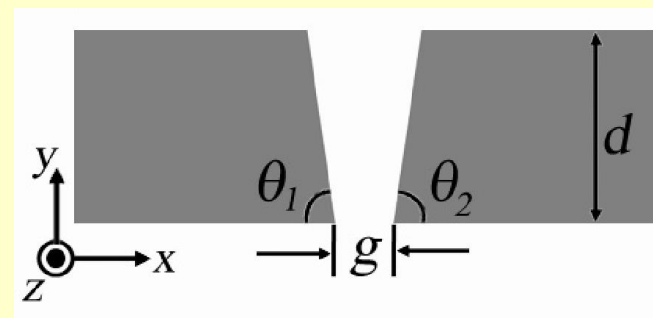
V型凹槽等离子激元波导的不足



具有梯形截面的狭缝等离子激元波导



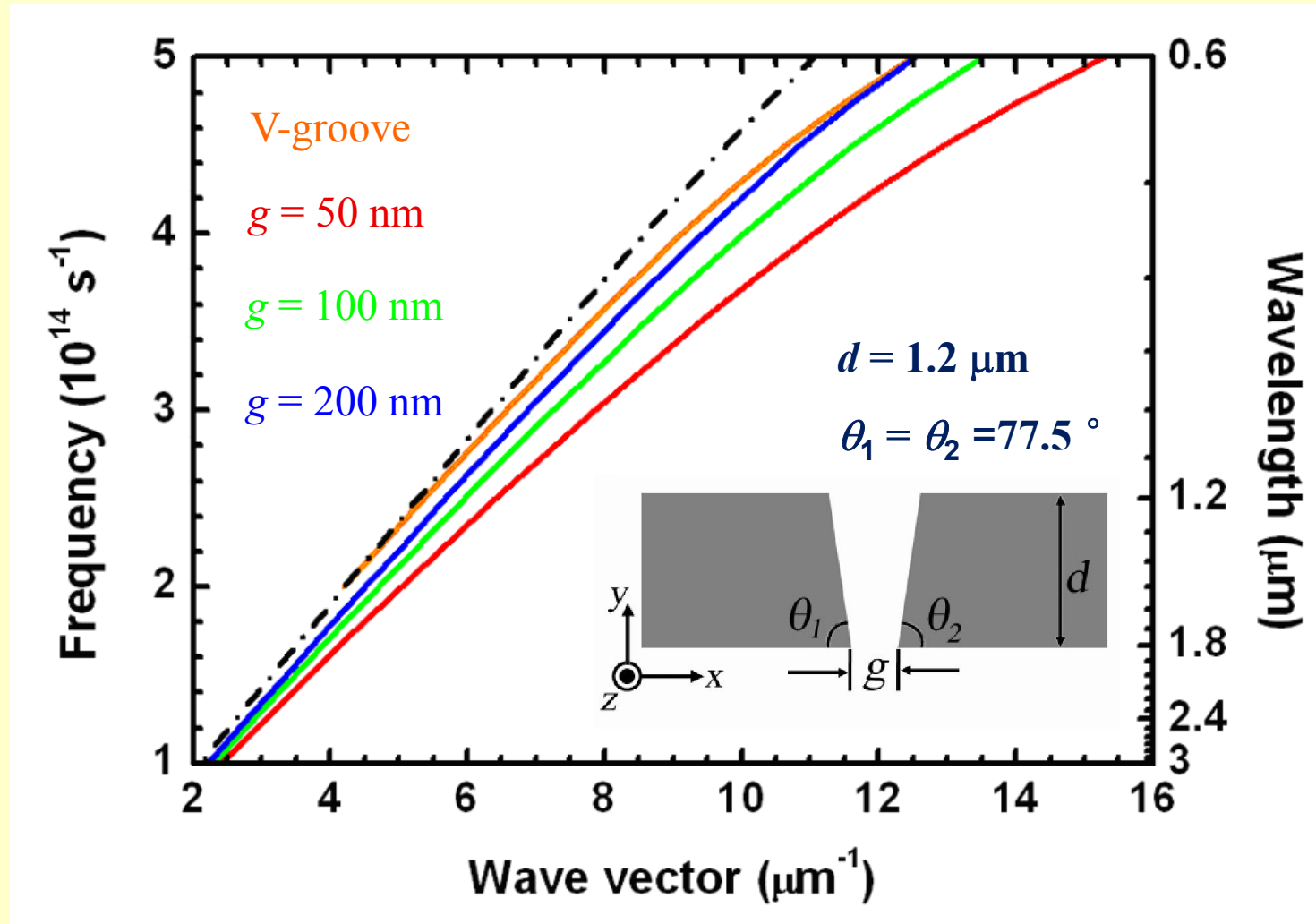
V-groove waveguide



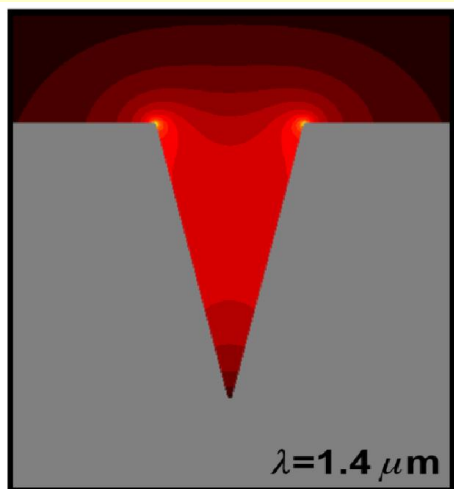
Split groove waveguide

Opt. Express 18, 16722 (2010)

