§3 电偶极辐射

$$\varphi(\vec{x},t)=\int\frac{1}{4\pi\varepsilon_0 r}\rho\left(\vec{x}',t-\frac{r}{c}\right)\mathrm{d}V'$$

③ 交变电流下的标势解

电荷密度与电流密度满足电荷守恒定律；

$$\vec{J}(\vec{x}',t)=\vec{J}(\vec{x}')\mathrm{e}^{-i\omega t}$$

$$\rho(\vec{x}',t)=\rho(\vec{x}')\mathrm{e}^{-i\omega t}$$

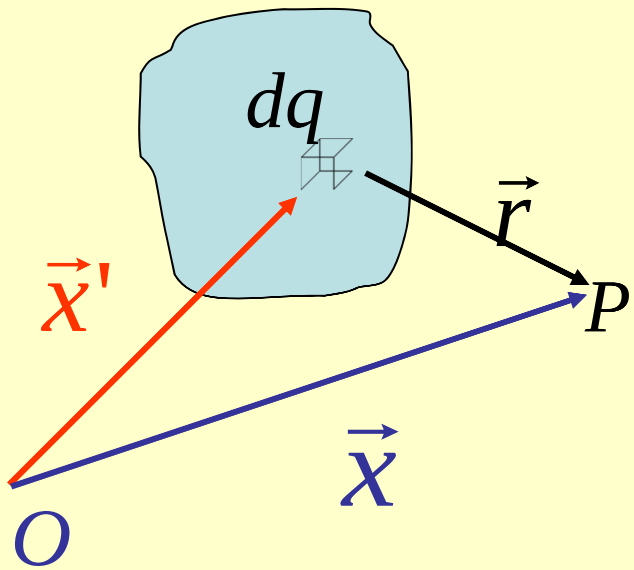
$$\nabla'\cdot\vec{J}(\vec{x}',t')+\frac{\partial\rho(\vec{x}',t')}{\partial t'}=0$$

$$\mathrm{i}\omega\rho(\vec{x}')=\nabla'\cdot\vec{J}(\vec{x}')$$

$$\rho(\vec{x}')=-\mathrm{i}\omega^{-1}\nabla'\cdot\vec{J}(\vec{x}')$$

2、矢势的展开式

假设源点：$\vec{J}(\vec{x}',t)=\vec{J}(\vec{x}')e^{-i\omega t}$



矢势：$\vec{A}(\vec{x},t)=\vec{A}(\vec{x})e^{-i\omega t}$

$$\vec{A}(\vec{x})=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{x}')}{r}e^{ikr}dV'$$

2）**远区电磁场　矢势的展开**

对于远场区域的辐射场，可以采用近似方法求解

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikR}}{R}\int \vec{J}(\vec{x}')\left[1 - ik\,\vec{n}\cdot\vec{x}' + \ldots\ldots\right]dV'$$

3. 偶极辐射（dipole radiation）

展开式中的各项对应电磁多极辐射

$$\dot{\vec{p}} = \frac{d\vec{p}}{dt} = \frac{dQ}{dt}\Delta l = I\Delta l = \int \vec{J}(\vec{x}\,')\mathrm{d}V\,'$$

$\vec{p}$ 为电荷系统的电偶极矩（仅含振幅部分）

$$A^{(0)}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\int \vec{J}(\vec{x}\,')\mathrm{d}V\,' = \frac{\mu_0}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}(-\mathrm{i}\omega\vec{p})$$

考虑时间振荡因子，则

$$\Rightarrow \vec{A}^{(0)}(\vec{x},t) = \frac{\mu_0}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}(-\mathrm{i}\omega\vec{p})\mathrm{e}^{-\mathrm{i}\omega t}$$

$$= \frac{\mu_0}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\dot{\vec{p}}$$

其中定义 $\dot{\vec{p}} = -\mathrm{i}\omega\vec{p}\mathrm{e}^{-\mathrm{i}\omega t}$

- To be continued

$$\vec{A}^{(0)}\left(\vec{x},t\right)$$

⇓

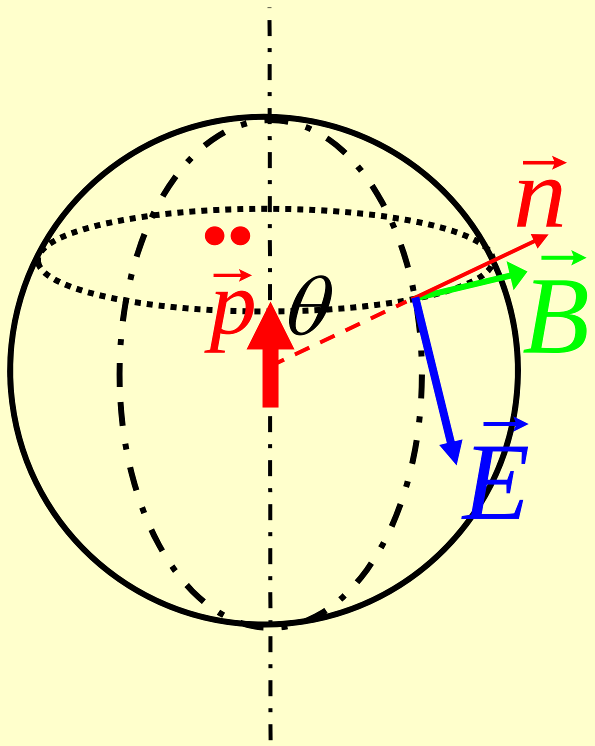
$$\vec{B}^{(0)}=\nabla\times A^{(0)}$$

⇓

$$\nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}=-\frac{i\omega}{c^2}\vec{E}$$

⇓

$$\vec{S}=\vec{E}\times\vec{H}$$



$$A^{(0)}(\vec{x},t)=\frac{\mu_0}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\dot{\vec{p}}$$

2）磁场

$$\vec{B}^{(0)}=\nabla\times A^{(0)}=\nabla\times\left[\frac{\mu_0}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\dot{\vec{p}}\right]$$

$$=\frac{\mu_0}{4\pi}\nabla\left(\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\right)\times\dot{\vec{p}}$$

$$\nabla\times\left(\varphi\vec{f}\right)=(\nabla\varphi)\times\vec{f}+\varphi\nabla\times\vec{f}$$

$$\nabla\left(\frac{e^{\mathrm{i}kR}}{R}\right)=\left(-\frac{1}{R^2}\vec{\mathrm{e}}_r+\mathrm{i}k\frac{1}{R}\vec{\mathrm{e}}_r\right)\mathrm{e}^{\mathrm{i}kR}\quad\approx\mathrm{i}k\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\vec{n}$$

$$\nabla\psi=\frac{\partial\psi}{\partial R}\vec{\mathrm{e}}_r+\frac{1}{R}\frac{\partial\psi}{\partial\theta}\vec{\mathrm{e}}_\theta+\frac{1}{R\sin\theta}\frac{\partial\psi}{\partial\varphi}\vec{\mathrm{e}}_\phi$$

$$\nabla\left(\frac{e^{ikR}}{R}\right)\approx \mathrm{i}k\frac{\mathrm{e}^{ikR}}{R}\vec{n}$$

$$\vec{B}^{(0)}=\frac{\mu_0}{4\pi}\nabla\left(\frac{\mathrm{e}^{ik\mathrm{R}}}{R}\right)\times\dot{\vec{p}}=\frac{\mathrm{i}\mu_0 k}{4\pi}\frac{\mathrm{e}^{ikR}}{R}\vec{n}\times\dot{\vec{p}}$$

$$=\frac{\mathrm{i}\mu_0 k}{4\pi}\left(\frac{1}{-\mathrm{i}\omega}\right)\frac{\mathrm{e}^{ikR}}{R}\vec{n}\times\left(-\mathrm{i}\omega\dot{\vec{p}}\right)$$

定义： $\ddot{\vec{p}}=-\mathrm{i}\omega\dot{\vec{p}}=-\omega^2\vec{p}\mathrm{e}^{-\mathrm{i}\omega t}$

$$\vec{B}^{(0)}=-\frac{\mu_0 k}{4\pi\omega}\frac{\mathrm{e}^{ikR}}{R}\vec{n}\times\ddot{\vec{p}}$$

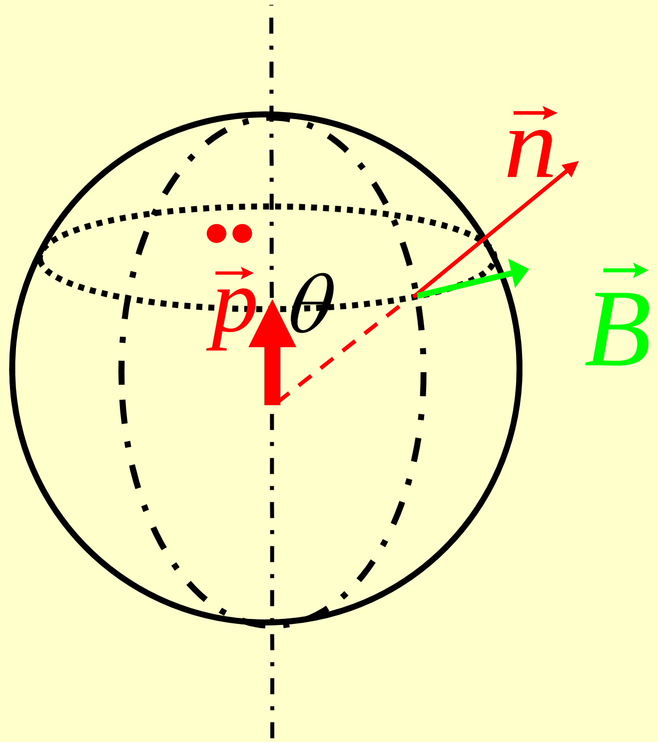
$$\vec{B}^{(0)} = -\frac{\mu_0 k}{4\pi\omega}\frac{e^{ikR}}{R}\vec{n}\times\ddot{\vec{p}}$$

$$\vec{B}^{(0)} = \frac{\mu_0}{4\pi c}\frac{e^{ikR}}{R}\ddot{\vec{p}}\times\vec{n}$$

$$= \frac{1}{4\pi\varepsilon_0 c^3}\frac{e^{ikR}}{R}\ddot{\vec{p}}\times\vec{n}$$

$$= \frac{1}{4\pi\varepsilon_0 c^3}\frac{e^{ikR}}{R}\ddot{p}\sin\theta\,\vec{e}_\phi$$

$$= B_\phi(R,\theta)\vec{e}_\phi$$

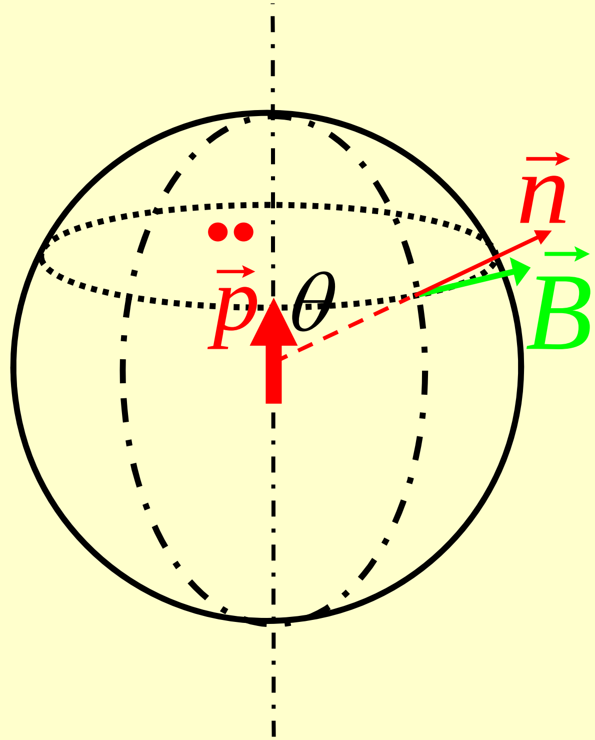


$$B_\phi(R,\theta) = \frac{1}{4\pi\varepsilon_0 c^3}\ddot{p}\frac{e^{ikR}}{R}\sin\theta \quad = C_1\frac{e^{ikR}}{R}\sin\theta$$

$$\vec{B}^{(0)} = C_1 \frac{e^{ikR}}{R} \sin\theta\, \vec{e}_\phi$$

2）电场

$$\vec{E} = \frac{ic}{k} \nabla \times \vec{B}_\phi(R,\theta)$$



$$\vec{E} = \frac{ic}{k} \frac{1}{R\sin\theta} \frac{\partial}{\partial\theta}\left(C_1 \frac{e^{ikR}}{R} \sin^2\theta\right) \vec{e}_R$$

小量可忽略 $\propto \frac{1}{R^2}$

$$+ \frac{ic}{k} \frac{-1}{R} \frac{\partial}{\partial R}\left(C_1 e^{ikR} \sin\theta\right) \vec{e}_\theta$$

