§4 （微波）谐振腔

谐振腔（resonator）——激发高频电磁波

① 低频电磁波一般利用LC电路组成的振荡器激发；

② 当频率升高时（例如微波段的电磁波），回路辐射损耗逐渐地增加；

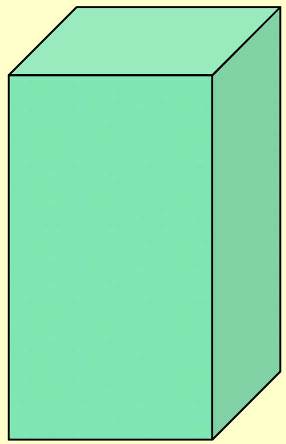
③ 高频电磁波可采用金属谐振腔来激发；

④ 相干光源－采用光学谐振腔来激发。

本节主要内容

1. 由理想金属导体构成的矩形谐振腔内的电场
2. 矩形谐振腔内的磁场
3. 矩形谐振腔的本征频率、最小本征频率

1、由理想金属导体构成的矩形谐振腔内的电场



1）谐振腔中为自由空间，时谐电磁波满足

$$\nabla^2 \vec{E}(\vec{x}) + k^2 \vec{E}(\vec{x}) = 0$$

——Helmholtz方程

$$\nabla \cdot \vec{E} = 0$$

2）理想导体与介质分界面电磁场边值关系：

① 导体表面外侧电场切向分量为零

$$\boxed{E_t = 0}$$

② 磁感应强度场的法向分量为零；

$$\boxed{B_n = 0}$$

对于波导（以TE模式为例），

$$E_z = 0, H_z(x, y) = H_0 \cos\left(\frac{m\pi}{L_1}x\right) \cos\left(\frac{m\pi}{L_2}y\right)$$

考虑到电磁场的传播因子 $e^{i(k_z z - \omega t)}$

$$H_z(x, y) = H_0 \cos\left(\frac{m\pi}{L_1}x\right) \cos\left(\frac{m\pi}{L_2}y\right) e^{i(k_z z - \omega t)}$$

对于谐振腔，第一中处理方法是表示成两个反向的波导模式的叠加：

$$H_z(x, y) = H_0 \cos\left(\frac{m\pi}{L_1}x\right) \cos\left(\frac{m\pi}{L_2}y\right) e^{i(k_z z - \omega t)} + H_0' \cos\left(\frac{m\pi}{L_1}x\right) \cos\left(\frac{m\pi}{L_2}y\right) e^{i(-k_z z - \omega t)}$$

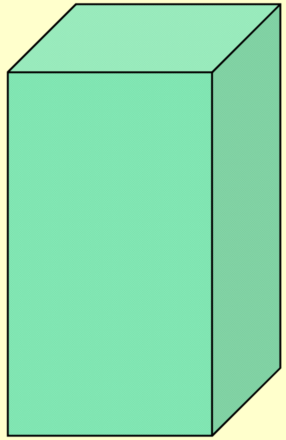
$$\nabla^2 \vec{E}(\vec{x}) + k^2 \vec{E}(\vec{x}) = 0$$

第二种处理方法：将谐振腔电场的三个分量都仿照波导中横向边界的处理：

$$\nabla^2 E_x(\vec{x}) + k^2 E_x(\vec{x}) = 0,$$

$$\nabla^2 E_y(\vec{x}) + k^2 E_y(\vec{x}) = 0,$$

$$\nabla^2 E_z(\vec{x}) + k^2 E_z(\vec{x}) = 0,$$



统一形式：$\boxed{\nabla^2 u(\vec{x}) + k^2 u(\vec{x}) = 0}$

$$\nabla^2 u(\vec{x}) + k^2 u(\vec{x}) = 0$$

2）采用分离变量法求解电场的任意一个分量

$$u(\vec{x}) = X(x)Y(y)Z(z)$$

$$\frac{\mathrm{d}^2 X}{\mathrm{d}x^2} + k_x^2 X = 0,$$

$$\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} + k_y^2 Y = 0,$$

$$\frac{\mathrm{d}^2 Z}{\mathrm{d}z^2} + k_z^2 Z = 0,$$

$$\left(k_x^2 + k_y^2 + k_z^2 = k^2\right)$$

$$\boxed{C\cos(kx) + D\sin(kx)}$$

特解

理想导体与介质分界面电磁场边值关系：

① 导体表面外侧电场切向分量为零

$$\boxed{E_t = 0}$$

$$\boxed{\nabla \cdot \vec{E} = 0 \rightarrow \frac{\partial E_n}{\partial n} = 0}$$

② 磁感应强度场的法向分量为零；

$$\boxed{B_n = 0}$$

3）电场任一分量的特解形式为

$$u(\vec{x})=\left[C_1\cos(k_x x)+D_1\sin(k_x x)\right]\cdot\left[C_2\cos(k_y y)+D_2\sin(k_y y)\right]\cdot\left[C_3\cos(k_z z)+D_3\sin(k_z z)\right]$$

考虑 电场沿 x轴方向上的分量：

$$E_x=\left[C_1\cos(k_x x)+D_1\sin(k_x x)\right]\cdot\left[C_2\cos(k_y y)+D_2\sin(k_y y)\right]\cdot\left[C_3\cos(k_z z)+D_3\sin(k_z z)\right]$$

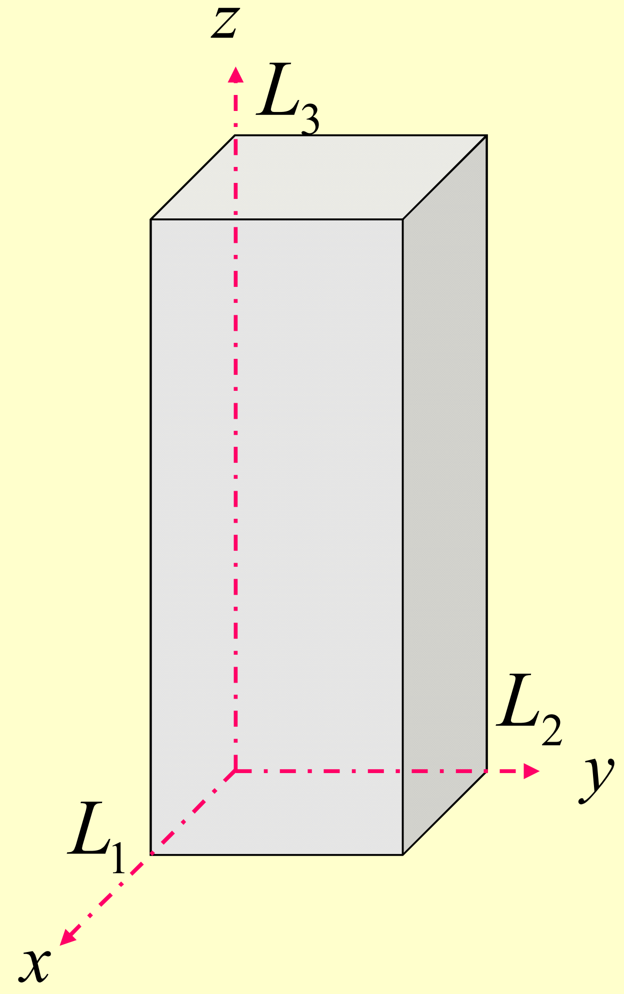
$$
\begin{aligned}
E_x = &\left[C_1 \cos(k_x x) + D_1 \sin(k_x x)\right] \\
&\cdot\left[C_2 \cos(k_y y) + D_2 \sin(k_y y)\right] \\
&\cdot\left[C_3 \cos(k_z z) + D_3 \sin(k_z z)\right]
\end{aligned}
$$