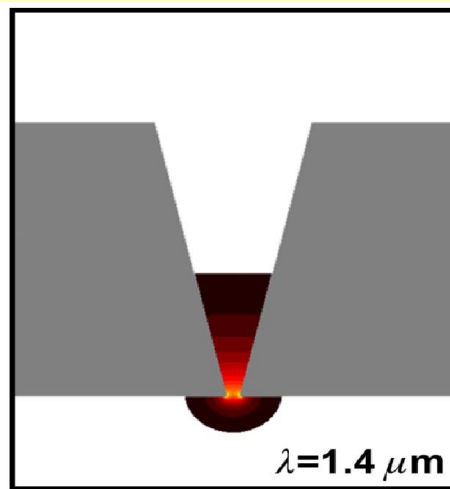
设计波导的传播特性



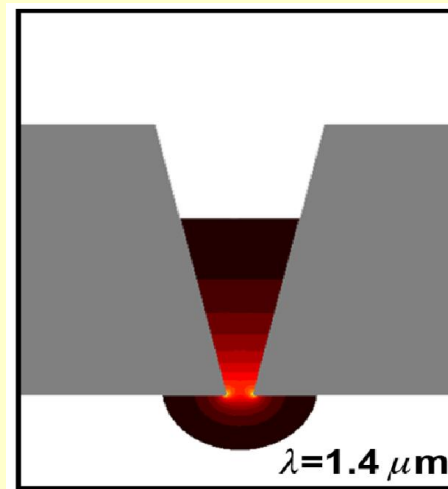
传播模式场的分布情况



V-groove



SGW $g = 50 \text{ nm}$



SGW $g = 100 \text{ nm}$

- 梯形截面狭缝等离子激元波导的传播模式场被很好地局域在纳米量级的狭缝之间；
- 梯形截面狭缝等离子激元波导的传播模式场由底端左右尖角上的劈尖等离子激元模式耦合而成。