$\propto \frac{1}{R}$

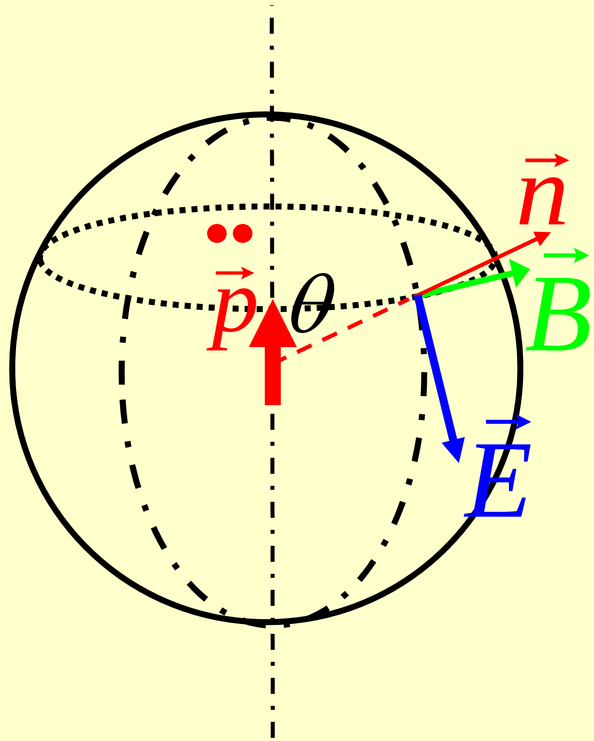
$$\nabla \times \vec{f} = \nabla \times \left[f_\phi \vec{e}_\phi\right] = \frac{1}{R\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta f_\phi\right) \vec{e}_R - \frac{1}{R} \frac{\partial}{\partial R}\left(R f_\phi\right) \vec{e}_\theta$$

$$C_1 = \frac{1}{4\pi\varepsilon_0 c^3}\ddot{p}$$

$$\vec{E} \approx \frac{\mathrm{i}c}{k}\frac{-1}{R}\frac{\partial}{\partial R}\left(C_1 \mathrm{e}^{\mathrm{i}kR}\sin\theta\right)\vec{\mathrm{e}}_\theta$$

$$\approx \frac{\mathrm{i}c}{k}\frac{1}{4\pi\varepsilon_0 c^3}\ddot{p}\,\frac{-\mathrm{i}\,k}{R}\mathrm{e}^{\mathrm{i}kR}\sin\theta\,\vec{\mathrm{e}}_\theta$$

$$= \frac{1}{4\pi\varepsilon_0 c^2 R}\mathrm{e}^{\mathrm{i}kR}\ddot{p}\sin\theta\,\vec{\mathrm{e}}_\theta$$



$$\ddot{\vec{p}} = -\mathrm{i}\omega\,\dot{\vec{p}} = -\omega^2\,\vec{p}\mathrm{e}^{-\mathrm{i}\omega t}$$

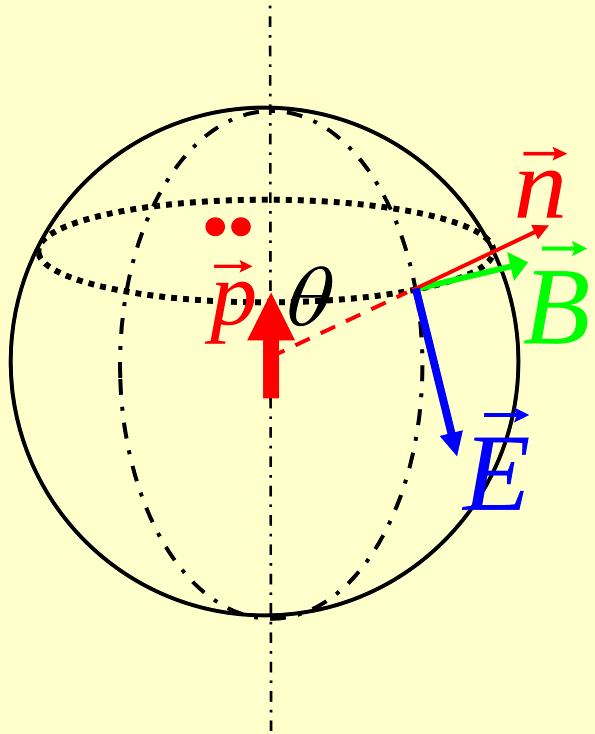
$$\vec{B} = \frac{1}{4\pi\varepsilon_0 c^3}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\ddot{p}\sin\theta\,\vec{\mathrm{e}}_\phi$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{\mathrm{e}^{\mathrm{i}kR}}{R}\ddot{p}\sin\theta\,\vec{\mathrm{e}}_\theta$$

考虑时间振荡因子：$\mathrm{e}^{-i\omega t}$

$$\vec{B} = \frac{-1}{4\pi\varepsilon_0 c^3}\frac{1}{R}\left|\ddot{\vec{p}}\right|\sin\theta\;\mathrm{e}^{\mathrm{i}(kR-\omega t)}\vec{\mathrm{e}}_\varphi$$

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0 c^2}\frac{1}{R}\left|\ddot{\vec{p}}\right|\sin\theta\;\mathrm{e}^{\mathrm{i}(kR-\omega t)}\vec{\mathrm{e}}_\theta$$



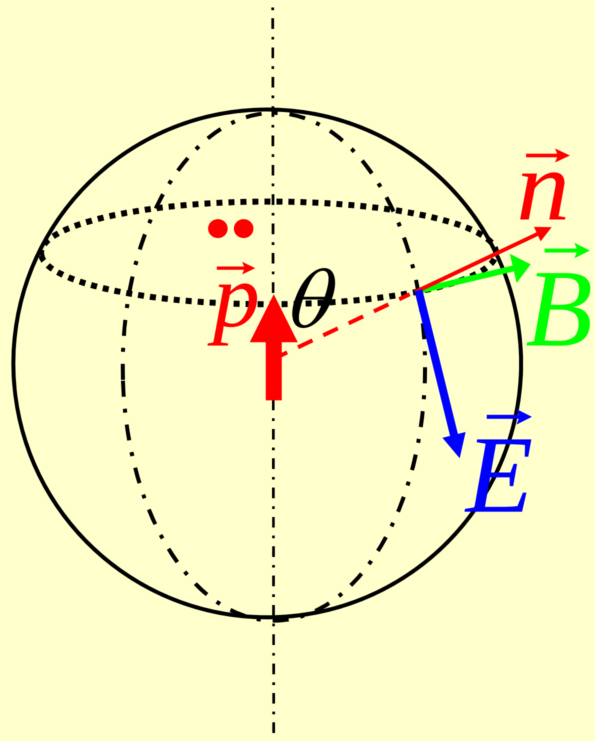
总结：

$$\ddot{\vec{p}} = -\omega^2 \vec{p} e^{-i\omega t}$$

$$\left|\ddot{\vec{p}}\right| = \omega^2 \left|\vec{p}\right|$$

$$\vec{B} = \frac{-1}{4\pi\varepsilon_0 c^3} \frac{1}{R} \left|\ddot{\vec{p}}\right| \sin\theta \, e^{i(kR-\omega t)} \vec{e}_\varphi$$

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0 c^2} \frac{1}{R} \left|\ddot{\vec{p}}\right| \sin\theta e^{i(kR-\omega t)} \vec{e}_\theta$$



① B线沿纬度线上振荡；E线沿经度线上振荡；

② 远场区，**电场和磁场振幅都具有 1/R 的特点**（*不同于静电场和静磁场*）；

③ 具有这种特性的场，在运动中伴随有能量的辐射，这样的场称为**辐射场**。

4. **时变电偶极矩**在远场区激发的电磁场辐射

——能流、辐射功率、角分布

1）平均能流密度：

$$\vec{B}=\frac{1}{4\pi\varepsilon_0 c^3}\frac{1}{R}\ddot{p}\sin\theta\, \mathrm{e}^{\mathrm{i}(kR-\omega t)}\vec{\mathrm{e}}_\varphi$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_0 c^2}\frac{1}{R}\ddot{p}\sin\theta\, \mathrm{e}^{\mathrm{i}(kR-\omega t)}\ \vec{\mathrm{e}}_\theta$$

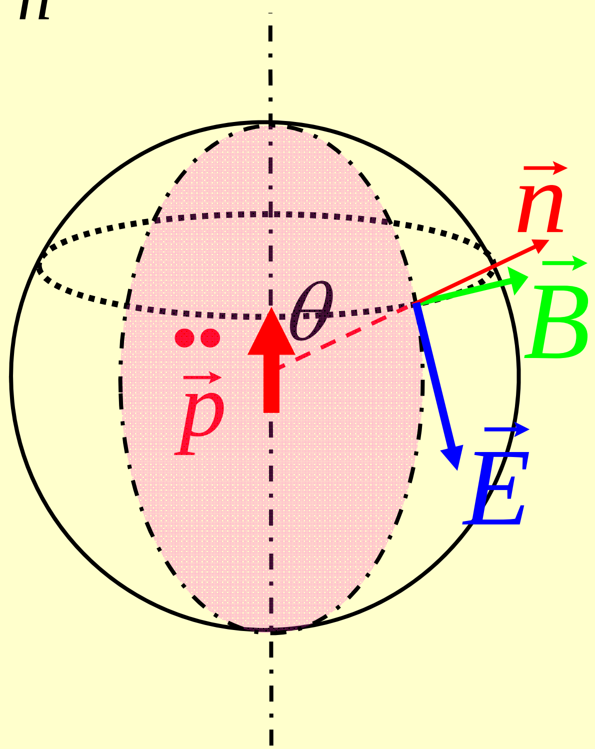
$$\vec{S}=\vec{E}\times\vec{H}=\mathrm{Re}\left(\vec{E}\right)\times\mathrm{Re}\left(\vec{H}\right)$$

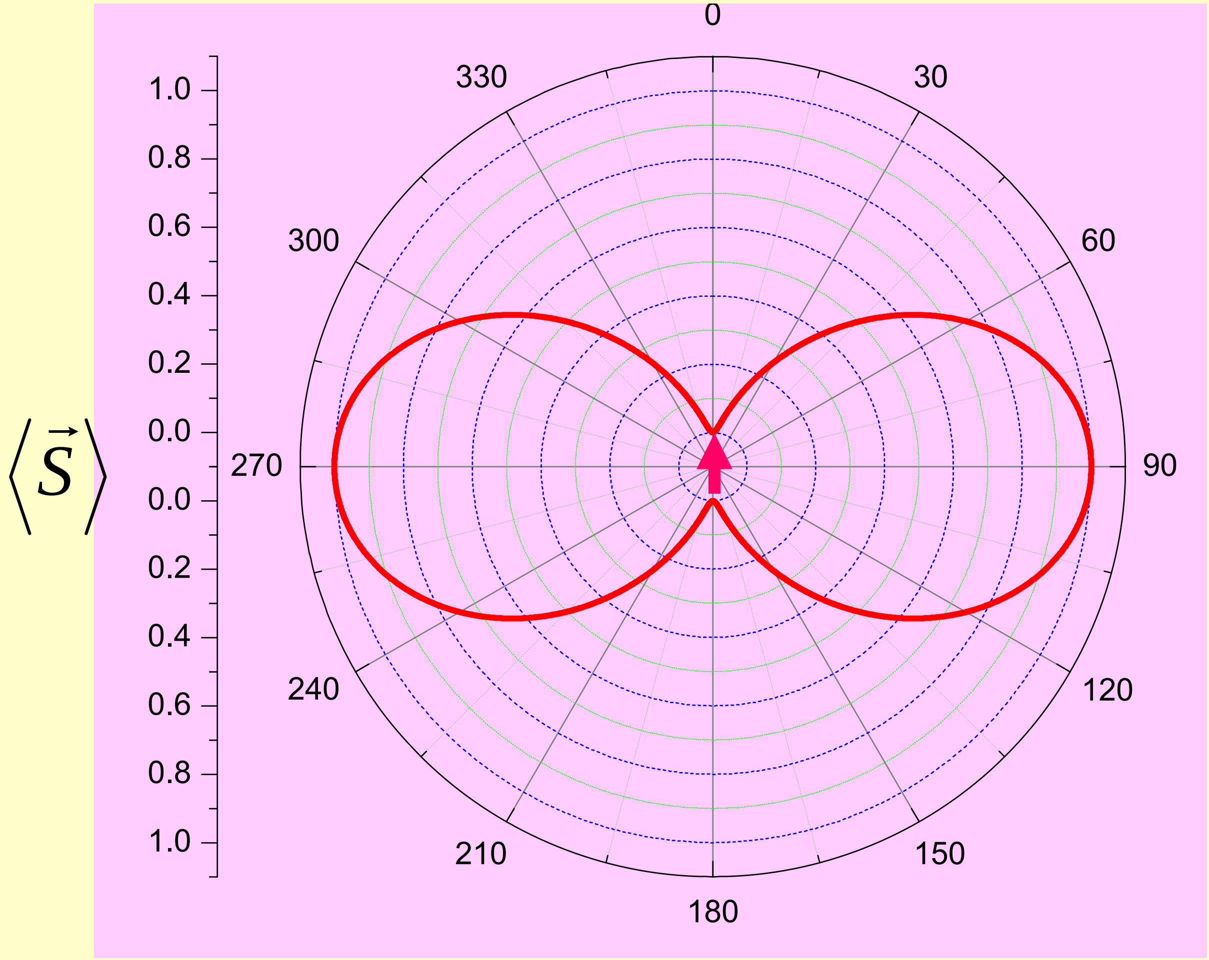
$$=\mathrm{Re}\left(\vec{E}\right)\times\frac{1}{\mu_0}\mathrm{Re}\left(\vec{B}\right)$$