六个边界面：

前、后：$x = L_1$ 及 $x = 0$

上、下：$z = L_3$ 及 $z = 0$

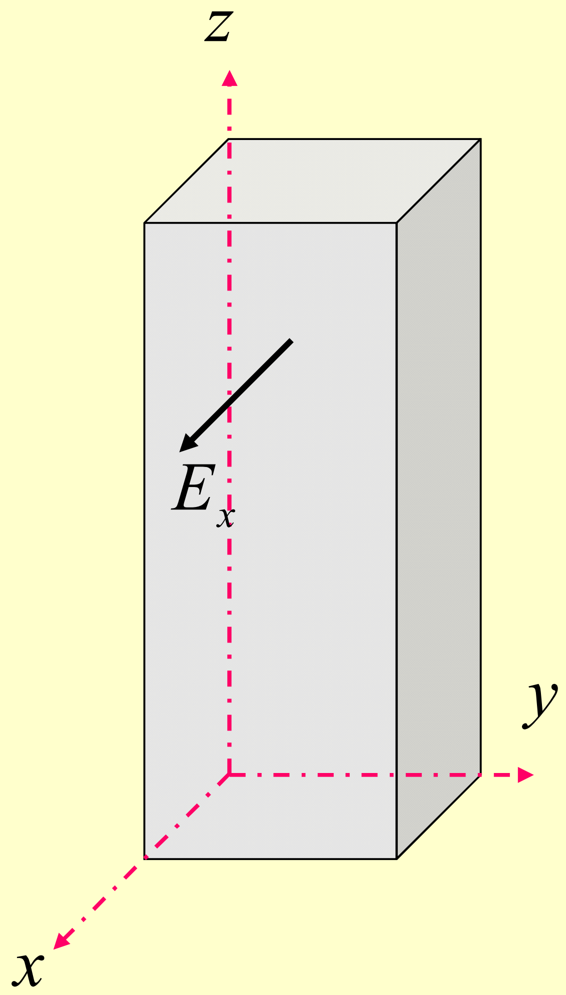
左、右：$y = 0$ 及 $y = L_2$



先考虑 x = 0, y = 0, z = 0 三个面上的边界条件：

$$E_x = \left[C_1\cos(k_x x) + D_1\sin(k_x x)\right] \cdot \left[C_2\cos(k_y y) + D_2\sin(k_y y)\right] \cdot \left[C_3\cos(k_z z) + D_3\sin(k_z z)\right]$$

$$E_x = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$



$$E_x = E_{0x}\cos\left(k_x x\right)\sin\left(k_y y\right)\sin\left(k_z z\right)$$

类似可得：
$$E_y = E_{0y}\sin\left(k_x x\right)\cos\left(k_y y\right)\sin\left(k_z z\right)$$

$$E_z = E_{0z}\sin\left(k_x x\right)\sin\left(k_y y\right)\cos\left(k_z z\right)$$

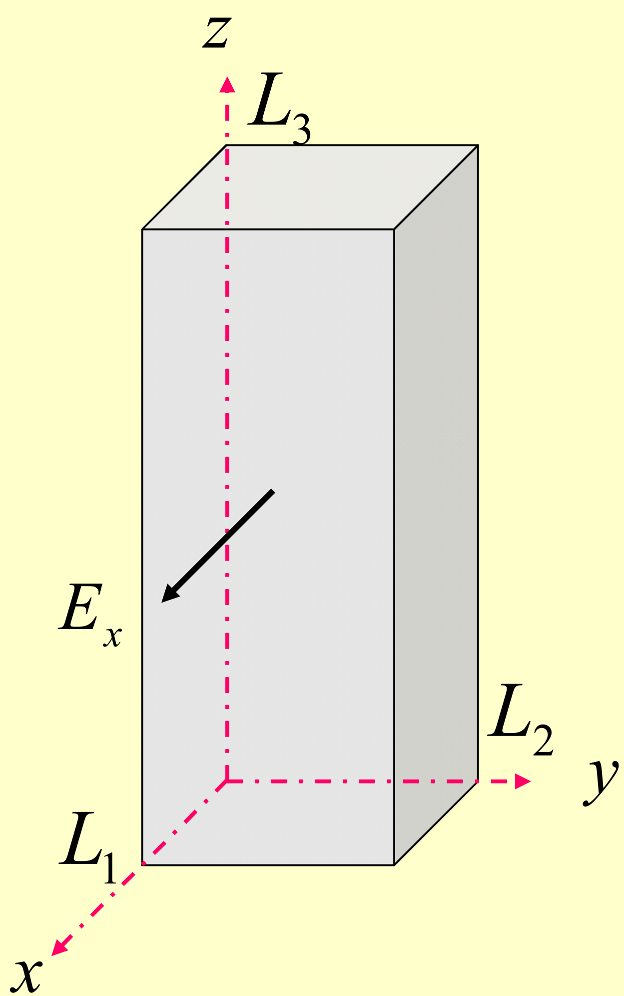
$$E_x = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

再考虑 $x = L_1, y = L_2, z = L_3$ 三个面上的边界条件：

$$\left.\frac{\partial E_x}{\partial x}\right|_{x=L_1} = 0$$

$$\left.E_x\right|_{y=L_2} = 0,$$

$$\left.E_x\right|_{z=L_3} = 0$$



求得：

$$k_x L_1 = m\pi,\quad m = 1,2,3,\cdots$$

$$k_y L_2 = n\pi,\quad n = 1,2,3,\cdots$$

$$k_z L_3 = l\pi,\quad l = 1,2,3,\cdots$$

矩形谐振腔内的电场：

$$E_x = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y = E_{0y}\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z = E_{0z}\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