DMD和MDM结构的SPP波导



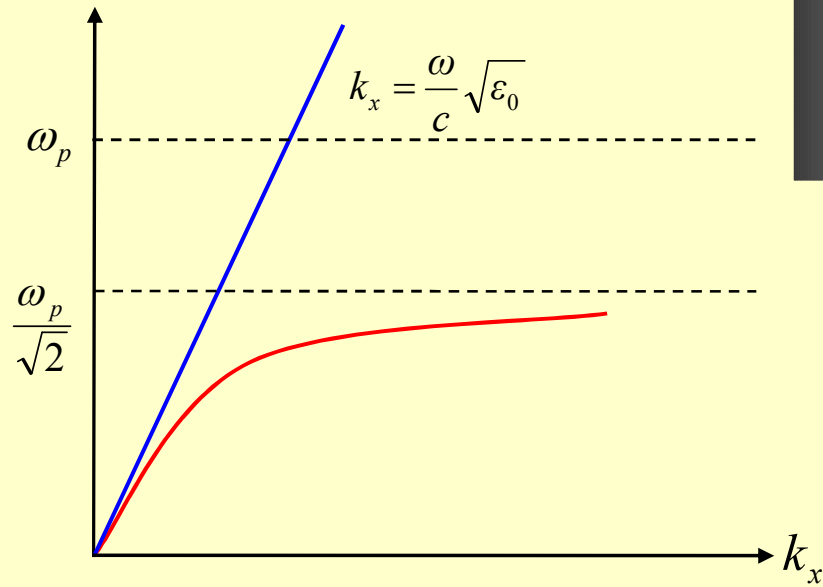
金属-介质结构

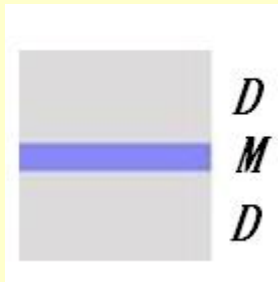


金属-介质-金属结构



介质-金属-介质结构

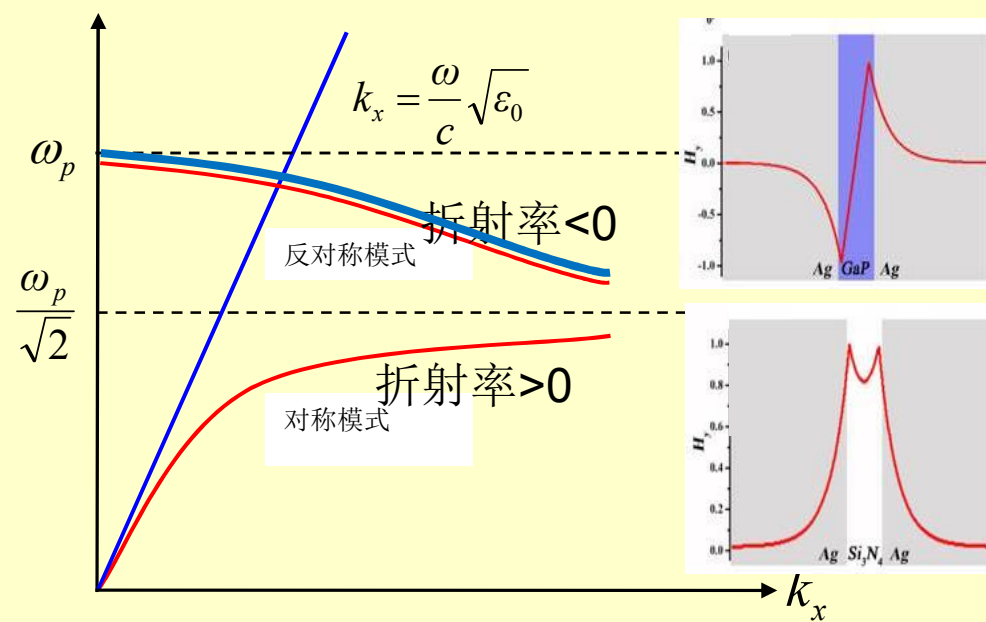
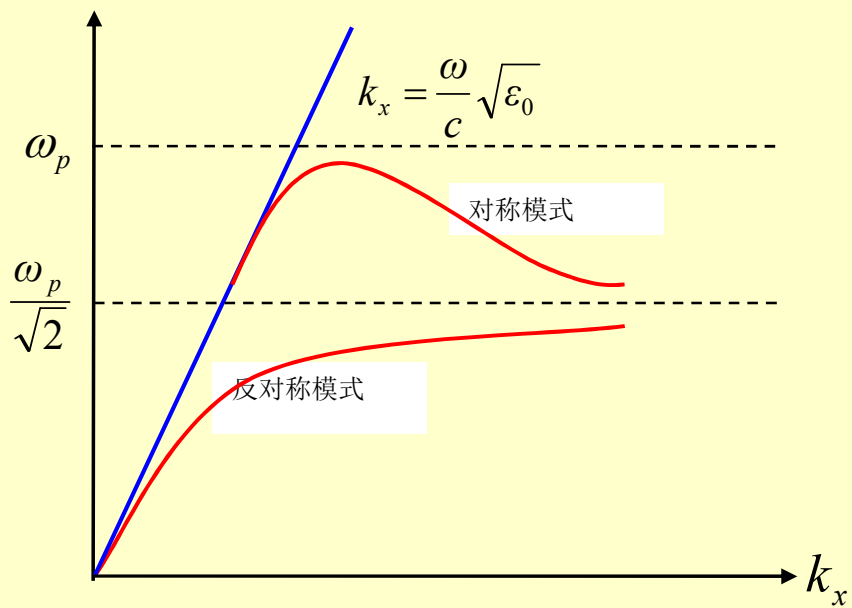