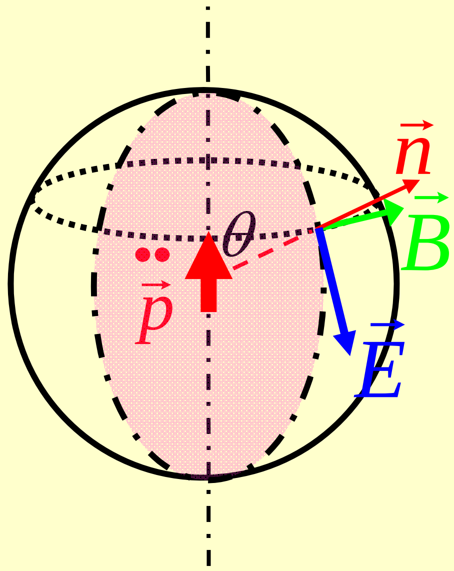
$$=\frac{1}{16\mu_0\pi^2\varepsilon_0^2c^5}\frac{1}{R^2}\left|\ddot{p}\right|^2\sin^2\theta\left[\cos\left(kR-\omega t\right)\right]^2\vec{n}$$

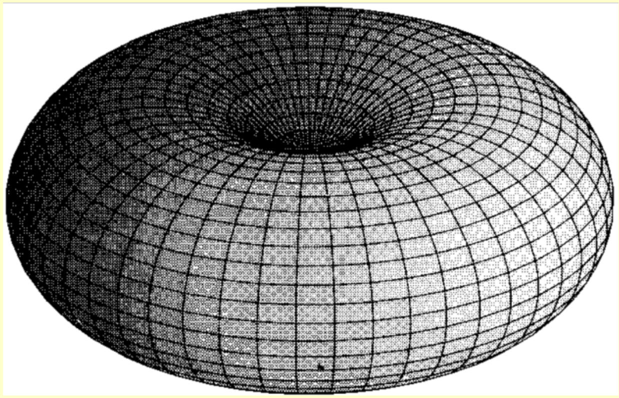
$$\left\langle\vec{S}\right\rangle=\frac{1}{32\mu_0\pi^2\varepsilon_0^2c^5}\frac{1}{R^2}\left|\ddot{p}\right|^2\sin^2\theta\ \vec{n}$$

$$=\frac{1}{32\pi^2\varepsilon_0c^3}\frac{1}{R^2}\left|\ddot{p}\right|^2\sin^2\theta\ \vec{n}$$









偶极子辐射：

- 在电偶极矩的轴线方向上没有辐射；
- 在ϑ=90°的平面上的辐射最强。

$$\langle \vec{S} \rangle = \frac{1}{32\pi^2\varepsilon_0 c^3}\frac{1}{R^2}\left|\ddot{p}\right|^2 \sin^2\theta\ \vec{n}$$

2）总辐射功率

总辐射功率为平均能流密度对球面积分

$$P = \oint \left|\langle \vec{S} \rangle\right| R^2 \mathrm{d}\Omega$$

$$= \frac{1}{32\pi^2\varepsilon_0 c^3}\left|\ddot{p}\right|^2 \oiint \sin^2\theta\ \mathrm{d}\Omega$$

$$= \frac{1}{32\pi^2\varepsilon_0 c^3}\left|\ddot{p}\right|^2 \int_0^{\pi} \sin^3\theta \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{3c^3}\left|\ddot{p}\right|^2$$

$$P = \frac{1}{4\pi\varepsilon_0}\frac{1}{3c^3}\left|\ddot{p}\right|^2$$

$$\ddot{\vec{p}} = -\mathrm{i}\omega\,\dot{\vec{p}} = -\omega^2\,\vec{p}\mathrm{e}^{-\mathrm{i}\omega t}$$

时变电偶极矩在远场区辐射的特点：

① 远场区球面上的总辐射功率与距离无关，因此电磁波可以传播很远；

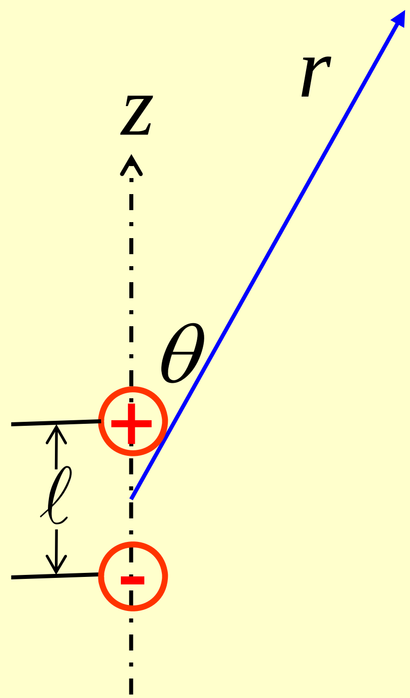
② 如果保持电偶极矩的振幅不变，频率增高时，辐射功率迅速增大。

例题：

假设两个很小的金属球，之间用细导线相连，两个球上的电量分别是q(t)和-q(t)。假设：

$$q(t) = q_0 \cos(\omega t)$$

计算平均能流密度和总辐射功率。



偶极子辐射

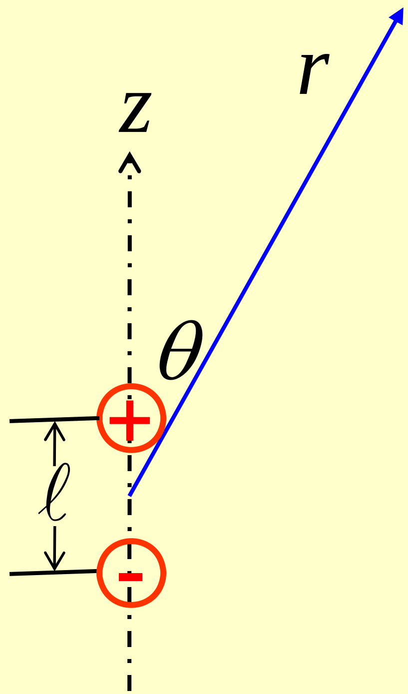
假设两个很小的金属球，之间用细导线相连，两个球上的电量分别是q(t)和-q(t)。假设：

$$q(t)=q_0\cos(\omega t)$$

电偶极矩：$\vec{p}(t)=q_0\ell\cos(\omega t)\vec{e}_z$

$$=p_0\cos(\omega t)\vec{e}_z$$

$$\vec{p}(t)=p_0 e^{-i\omega t}\vec{e}_z \quad (p_0=q_0\ell)$$



$$\Rightarrow \dot{\vec{p}}=-i\omega p_0 e^{-i\omega t}\,\vec{e}_z$$

$$\Rightarrow \ddot{\vec{p}}=-i\omega\,\dot{\vec{p}}=-\omega^2 p_0 e^{-i\omega t}\vec{e}_z$$

$$\vec{B}=\frac{1}{4\pi\varepsilon_0 c^3}\frac{1}{R}\ddot{p}\sin\theta\, e^{i(kR-\omega t)}\vec{e}_\varphi$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_0 c^2}\frac{1}{R}\ddot{p}\sin\theta\, e^{i(kR-\omega t)}\,\vec{e}_\theta$$

$$\ddot{\vec{p}} = -\omega^2 p_0 \mathrm{e}^{-\mathrm{i}\omega t} \vec{\mathrm{e}}_z$$

$$\vec{B} = \frac{-1}{4\pi\varepsilon_0 c^3} \frac{1}{R} \omega^2 p_0 \sin\theta \, \mathrm{e}^{\mathrm{i}(kR-\omega t)} \vec{\mathrm{e}}_\phi$$

$$= \frac{-\mu_0 p_0 \omega^2}{4\pi c} \frac{\sin\theta}{R} \mathrm{e}^{\mathrm{i}(kR-\omega t)} \vec{\mathrm{e}}_\phi$$

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0 c^2} \frac{1}{R} \omega^2 p_0 \sin\theta \mathrm{e}^{\mathrm{i}(kR-\omega t)} \vec{\mathrm{e}}_\theta$$

$$= \frac{-\mu_0 \omega^2 p_0}{4\pi} \frac{\sin\theta}{R} \mathrm{e}^{\mathrm{i}(kR-\omega t)} \vec{\mathrm{e}}_\theta$$

$$\left\langle \vec{S} \right\rangle = \frac{\mu_0 \omega^4 p_0^2}{32\pi^2 c} \frac{\sin^2\theta}{R^2} \vec{n}$$

$$P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$

$$\vec{B} = \frac{1}{4\pi\varepsilon_0 c^3} \frac{1}{R} \ddot{p} \sin\theta \, \mathrm{e}^{\mathrm{i}(kR-\omega t)} \vec{\mathrm{e}}_\phi$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 c^2} \frac{1}{R} \ddot{p} \sin\theta \mathrm{e}^{\mathrm{i}(kR-\omega t)} \vec{\mathrm{e}}_\theta$$

$$\left\langle \vec{S} \right\rangle = \frac{1}{32\pi^2 \varepsilon_0 c^3} \frac{1}{R^2} \left(\ddot{p} \right)^2 \sin^2\theta \, \vec{n}$$

$$P = \frac{1}{4\pi\varepsilon_0} \frac{1}{3c^3} \left(\ddot{p} \right)^2$$

5、短天线的辐射　辐射电阻

1）短天线辐射的偶极辐射近似

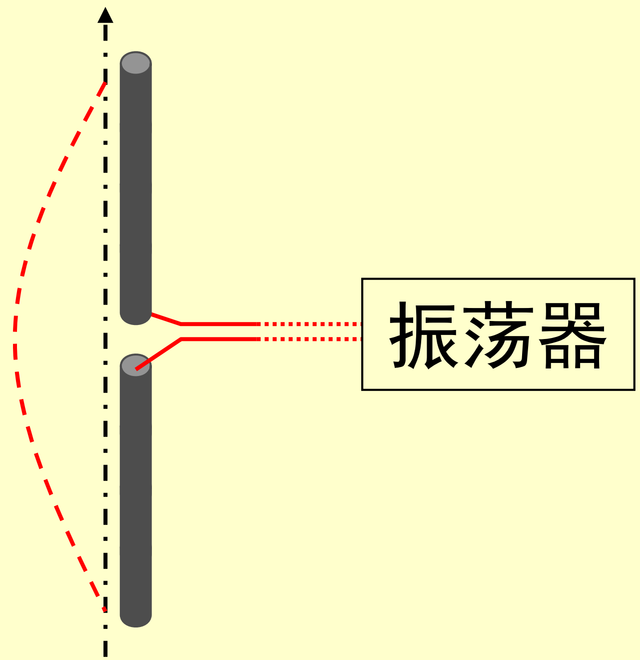
电偶极辐射近似：电荷体系的分布尺寸远小于辐射电磁波的波长

源点： $\vec{J}(\vec{x}',t)=\vec{J}(\vec{x}')e^{-i\omega t}$

2）短天线的辐射功率

对于中心馈电的直线短天线，

$$I_z = I_0\left(1-\frac{2}{\ell}|z|\right)$$



①馈电点处的电流最大；

②天线两端的电流为零。

$$\int \vec{J}(\vec{x}\,')\,\mathrm{d}V' = -\mathrm{i}\omega\vec{p}$$

$$\dot{\vec{p}} = -\mathrm{i}\omega\vec{p}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

$$\left|\dot{\vec{p}}\right| = \int \vec{J}\left(\vec{x}\,'\right)\mathrm{d}V'$$

$$\left|\dot{\vec{p}}\right| = \int_{-\ell/2}^{\ell/2} I\left(z\right)\mathrm{d}z$$

$$= \int_{-\ell/2}^{\ell/2} I_0\left(1-\frac{2}{\ell}\left|z\right|\right)\mathrm{d}z \quad = \frac{1}{2}I_0\ell$$

$$\ddot{\vec{p}} = -\mathrm{i}\omega\,\dot{\vec{p}}$$

$$P = \frac{1}{4\pi\varepsilon_0}\frac{1}{3c^3}\left|\ddot{p}\right|^2$$

总辐射功率：

$$P = \frac{1}{4\pi\varepsilon_0}\frac{\omega^2}{3c^3}\left|\dot{\mathrm{p}}\right|^2 \quad = \frac{1}{4\pi\varepsilon_0}\frac{\omega^2}{3c^3}\frac{1}{4}I_0^2\ell^2 \quad = \frac{\omega^2}{48\pi\varepsilon_0 c^3}I_0^2\ell^2$$

$$= \frac{4\pi^2}{48\pi\varepsilon_0 c\left(cT\right)^2}I_0^2\ell^2 \quad = \frac{\pi}{12}\sqrt{\frac{\mu_0}{\varepsilon_0}}I_0^2\frac{\ell^2}{\lambda^2}$$

$$P=\frac{\pi}{12}\sqrt{\frac{\mu_0}{\varepsilon_0}}I_0^2\frac{\ell^2}{\lambda^2}$$