其中k_x，k_y，k_z须满足：

$$k_x = \frac{m\pi}{L_1},\quad k_y = \frac{n\pi}{L_2},\quad k_z = \frac{l\pi}{L_3},$$

$$m,n,l = 1,2,3,\cdots$$

$$E_x = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y = E_{0y}\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z = E_{0z}\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

4）作为电磁场的解，还要求：

$$\nabla\cdot\vec{E}(\vec{x}) = 0$$

$$\Downarrow$$

$$\boxed{k_x E_{0x} + k_y E_{0y} + k_z E_{0z} = 0}$$

即：E_{0x}, E_{0y}, E_{0z} 中只有两个分量是独立的

总结： 对于矩形谐振腔，当波矢的各分量满足：

$$k_x = \frac{m\pi}{L_1}, \quad k_y = \frac{n\pi}{L_2}, \quad k_z = \frac{l\pi}{L_3},$$
$$m, n, l = 1, 2, 3, \cdots$$

同时电场分量的振幅满足： $k_x E_{0x} + k_y E_{0y} + k_z E_{0z} = 0$

时谐电磁波的电场：

$$E_x = E_{0x} \cos(k_x x) \sin(k_y y) \sin(k_z z)$$

$$E_y = E_{0y} \sin(k_x x) \cos(k_y y) \sin(k_z z)$$

$$E_z = E_{0z} \sin(k_x x) \sin(k_y y) \cos(k_z z)$$

代表腔中的一种谐振模，也称为矩形谐振腔的本征模。

2、谐振腔中本征模的磁场

$$E_x = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y = E_{0y}\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z = E_{0z}\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

$$\vec{H} = \frac{1}{\mathrm{i}\omega\mu_0}\nabla\times\vec{E}$$

$$H_x = \frac{1}{\mathrm{i}\omega\mu_0}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

$$= \frac{1}{\mathrm{i}\omega\mu_0}\left(E_{0z}k_y - E_{0y}k_z\right)\sin(k_x x)\cos(k_y y)\cos(k_z z)$$

$$= -\mathrm{i}H_{0x}\sin(k_x x)\cos(k_y y)\cos(k_z z)$$

其中： $H_{0x} = \dfrac{1}{\omega\mu_0}\left(E_{0z}k_y - E_{0y}k_z\right)$

类似地可得到

$$H_y = -\mathrm{i}H_{0y}\cos(k_x x)\sin(k_y y)\cos(k_z z),$$

$$H_z = -\mathrm{i}H_{0z}\cos(k_x x)\cos(k_y y)\sin(k_z z)$$

3、矩形谐振腔的本征频率、最小本征频率

$$k^2 = k_x^2 + k_y^2 + k_z^2$$
$$= \pi^2 \left(\frac{m^2}{L_1^2} + \frac{n^2}{L_2^2} + \frac{l^2}{L_3^2} \right)$$

1）谐振腔的本征频率（圆频率）为

$$\omega_{mnl} = ck = \pi c \sqrt{\frac{m^2}{L_1^2} + \frac{n^2}{L_2^2} + \frac{l^2}{L_3^2}}$$

2）如果谐振腔中充满绝缘介质，

$$\omega_{mnl} = vk = \frac{\pi}{\sqrt{\mu\varepsilon}} \sqrt{\frac{m^2}{L_1^2} + \frac{n^2}{L_2^2} + \frac{l^2}{L_3^2}}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \pi^2\left(\frac{m^2}{L_1^2} + \frac{n^2}{L_2^2} + \frac{l^2}{L_3^2}\right)$$

3）从上面的公式中可以看出，（m，n，l）不能有两个同时为零；

$$E_x = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y = E_{0y}\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z = E_{0z}\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

4）假设谐振腔的尺寸满足：

$$L_1 > L_2 > L_3$$

则，本征模的**最低频率**模为：

$$\boxed{(m,n,l)=(1,1,0)}$$

（1,1,0）本征模的圆频率为：

$$\omega_{1,1,0}=\pi c\sqrt{\frac{1}{L_1^2}+\frac{1}{L_2^2}}$$

$$k^2=k_x^2+k_y^2+k_z^2=\pi^2\left(\frac{m^2}{L_1^2}+\frac{n^2}{L_2^2}+\frac{l^2}{L_3^2}\right)$$

$$\omega_{mnl}=\pi c\sqrt{\frac{m^2}{L_1^2}+\frac{n^2}{L_2^2}+\frac{l^2}{L_3^2}}$$

$$\omega_{mnl} = \pi c\sqrt{\frac{m^2}{L_1^2}+\frac{n^2}{L_2^2}+\frac{l^2}{L_3^2}}$$

（1,1,0）本征模的圆频率:

$$\omega_{110} = \pi c\sqrt{\frac{1}{L_1^2}+\frac{1}{L_2^2}}$$

（1,1,0）本征模的波长为:

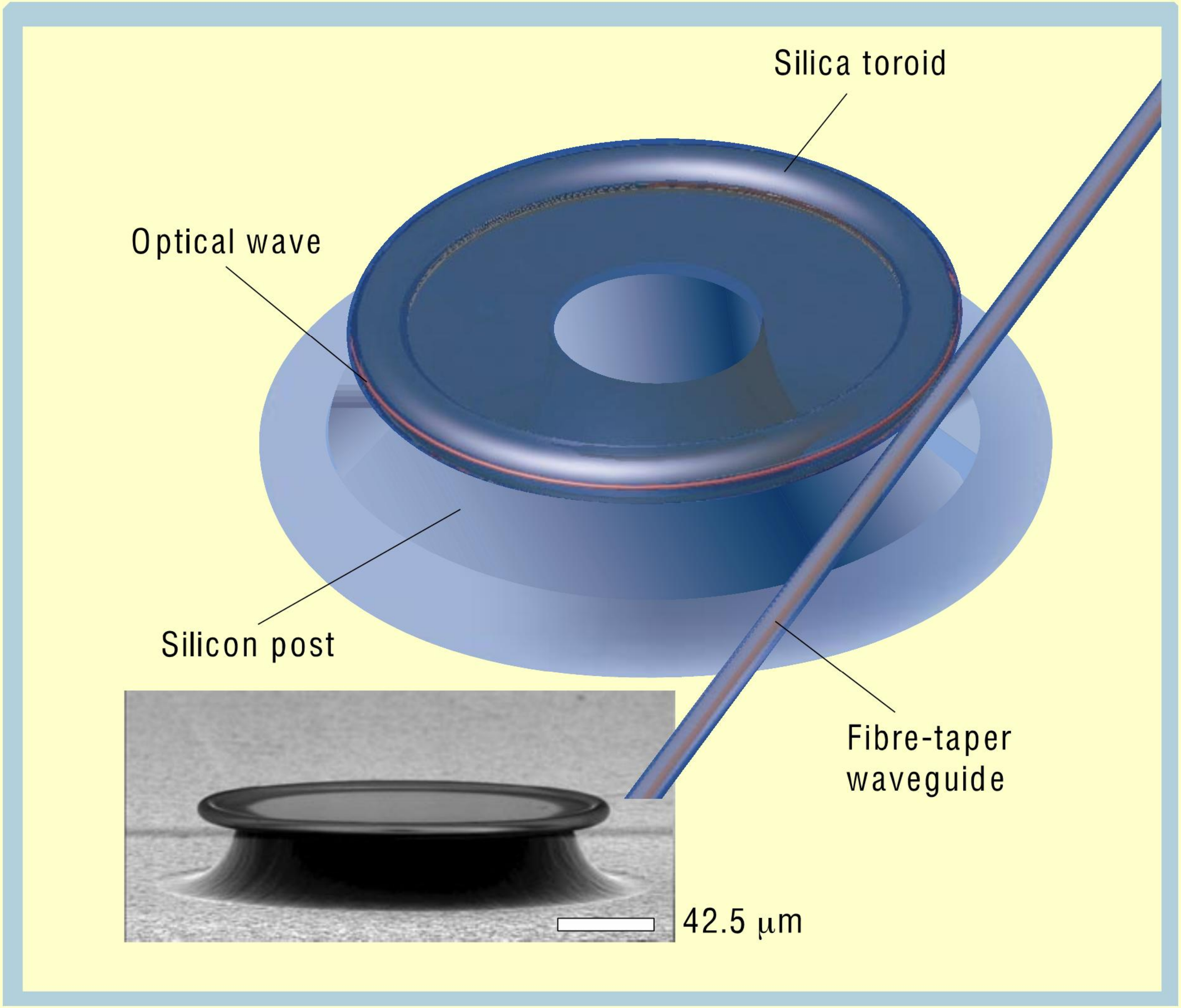
$$\lambda_{110} = \frac{2\pi c}{\omega_{110}} = \frac{2}{\sqrt{\frac{1}{L_1^2}+\frac{1}{L_2^2}}}$$