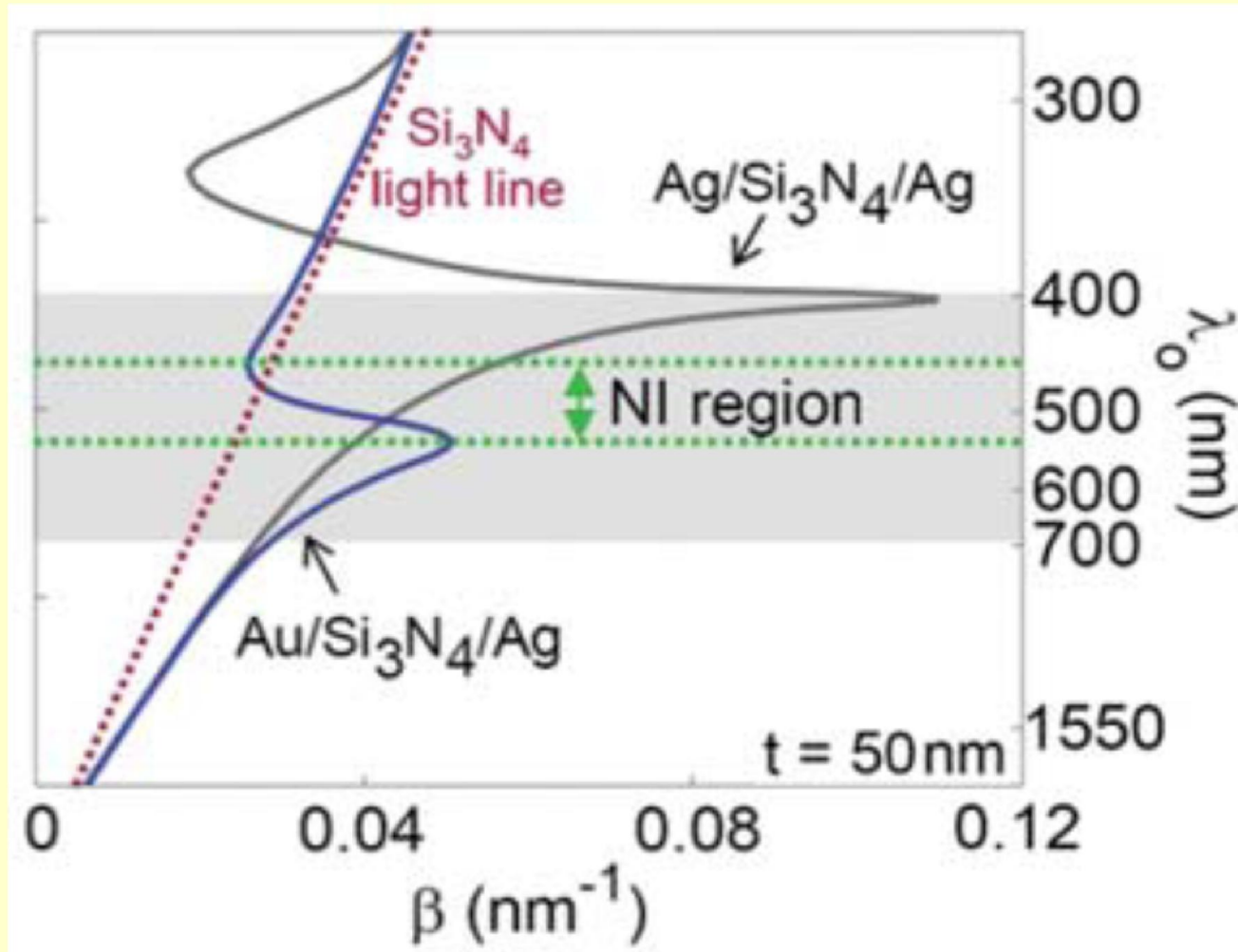
金属-介质-金属结构



介质-金属-介质结构

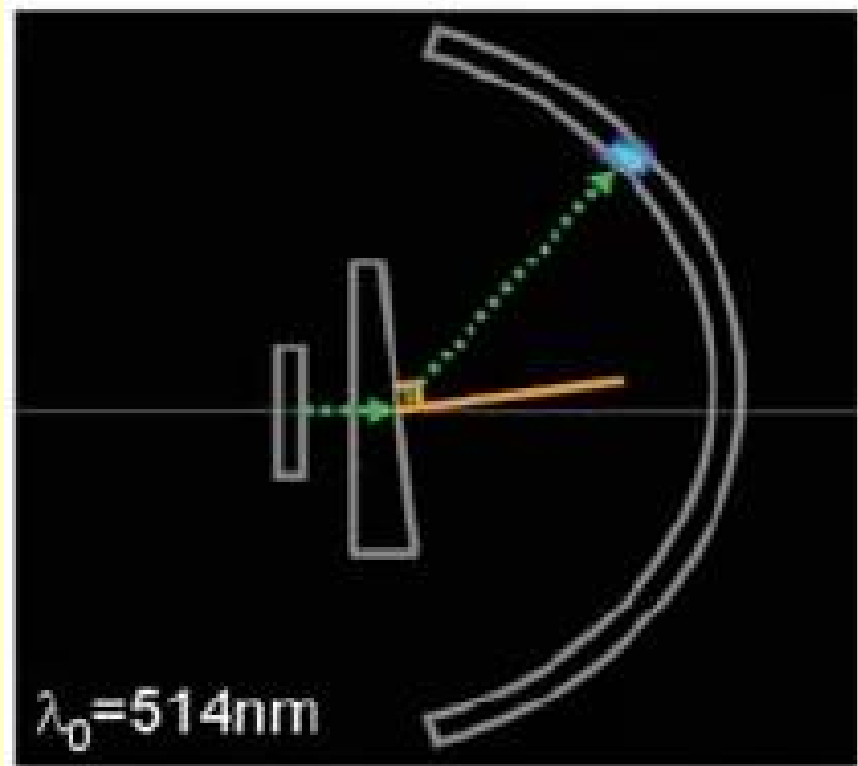
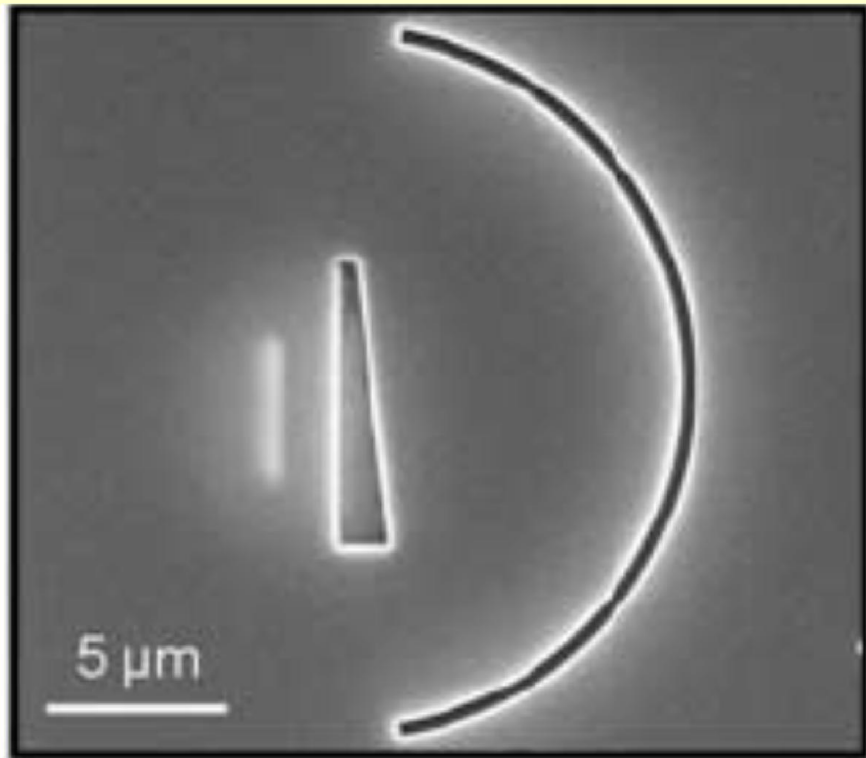


SPP的负折射现象



Science 316, 430 (2007)

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对于负折射效应，则相速度和能流方向相反！