① 在l<<λ 的情况下，对于给定的I_0，天线的辐射功率正比于 $(l/\lambda)^2$；

② 由于辐射功率正比于 I_0^2 ，如果把这种辐射等效成一个电阻上的损耗功率，则这个等效的电阻称为辐射电阻 R_r；

③ 能量不断的以电磁波的形式向外辐射，因此电源需要供给一定的功率来维持辐射。

$$P=\frac{\pi}{12}\sqrt{\frac{\mu_0}{\varepsilon_0}}I_0^2\frac{\ell^2}{\lambda^2}$$

3）短天线的辐射电阻：

$$P=\frac{1}{2}I_0^2R_r$$

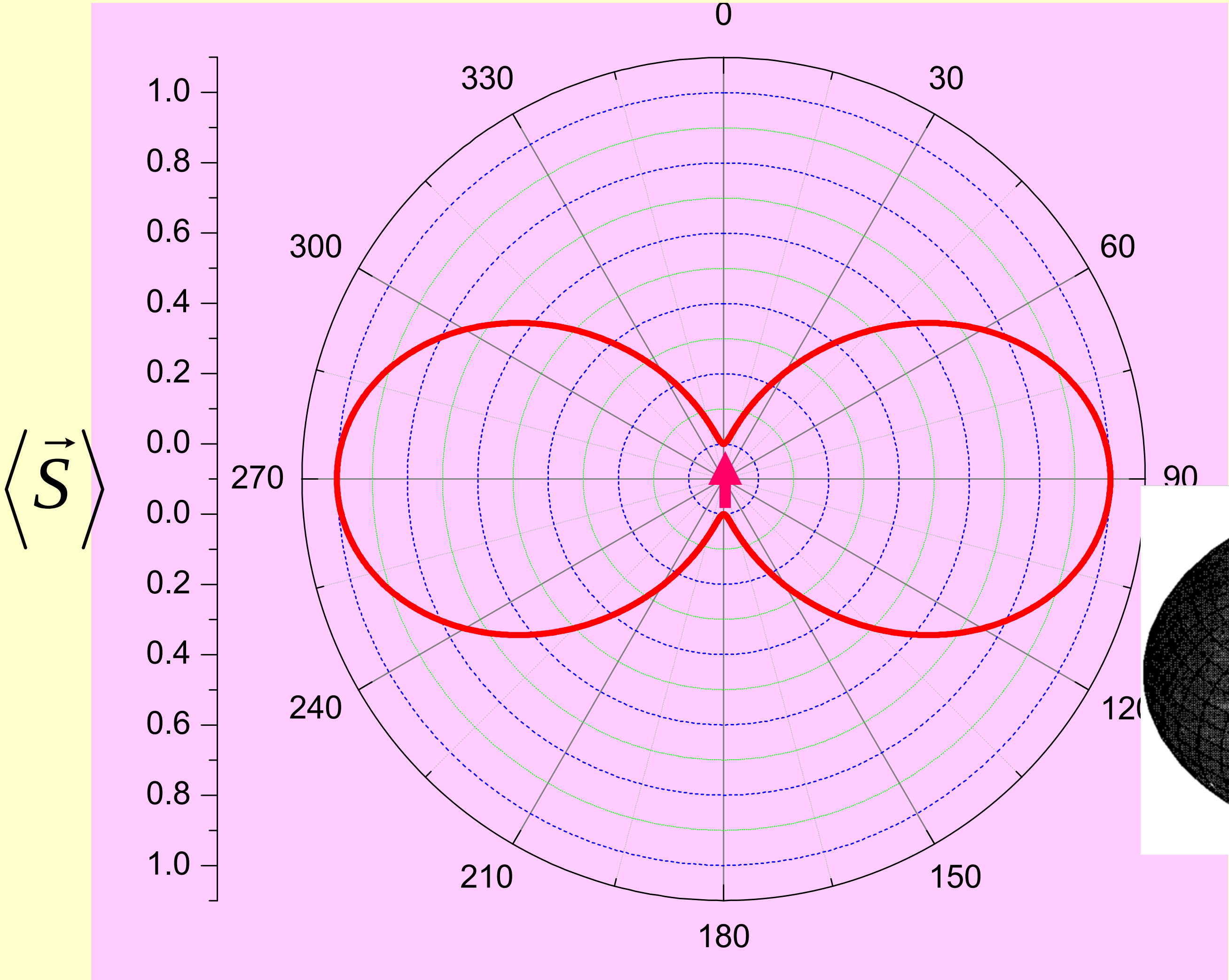
$$\sqrt{\frac{\mu_0}{\varepsilon_0}}=376.7\Omega$$

$$\Rightarrow\quad R_r=\frac{\pi}{6}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{\ell}{\lambda}\right)^2=187\left(\frac{\ell}{\lambda}\right)^2\Omega$$

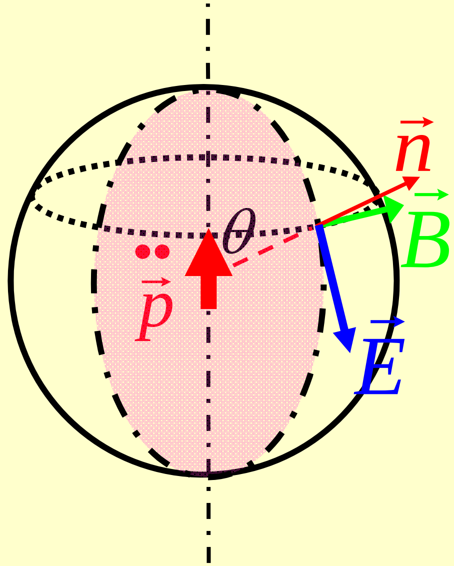
① 短天线的辐射能力有限；

② 要提高辐射功率，需将天线的长度增加到波长量级，但此时的辐射特性已经不能用偶极辐射来描述

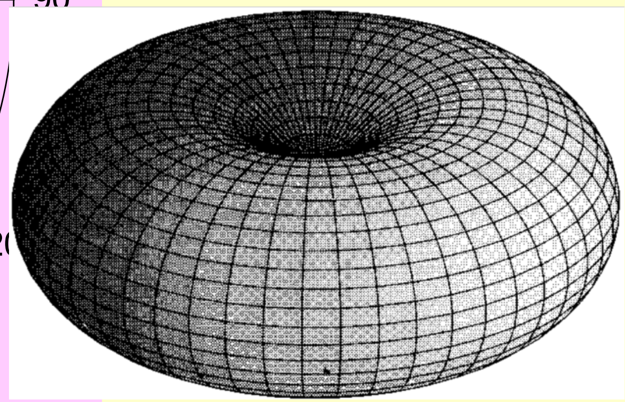
偶极子辐射



0
30
60
90
120
150
180
210
240
270
300
330
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\langle\vec{S}\rangle$



$\vec{n}$
$\vec{B}$
θ
$\ddot{\vec{p}}$
$\vec{E}$



Definition

Antenna (radio): An antenna (or aerial) is a transducer that transmits or receives electromagnetic waves.

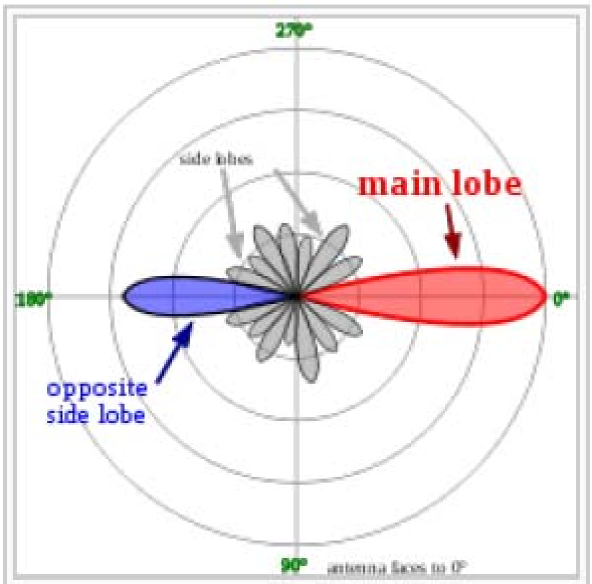
"Rabbit ears" dipole antenna for television reception

Cell phone base station antennas

Parabolic antenna for communicating with spacecraft, Canberra, Australia

Wikipedia

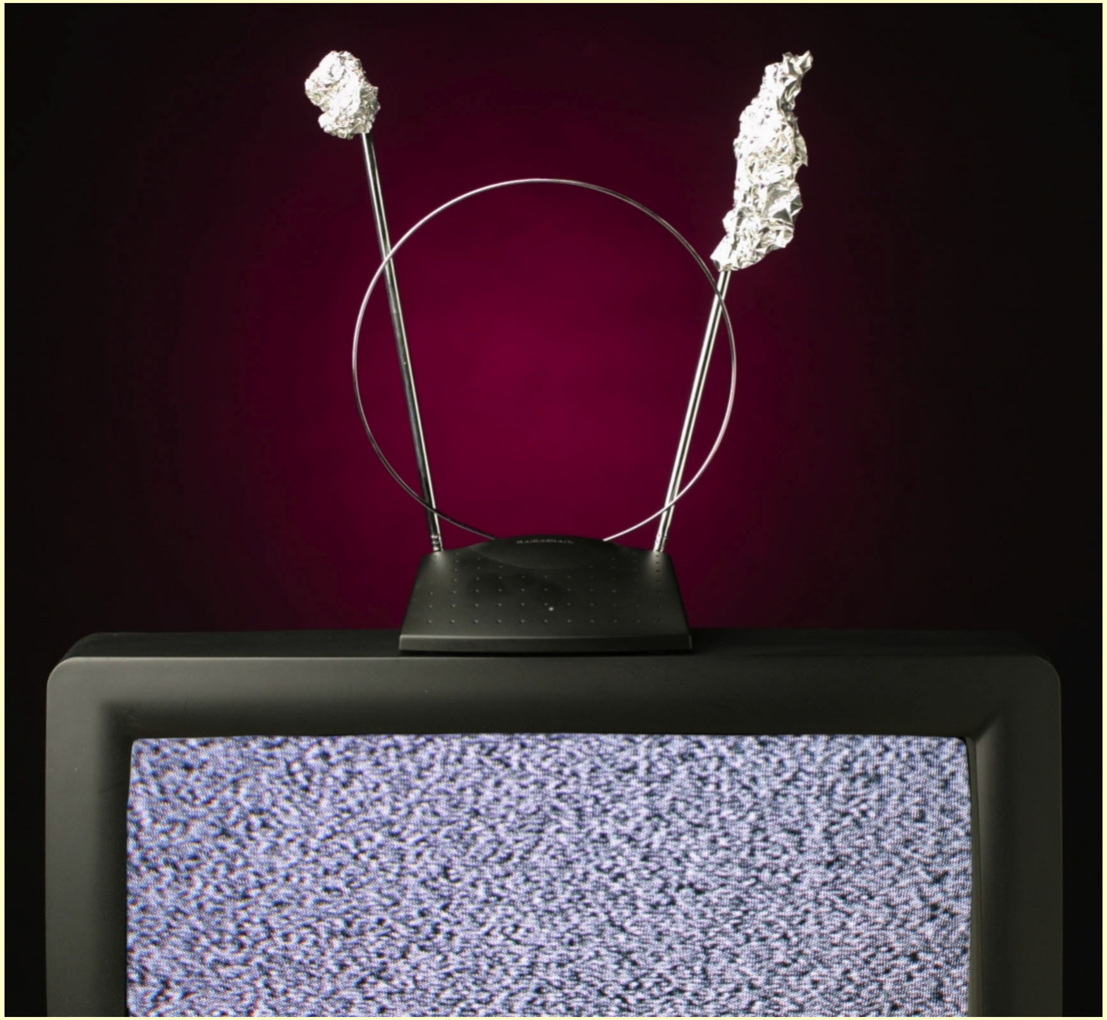
Yagi-Uda Antenna(八木宇田天线)



Yagi antenna used for mobile military communications station, Dresden, Germany, 1955