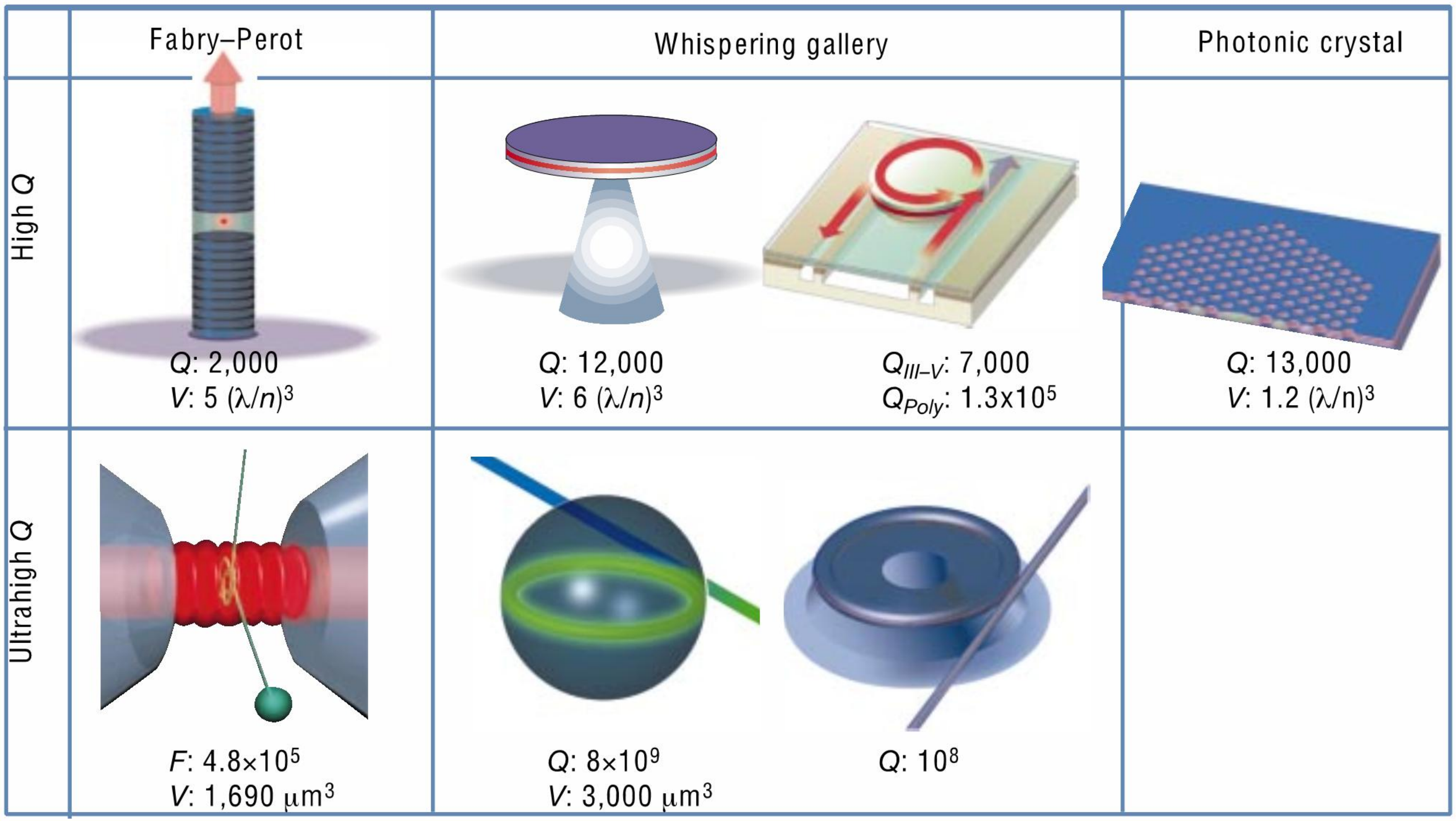
此本征模波长与谐振腔的线度处于同一数量级上。

微光学谐振腔

微光学谐振腔

- 在高频波段，金属的损耗增加，所以人们一般不采用金属来实现光学波段的谐振腔；
- 几种代表性的介质型微光学谐振腔。利用光学全反射机制，共振模式是传播的光学模。

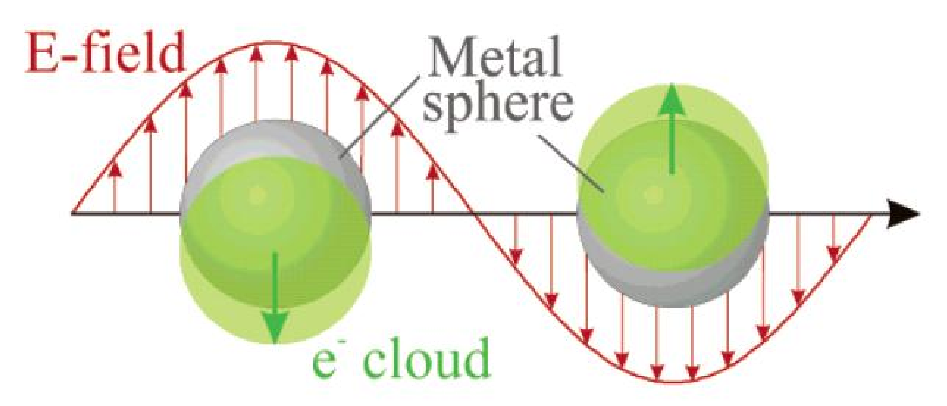

Silica toroid
Optical wave
Silicon post
Fibre-taper
waveguide
42.5 μm

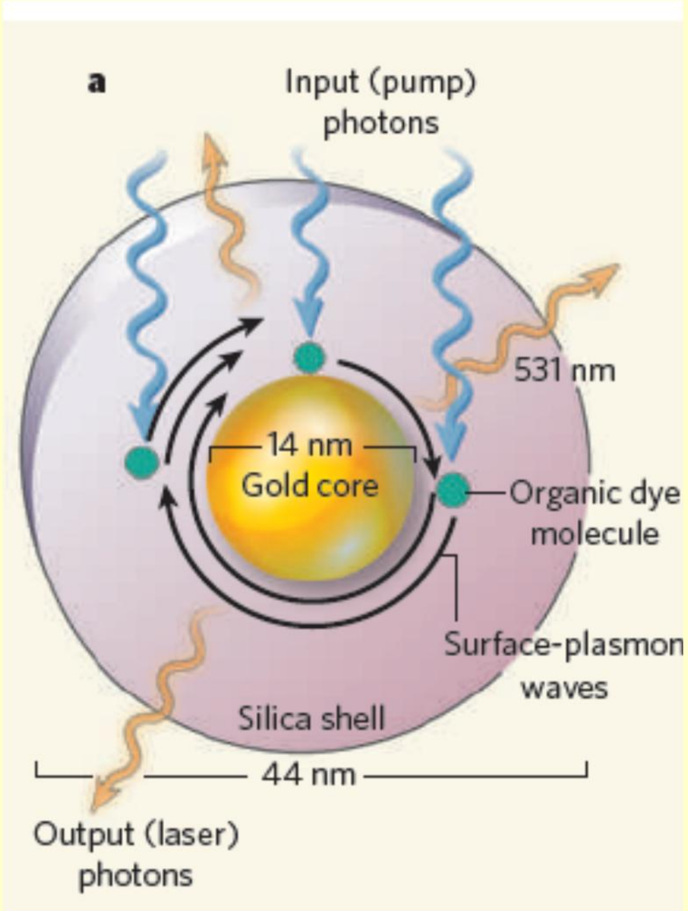


Fabry–Perot
Whispering gallery
Photonic crystal
High Q
Q: 2,000
V: 5 $(\lambda/n)^3$
Q: 12,000
V: 6 $(\lambda/n)^3$
Q_{III-V}: 7,000
Q_{Poly}: 1.3x10^5
Q: 13,000
V: 1.2 $(\lambda/n)^3$
Ultrahigh Q
F: 4.8×10^5
V: 1,690 μm^3
Q: 8×10^9
V: 3,000 μm^3
Q: 10^8

等离激元微腔，及纳米激光器
surface plasmons and nanolasers

基于等离激元共振腔的纳米激光器



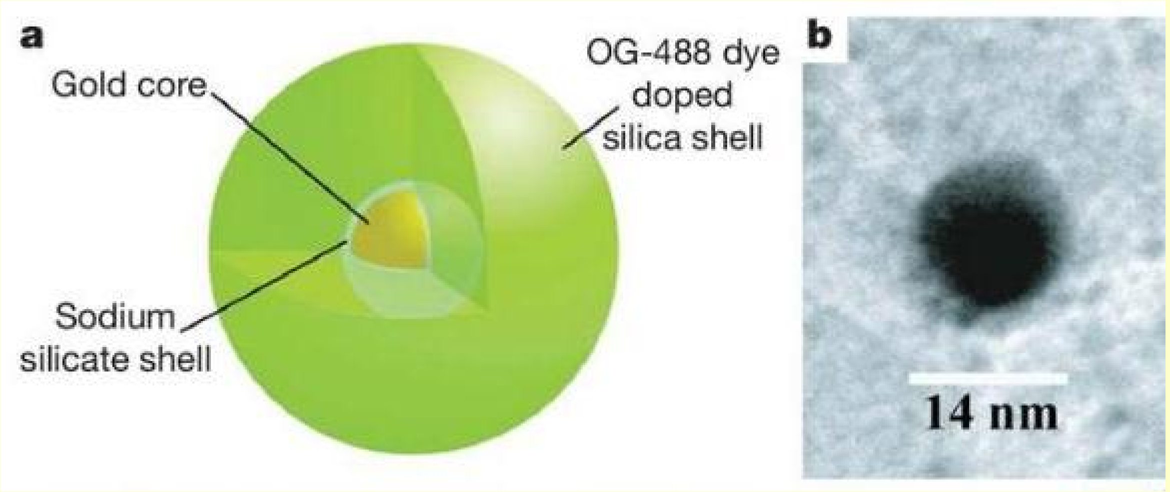


等离激元共振腔可以将模式场局域在亚波长尺寸空间内，能够有效地提升光学器件的集成度!