Wikipedia

"Rabbit ears" dipole antenna *for television reception*



Wikipedia

Cell phone base station antennas

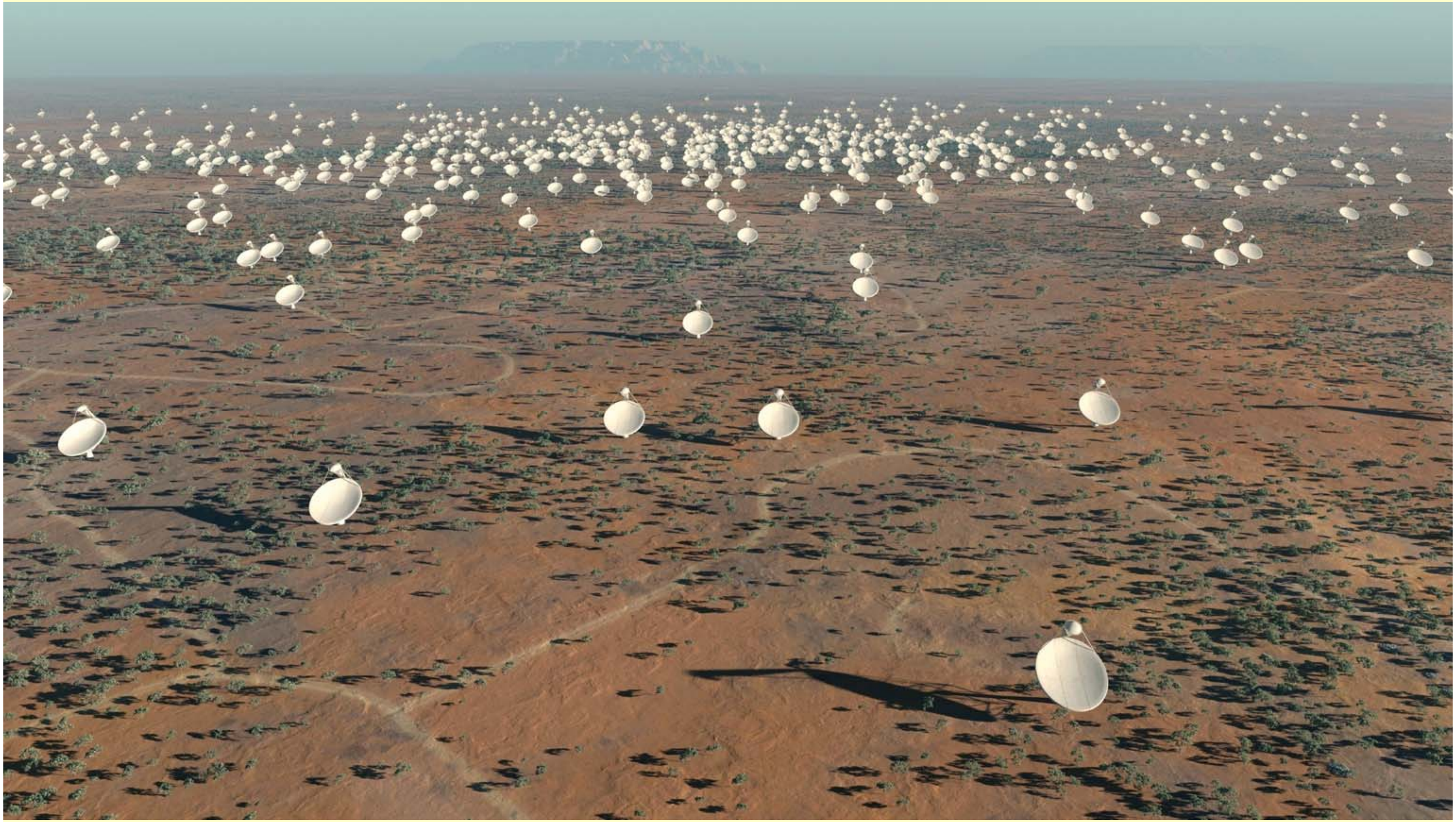
Parabolic antenna

Cell phone base station antennas

Parabolic antenna for communicating with spacecraft, Canberra, Australia

Wikipedia

天线阵列
(Antenna array)



相控阵天线

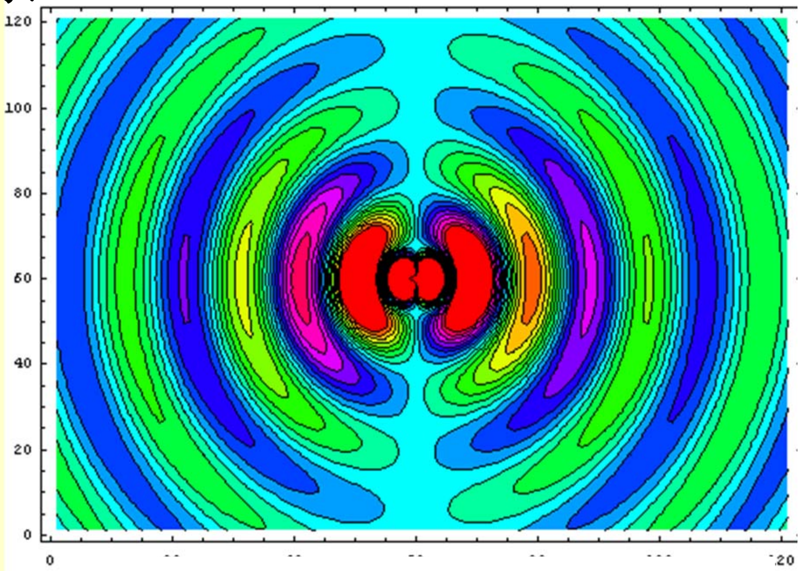
- 天线阵是将若干个天线按一定规律排列组成的天线系统。
-
- 利用天线阵可以获得所期望的辐射特性，诸如更高的增益、需要的方向性图等。
- 组成天线阵的独立单元称为阵元，排列的方式有直线阵、平面阵等。
- 天线阵的辐射特性取决于阵元的型式、数目、排列方式、间距，以及各阵元上的电流振幅和相位等。

偶极天线的辐射场

根据电偶极子辐射场公式， 在观察点产生的辐射电场为

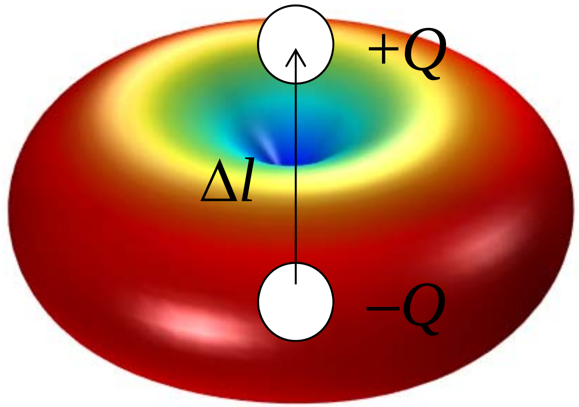
$$\vec{E} = \frac{|\ddot{p}|}{4\pi\varepsilon_0 c^2 R} e^{ikR} \sin\theta \vec{e_\theta}$$

$$\vec{B} = \frac{|\ddot{p}|}{4\pi\varepsilon_0 c^3 R} e^{ikR} \sin\theta \vec{e_\phi}$$



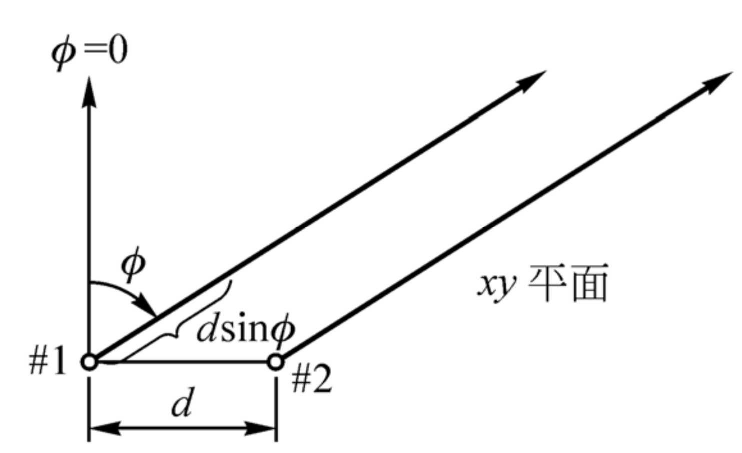
偶极天线的平均能流密度为

$$\overline{S} = \frac{|\ddot{p}|^2}{32\pi^2\varepsilon_0 c^3 R^2} \sin^2\theta \vec{n}$$



二元阵方向图相乘原理

如图所示两个沿 z 轴取向、沿 $\boldsymbol{x}$ 轴排列的对称天线构成的二元阵，间距为$\boldsymbol{d}$ 。设阵元**1**和 阵元**2**的相位差为 ξ，则



$$\left|p_2\right| = \left|p_1\right| \mathrm{e}^{i\xi}$$

两个阵元在观察点产生的电场

$$\vec{E}_1 = \frac{\left|\ddot{p}_1\right|}{4\pi\varepsilon_0 c^2 r_1} \mathrm{e}^{ikr_1} \sin\theta \vec{e}_\theta \qquad \vec{E}_2 = \frac{\left|\ddot{p}_2\right|}{4\pi\varepsilon_0 c^2 r_2} \mathrm{e}^{ikr_2} \sin\theta \vec{e}_\theta$$

二元阵的辐射场等于两个阵元的辐射场的矢量和。

两个阵元在观察点产生的电场

$$\vec{E}_1=\frac{|\ddot{\vec{p}}_1|}{4\pi\varepsilon_0 c^2 r_1}\mathrm{e}^{ikr_1}\sin\theta\vec{e}_\theta \qquad \vec{E}_2=\frac{|\ddot{\vec{p}}_2|}{4\pi\varepsilon_0 c^2 r_2}\mathrm{e}^{ikr_2}\sin\theta\vec{e}_\theta$$

对于远离天线阵的观察点，可作如下近似：

$$\frac{1}{r_1}\approx\frac{1}{r_2}\ (\text{对振幅项}) \qquad r_2\approx r_1-d\sin\phi\quad(\text{对相位项})$$

因此 $$e^{ikr_2}\approx e^{ik(r_1-d\sin\varphi)}=e^{ikr_1}e^{i\Psi}$$

$$\vec{E}_2=\vec{E}_1e^{i\Psi} \quad\Rightarrow\quad \boxed{\vec{E}_{\mathrm{rad}}=\vec{E}_1(1+e^{i\Psi})}$$

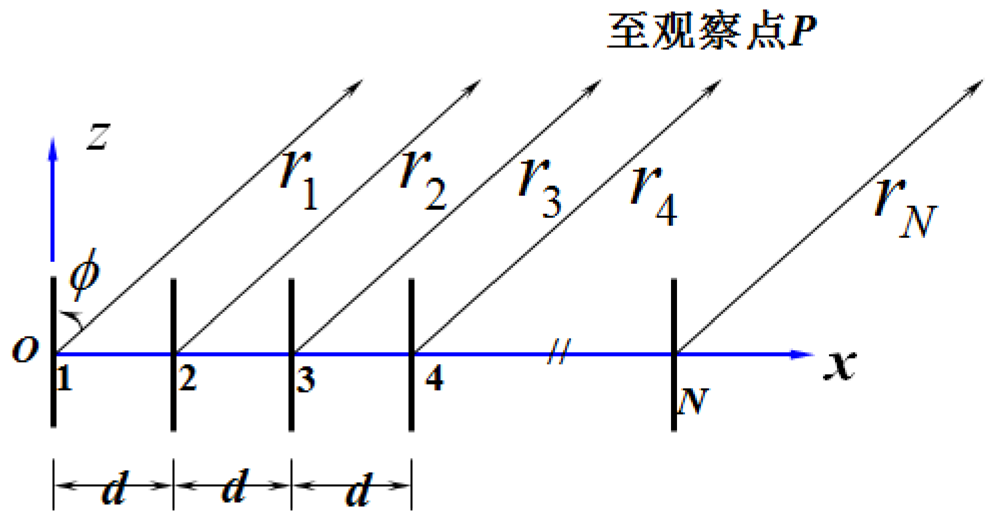
式中 $\psi=\xi-kd\sin\phi$ 是观察点$\boldsymbol{P}$处的电场$\boldsymbol{E_1}$和$\boldsymbol{E_2}$由的相位差。

均匀直线式天线阵

均匀直线阵是指天线阵的各阵元结构相同，并以相同的取向和相等的间距排列成直线，各个阵元的激励电流振幅相等、相位则沿阵的轴线以相同的比例递增或递减的天线阵。

N个阵元沿 ***x*** 轴排列，两相邻阵元的间距为***d***，激励电流相位差为ξ，则相邻两阵元辐射场的相位差为

$$\psi = \xi - kd\sin\phi$$



均匀直线阵

以阵元1为参考，则阵元 2 的辐射场的相位差为ψ，阵元3的辐射场的相位差为2ψ，依此类推。天线阵的辐射场为

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_N$$

$$= \vec{E}_1 \left[1 + e^{i\psi} + e^{i2\psi} + e^{i3\psi} + \cdots + e^{i(N-1)\psi} \right]$$

则 $$\left|\vec{E}\right| = \left|\vec{E}_1\right| \left| \frac{1 - e^{iN\psi}}{1 - e^{i\psi}} \right| = \left|\vec{E}_1\right| f_N(\psi) \qquad f_N(\psi) = \sin\frac{N\psi}{2} \Big/ \sin\frac{\psi}{2}$$

N 元均匀直线阵的阵因子

因 $$f_{N\max} = \lim_{\psi \to 0} \left(\sin\frac{N\psi}{2} \Big/ \sin\frac{\psi}{2}\right) = N$$

故 ***N*** 元均匀直线阵的归一化阵因子 $$F_N(\psi) = \frac{1}{N} \sin\frac{N\psi}{2} \Big/ \sin\frac{\psi}{2}$$