Nature news & views, 461, 604 (2009) – *Lasers go nano*

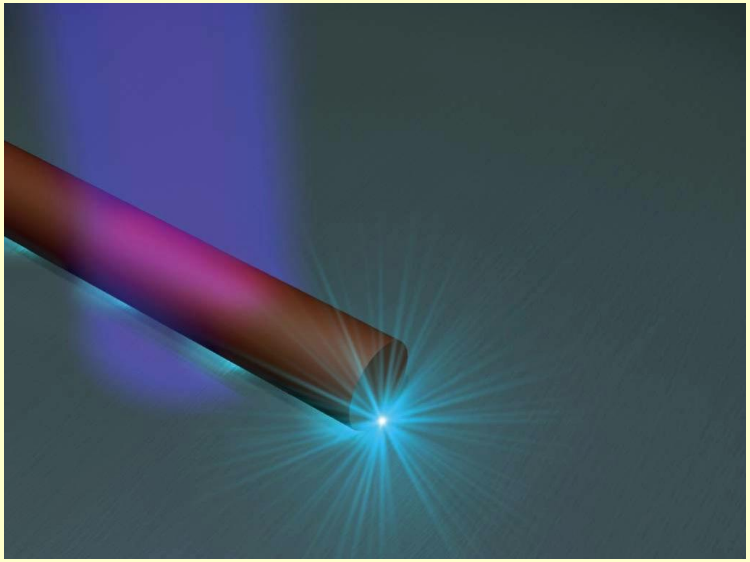
目前正在研究的等离激元纳米激光器

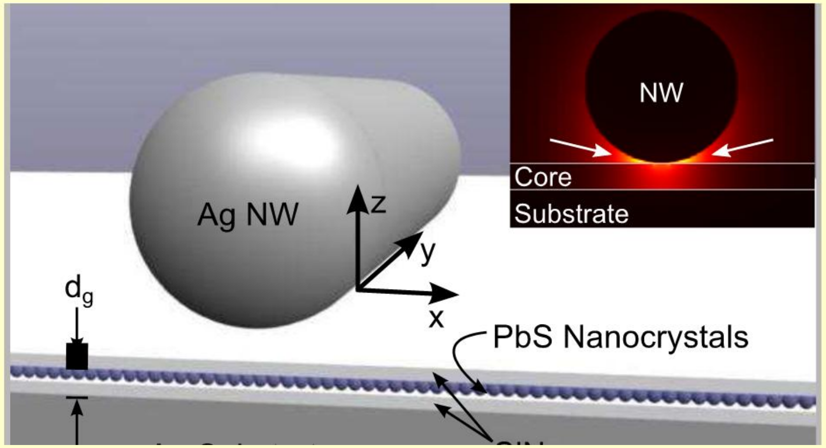
- 金纳米球外包裹光学增益材料的核-壳结构



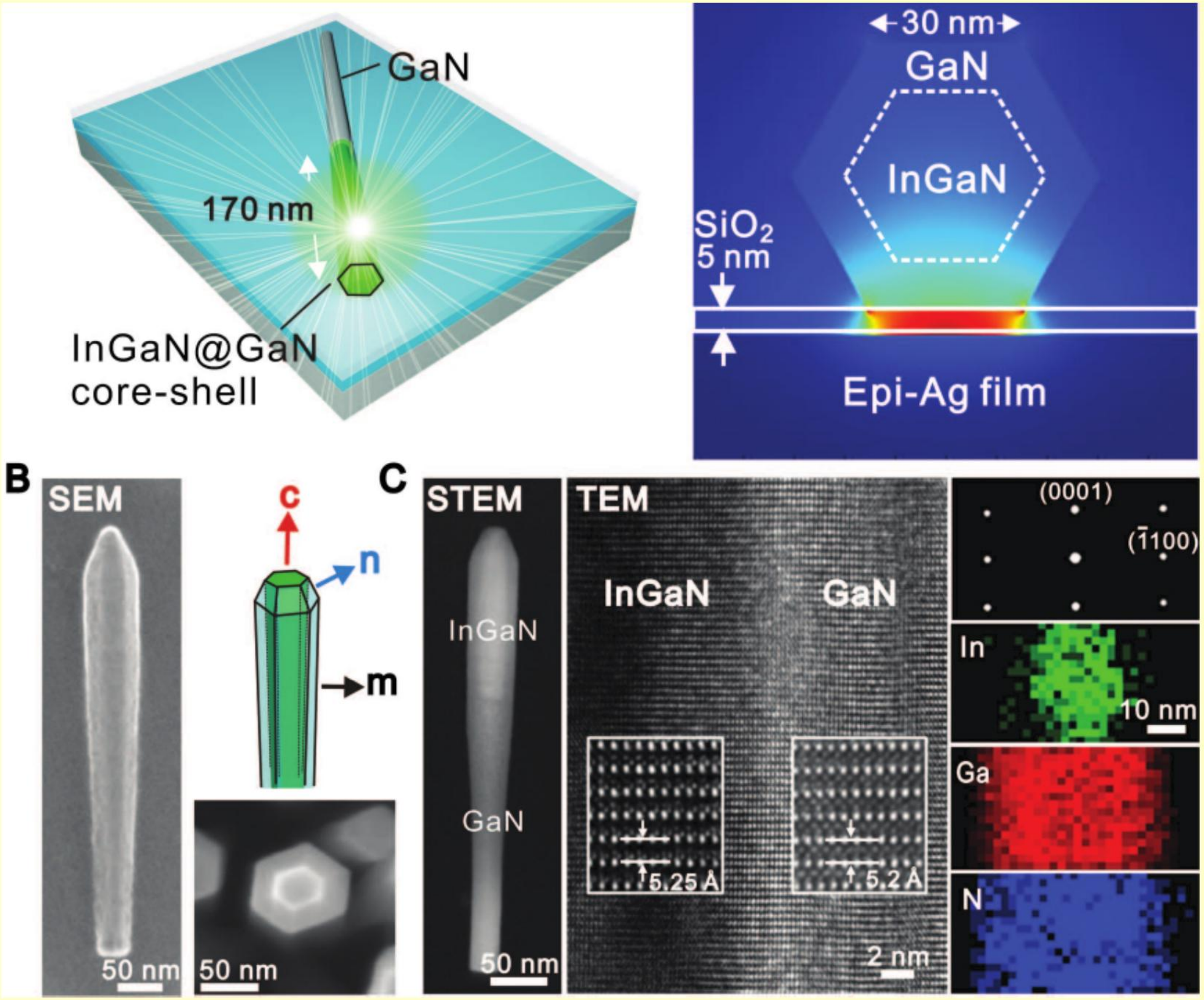
金属损耗过大，导致受激辐射阈值过高，难以在室温环境下工作

Nature 460, 1110 (2009); Nano Lett. 10, 3679 (2010).