一维天线阵列的数值讨论

$$\text{Array Factor} = \left| \frac{\sin \frac{N\Psi}{2}}{\sin \frac{\Psi}{2}} \right|$$

$$\psi = -kd \sin \phi$$

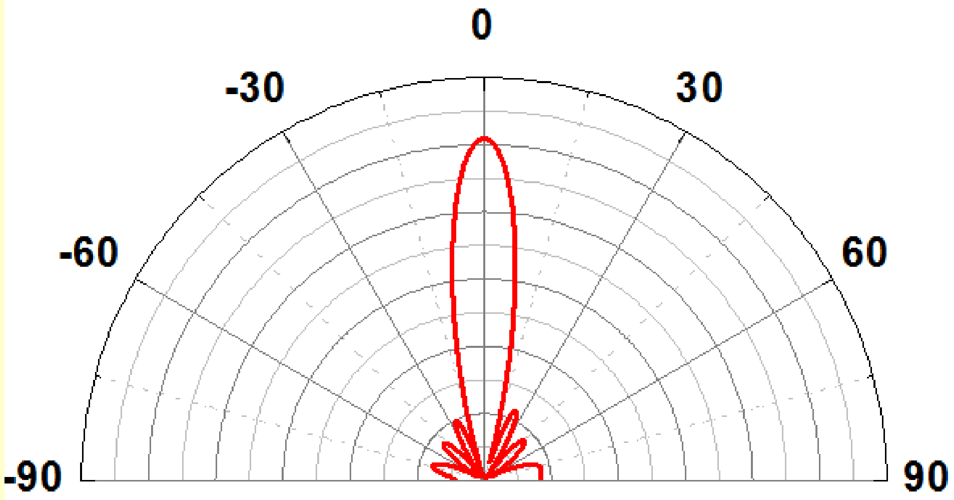
- 阵列因子对ψ=0成对称(因ψ和$-\psi$代入结果相同)
- 无效角由 $\sin\left(\frac{Nkd \sin \phi}{2}\right) = 0$ 决定

即 $\sin \phi_{null} = \frac{n\lambda_0}{Nd}$ ，$n = 0$，± 1，± 2，. . . .，须保持

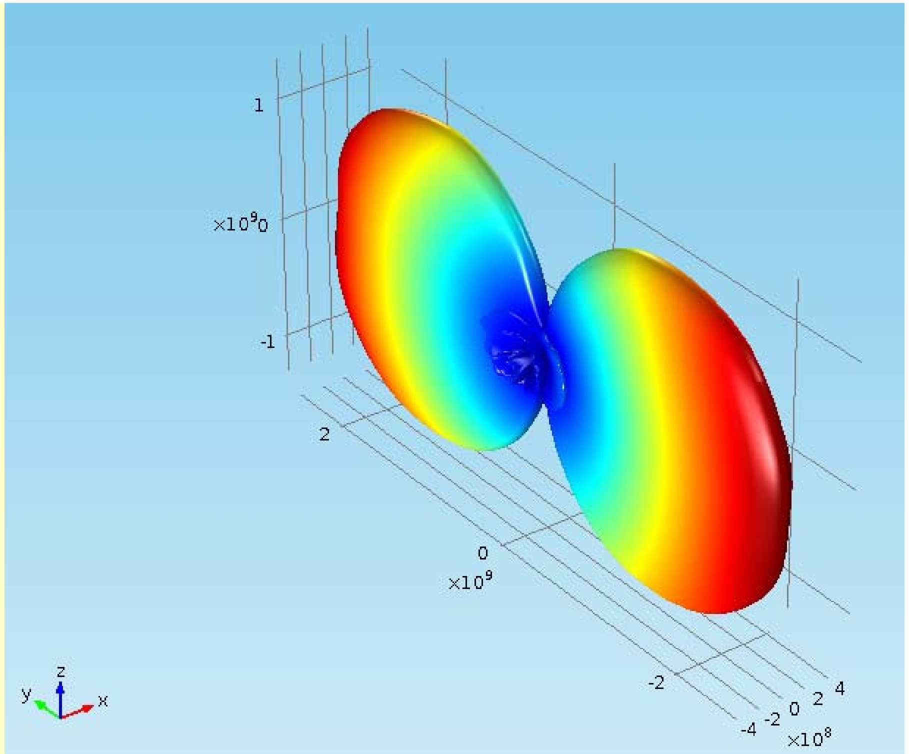
$$\frac{n\lambda_o}{Nd} \leq 1$$

- 最大值 φ=0时, $\left| \frac{\sin \frac{N\Psi}{2}}{\sin \frac{\Psi}{2}} \right| \to N$

一维天线阵列计算例子



天线阵列的辐射方向



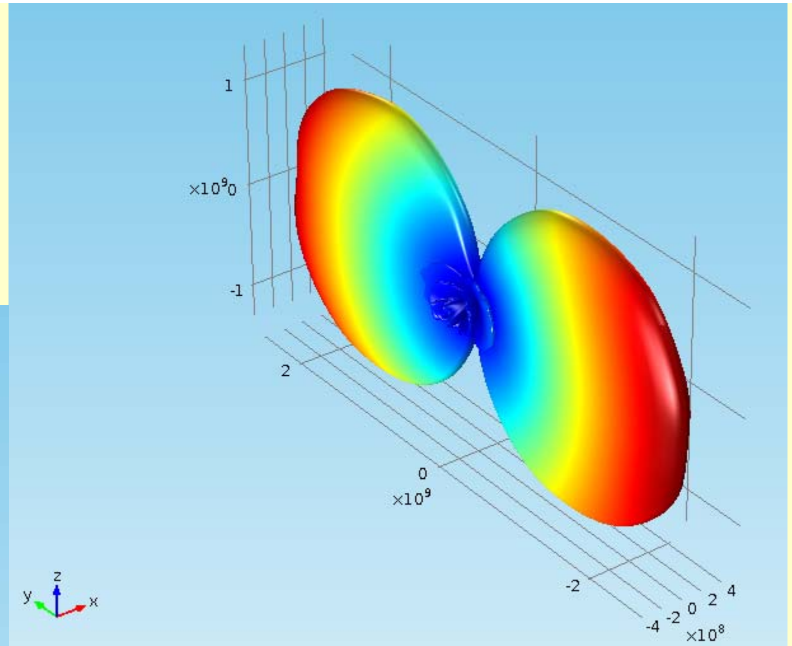
- N=7
- $d = \lambda_0/2$
- 求第一个无效角

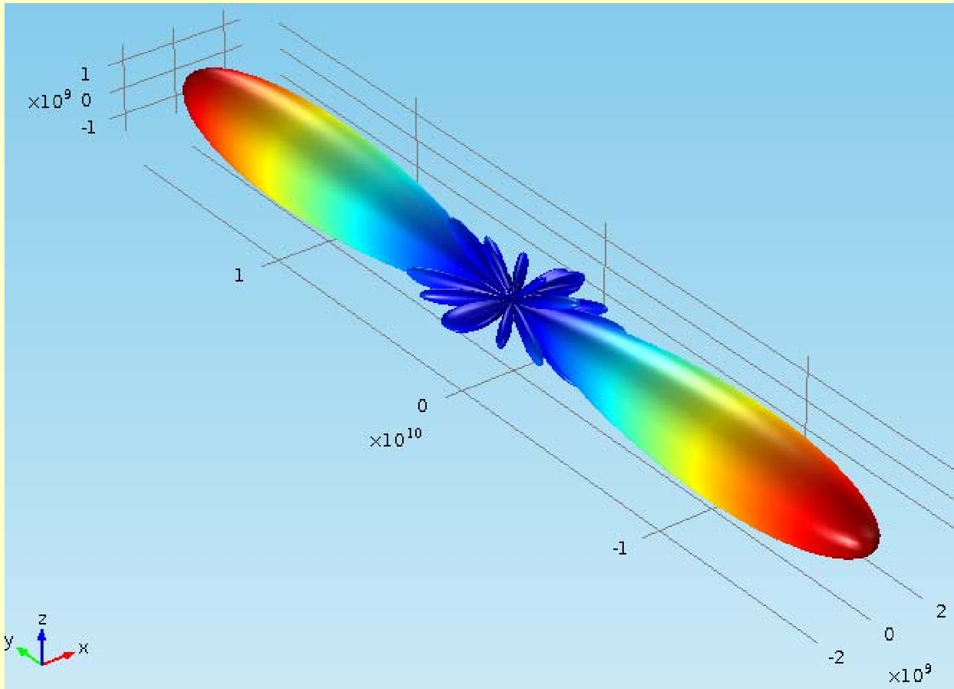
$$\text{Array Factor} = \left| \frac{\sin \frac{N\Psi}{2}}{\sin \frac{\Psi}{2}} \right|$$

$$\sin\left(\frac{Nkd\sin\phi}{2}\right) = 0$$

$$\sin\varphi_{\text{null}} = \frac{\pm\lambda_0}{Nd} = \pm\frac{2}{7}, \quad \varphi_{\text{null}} \approx \pm 16.6°$$

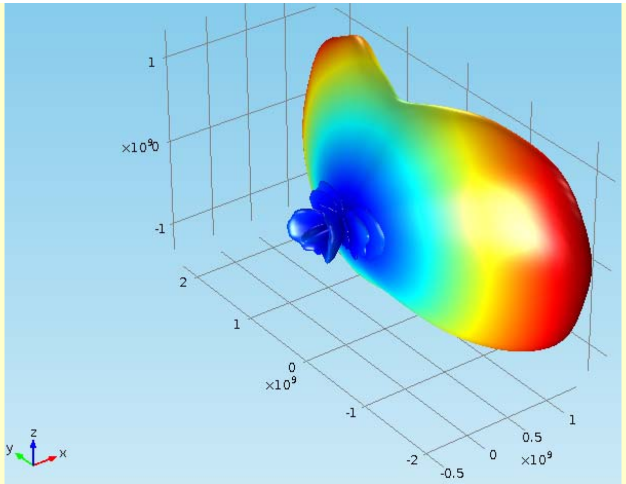
扇形波束和笔形波束

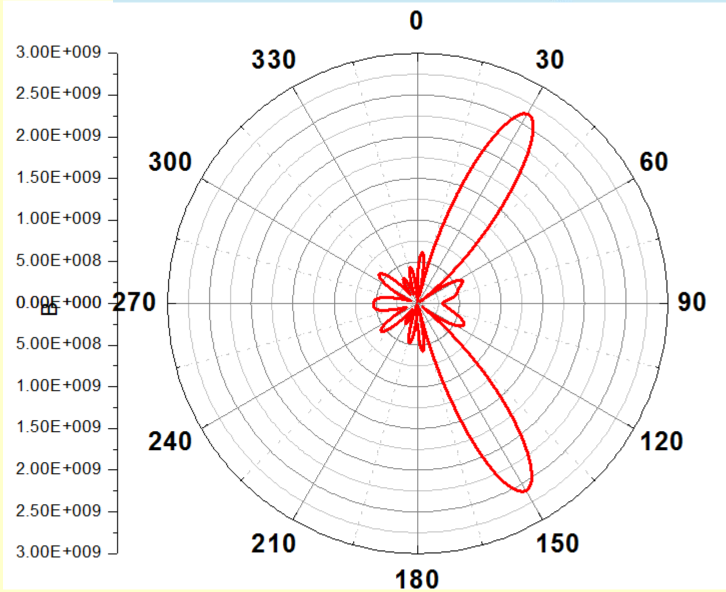




-
- ，但是在 ϕ为定值的平 波束犹如薄扇,所以称为
- 取7个一模一样的上例一维阵列排列成7×7的方阵，可预期波束在方向和方向的夹角都都差不多只有16°，这种波束，称为笔形波束,定向性极佳

天线电流的相位与指向性





- $|\sin(N\Psi/2)/\sin(\Psi/2)|$ 之最大仍在

  $\Psi=0$，亦即

  $$\psi = \xi - kd\sin\phi = 0$$

  $$\sin\phi_m = \frac{\xi}{kd}$$

- 波束指向转到角度ϕ_m

- 例如 $N=7$，$\xi = \pi/2, d = \lambda/2$ ，求天线的指向方向

  $$\sin\phi_m = \frac{\xi}{kd} = 1/2 \quad \Rightarrow \quad \phi_m = 30^\circ, 150^\circ$$

The definition of Optical Antenna

IEEE standard definitions of terms for antennas:

a means for radiating or receiving radio waves

In analogy

Optical Antenna:

A device designed to efficiently convert free-propagating optical radiation to localized energy, and vice versa

Motivations of radio and optical antenna

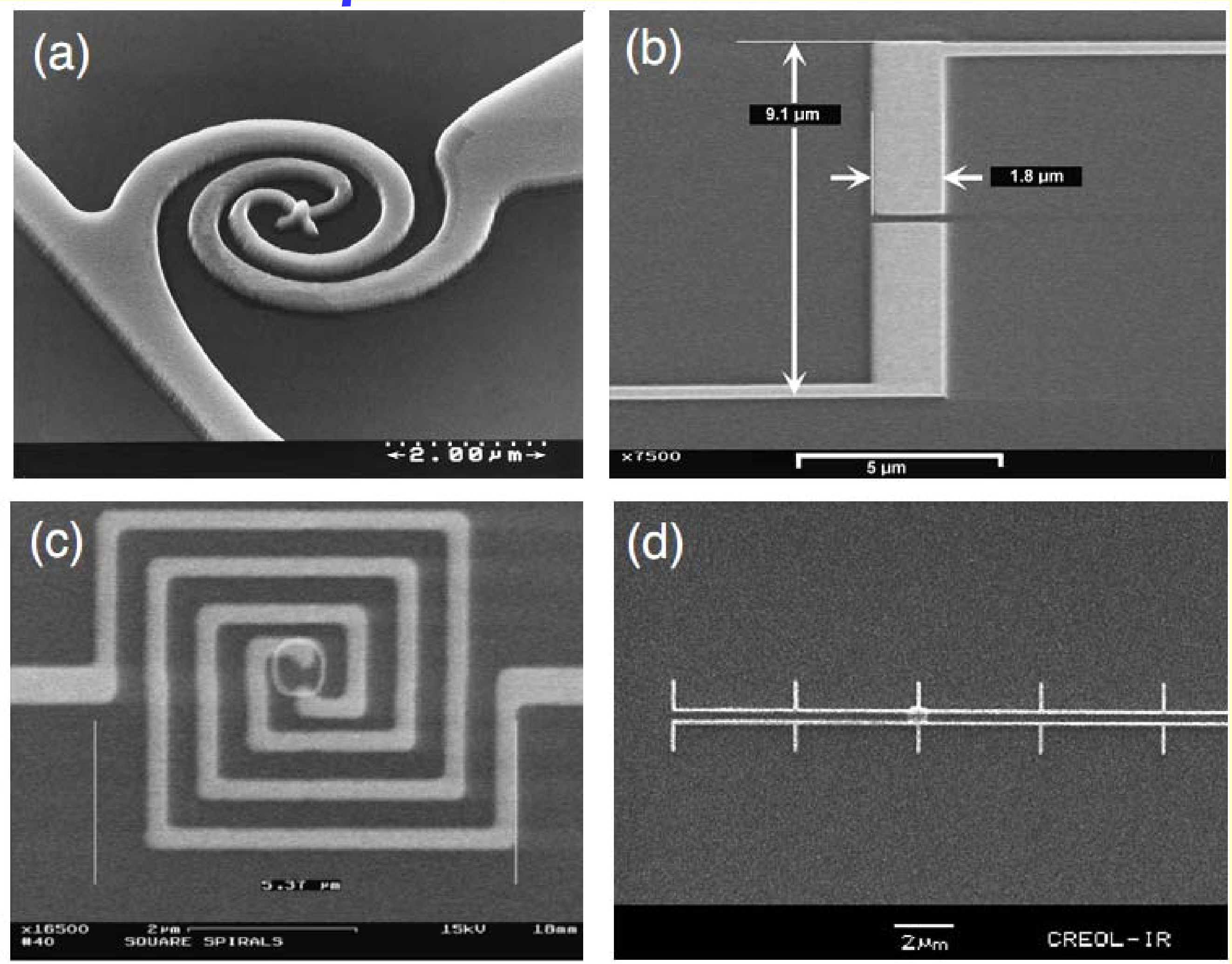
- Radio antennas: developed as solutions to a communication problem
- Optical antennas: motivated by microscopy

Why?

Enable us to concentrate external radiation to dimensions smaller than diffraction limit

Some earlier IR optical antennas

Field confinement $\sim\lambda / 2.7$



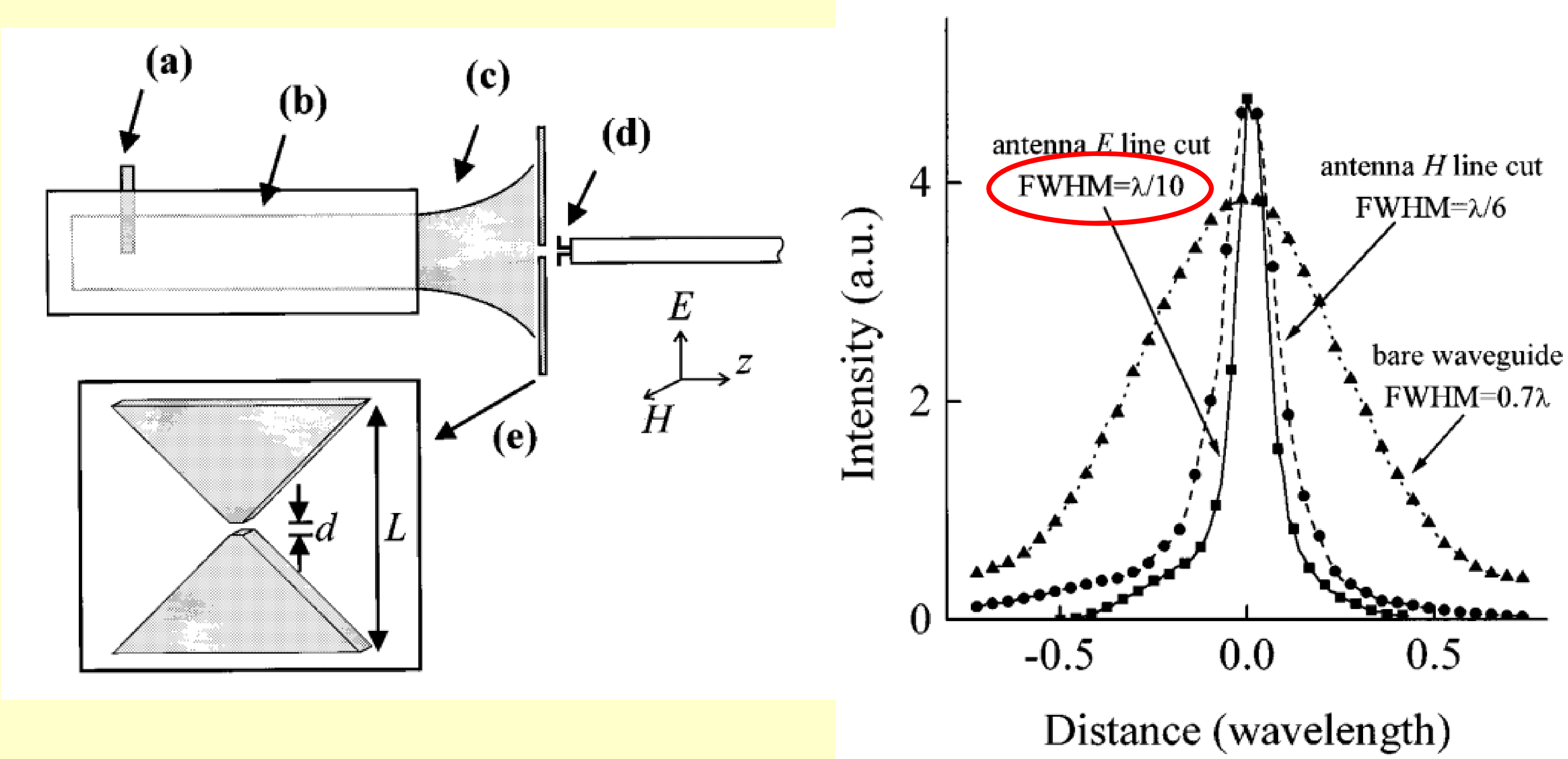
Fabricated by Boreman etc. since late 1990s

Advances in Optics and Photonics 1, 438–483 (2009)

Optical antenna: Towards a unity efficiency near-field optical probe

Robert D. Grober,[a] Robert J. Schoelkopf, and Daniel E. Prober
Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06520

(Received 22 October 1996; accepted for publication 13 January 1997)



However, its size ~cm

The spectrum we interested in is optical wavelength which means we must shrink the element's size

But *at optical frequencies, metals are no longer perfect conductors while at microwave frequencies metals act like a mirror*

Localized electronic oscillations-Surface Plasmon

What do we want?

- When receiving electromagnetic field, we want that the incoming field can be effectively confined
- When emitting electromagnetic field, we want that the radiating field can be directionally controlled

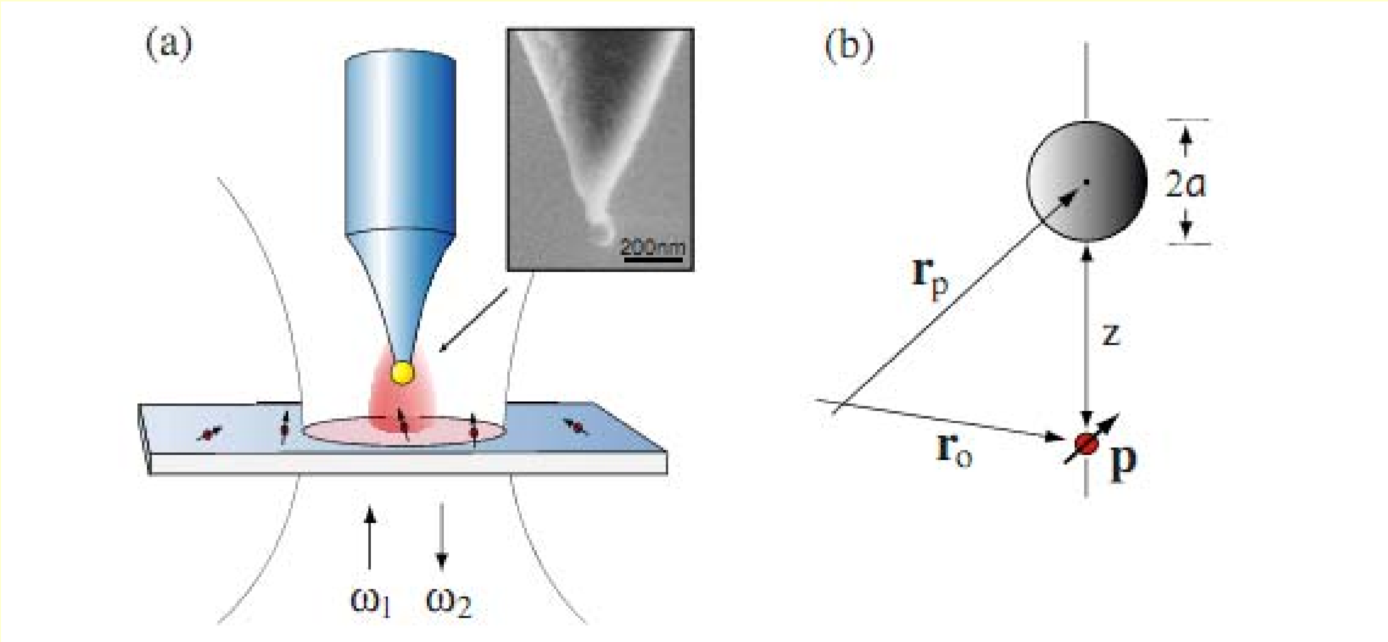
A simple example: Spherical Nanoparticles

"The particle serves as an antenna that receives an incoming electromagnetic field"

John Wessel in 1985

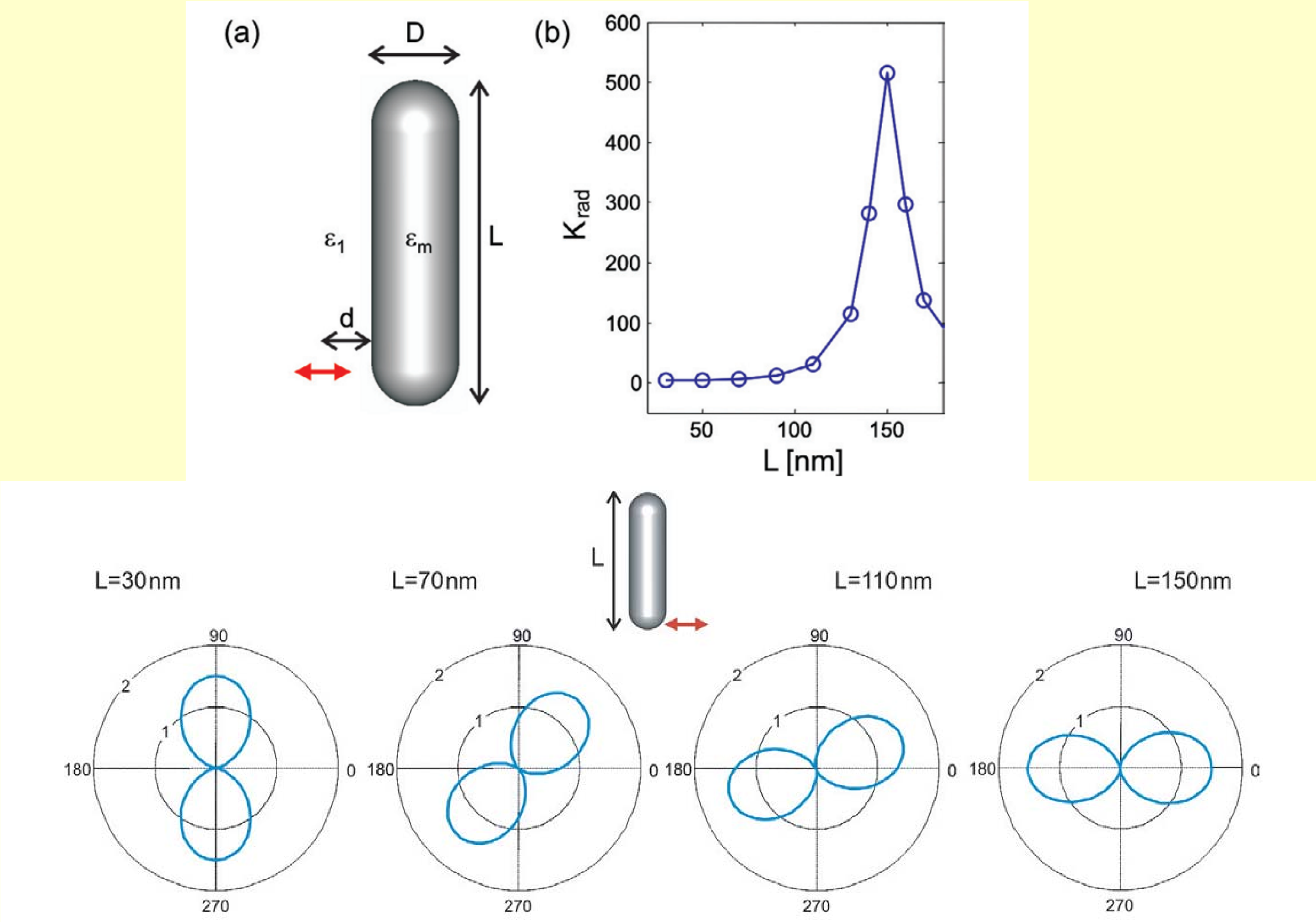
The polarizability $\alpha = 4\pi\varepsilon_o a^3[(\varepsilon(\omega)-1)/(\varepsilon(\omega)+2)]$ (Quasi-static limit)