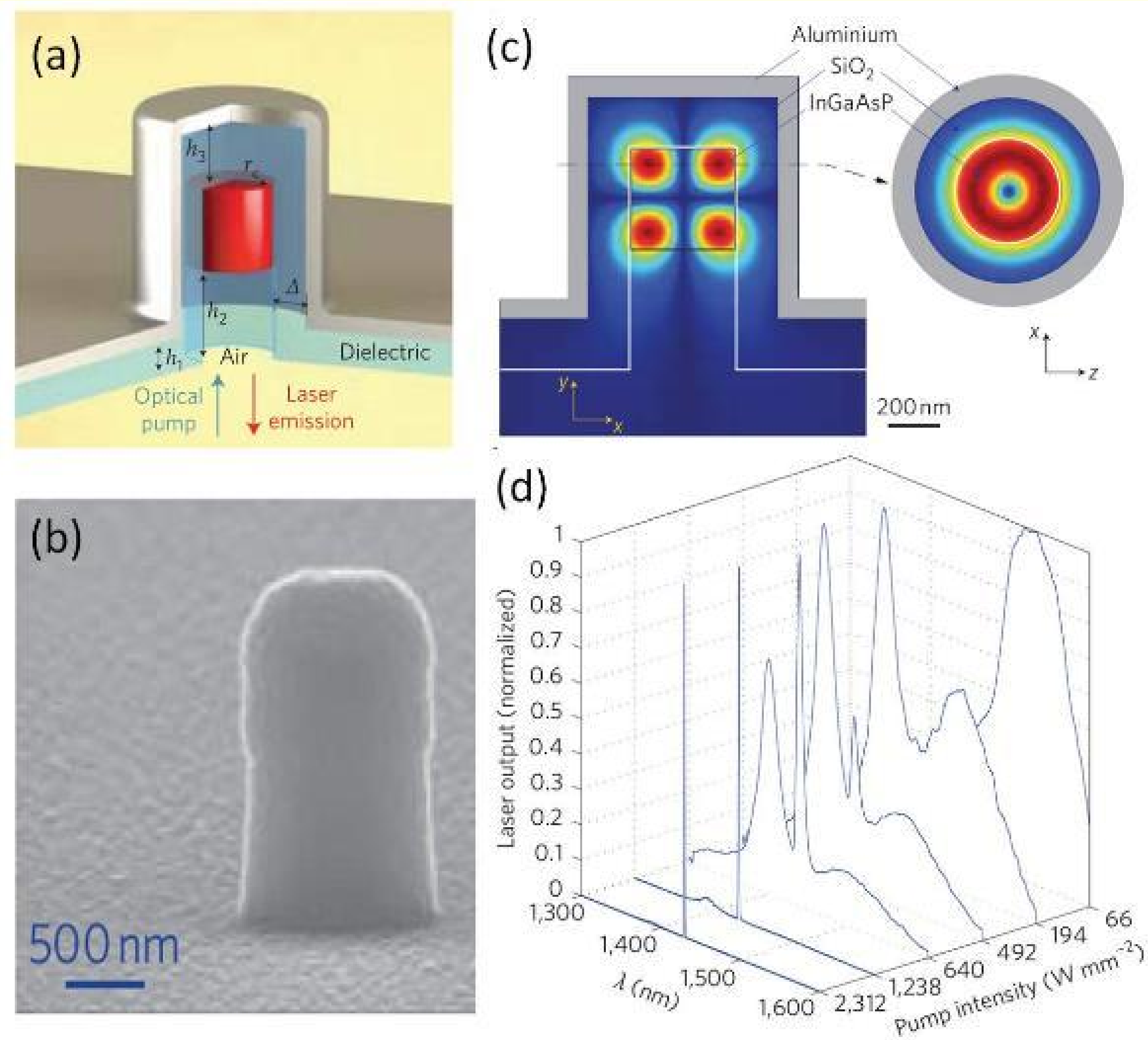


Nature 461, 629 (2009)



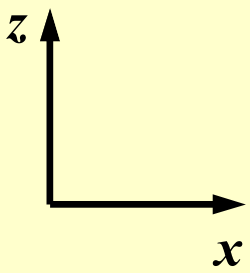
Lu et al., Science, 337, 450 (2012)

室温条件工作的纳米激光器

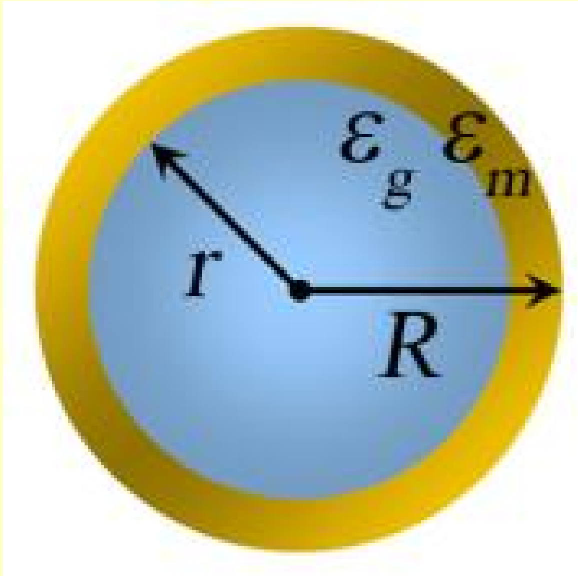


Nature Photonics, 4, 395 (2010)

我们的模型——银纳米球壳，作为SP的共振微腔



R = 250 nm
r = 200 nm

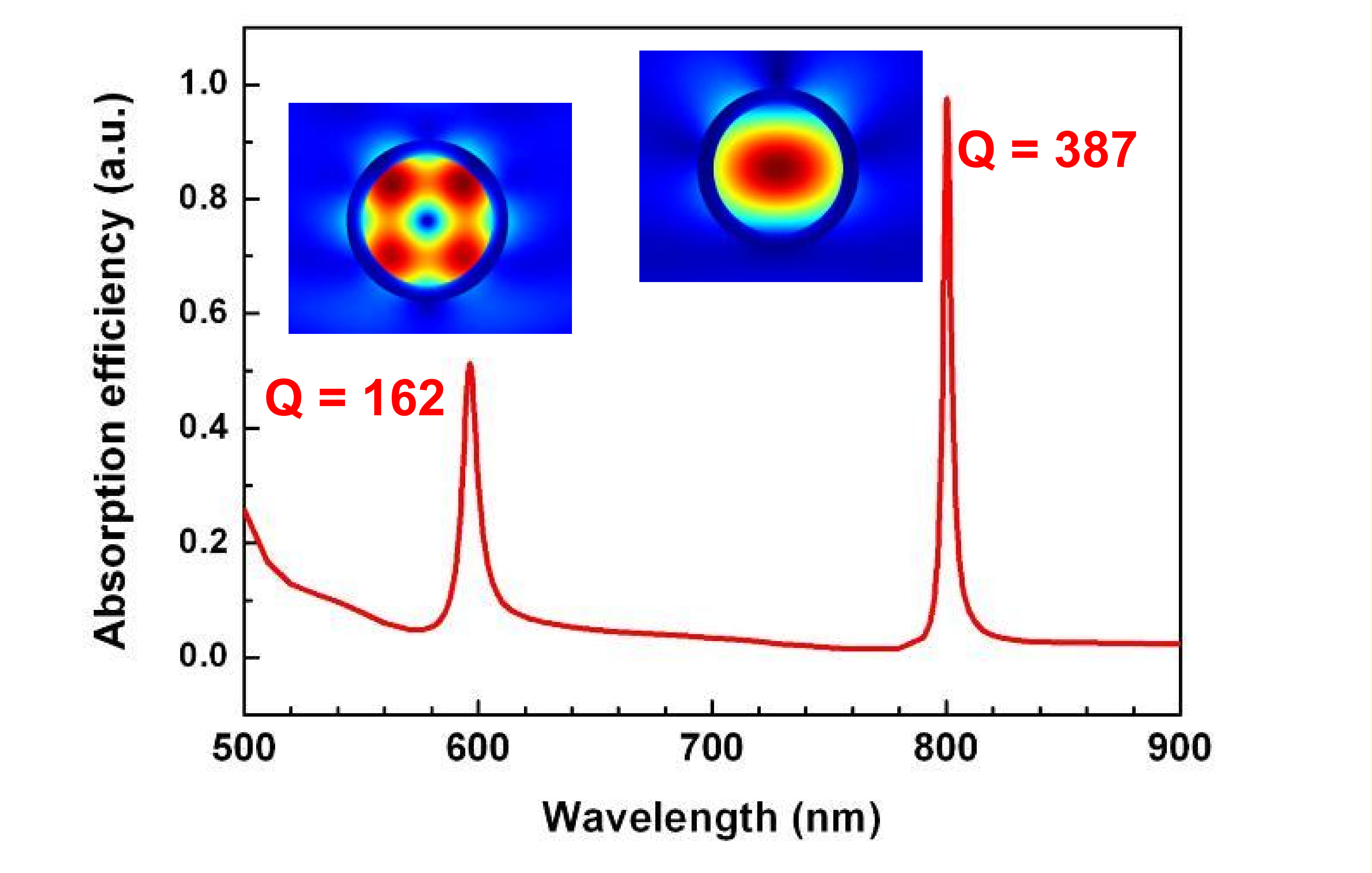


$$\varepsilon_{\mathrm{m}} = \varepsilon'_{\mathrm{m}} + i\varepsilon''_{\mathrm{m}}$$

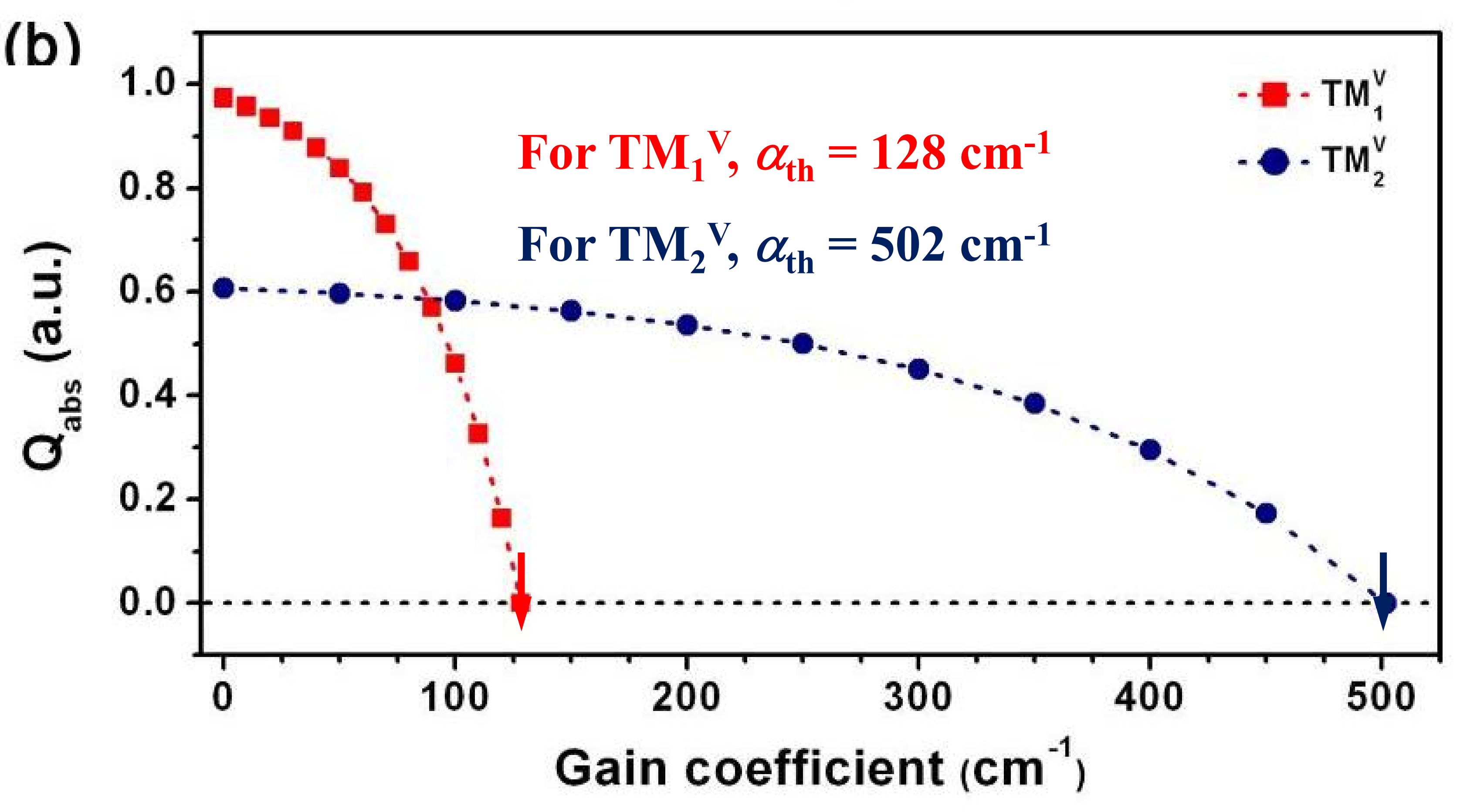
$$\varepsilon_{\mathrm{g}} = \varepsilon'_{\mathrm{g}} + i\varepsilon''_{\mathrm{g}}$$

Opt. Lett. ***37***, 1181 (2012)

银纳米球壳的吸收谱和腔模的场分布



受激辐射阈值的确定



以纳米银球外包裹增益材料的结构为例，其阈值大约为1500 cm^{-1}

所得的阈值结果明显小于其它类型等离激元纳米激光器的阈值！

结构	受激辐射模式	阈值 (cm^{-1})	波长
Asymmetric SRRs (N)	Trapped-mode	~ 2000	1650 nm
MIM WG (E)	MIM plasmonic TM0 WG mode	~ 5000	1480 nm
Fishnet (N)	Magnetic resonance	~ 1500	1485 nm
Gold NP (E)	Electric dipole LSPR	~ 1500	525 nm
Nanopan (E)	Whispering Gallery mode	4200	1338 nm
Hybrid WG (E)	Hybrid WG mode	~ 1000	490 nm
Nanoshell	**Void mode**	**128**	**800 nm**