$\mathrm{Re}\{\varepsilon\} = -2$ LSPR condition



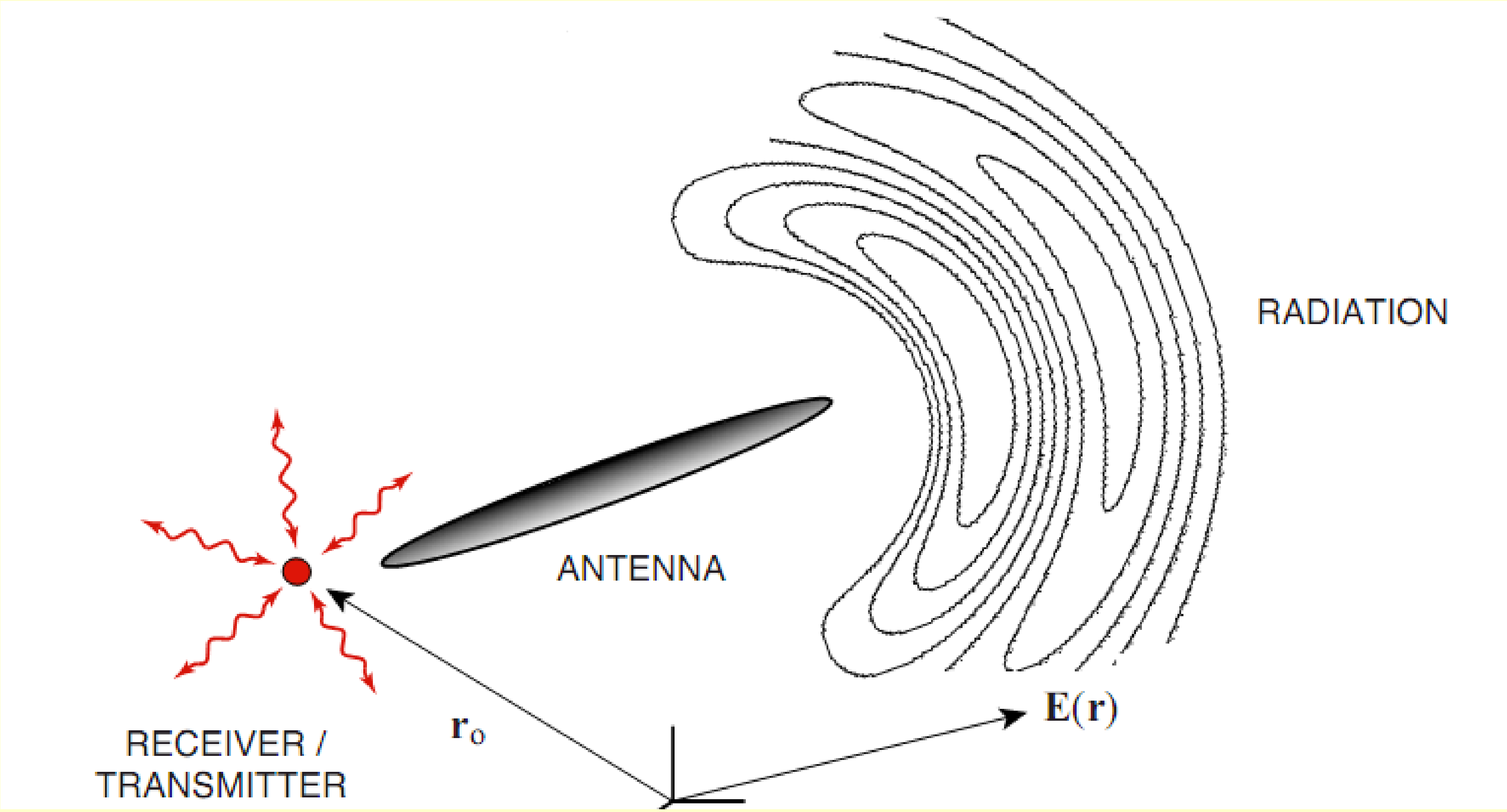
Advances in Optics and Photonics 1, 438–483 (2009)

Nanorods



New Journal of Physics 10 (2008) 105005

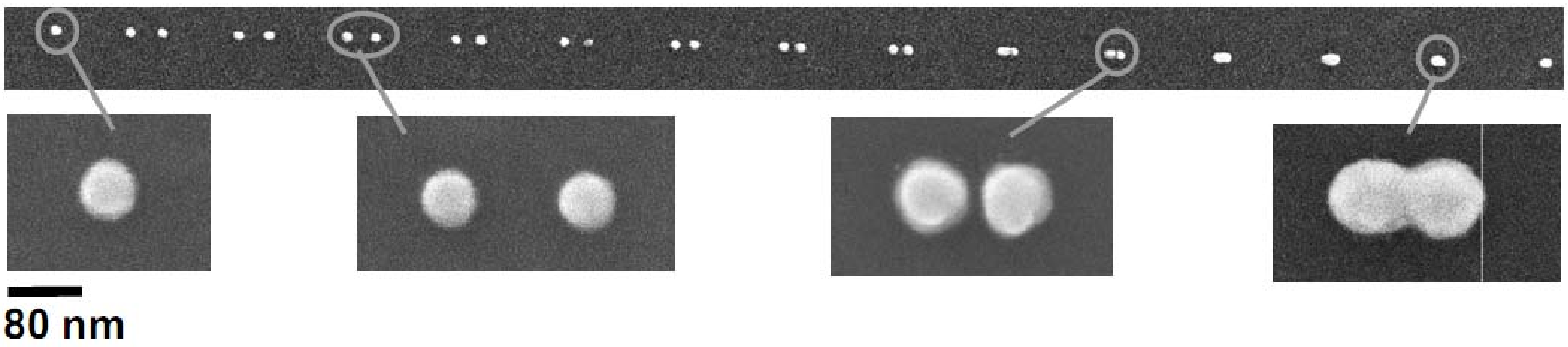
General problem statement

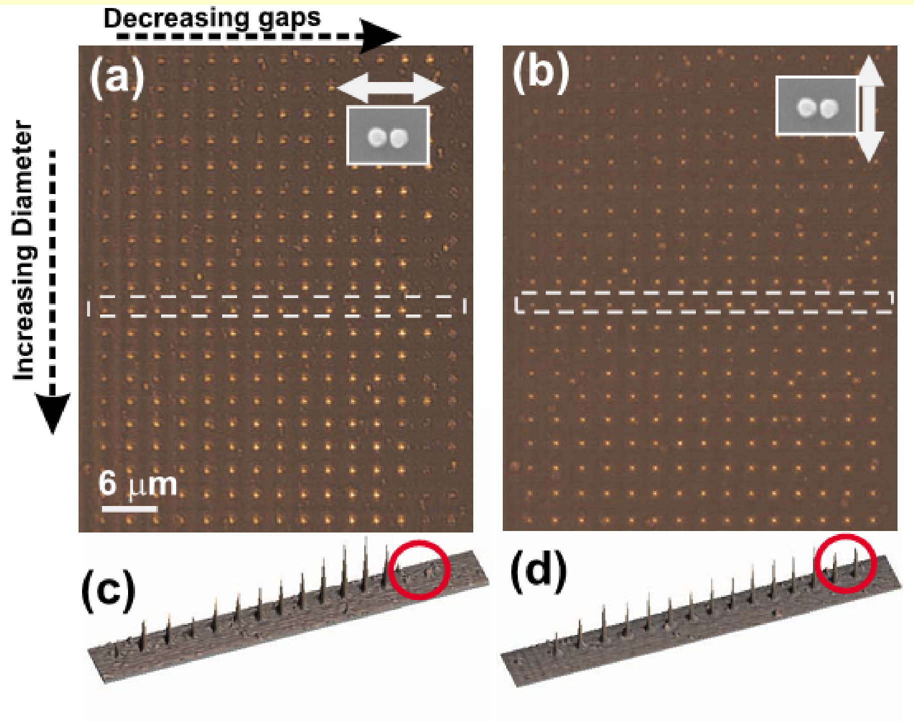


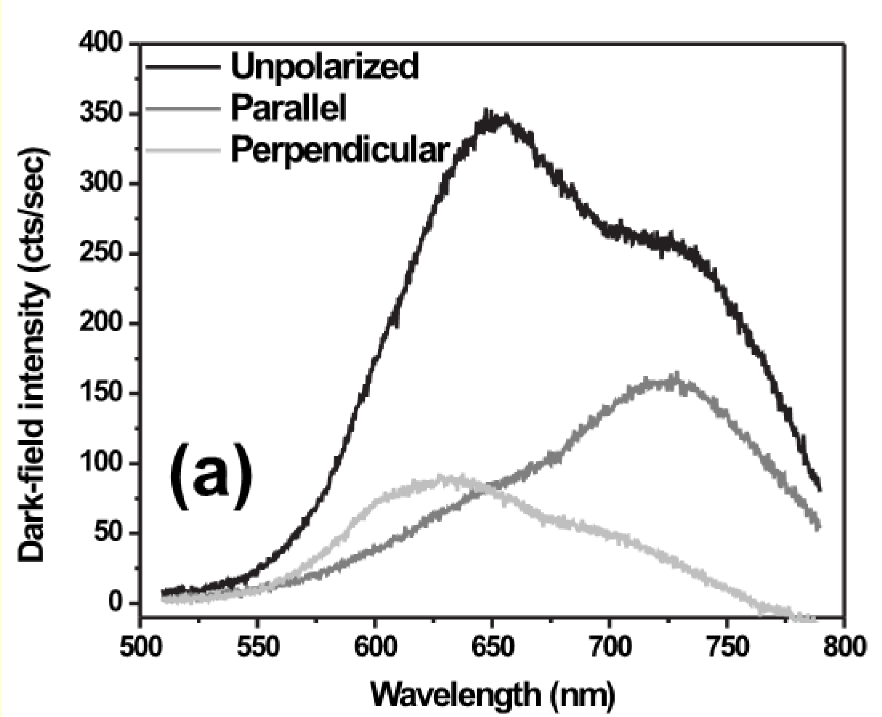
Control light-matter interaction of a single quantum system

Advances in Optics and Photonics 1, 438–483 (2009)

Particle pair(large field enhancement)







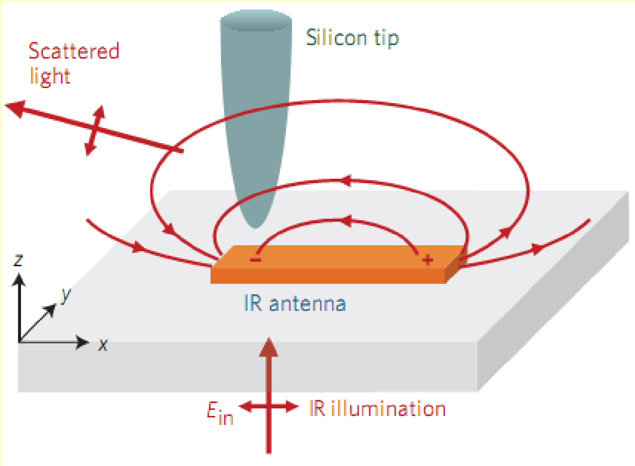
PRB 78, 155407 2008

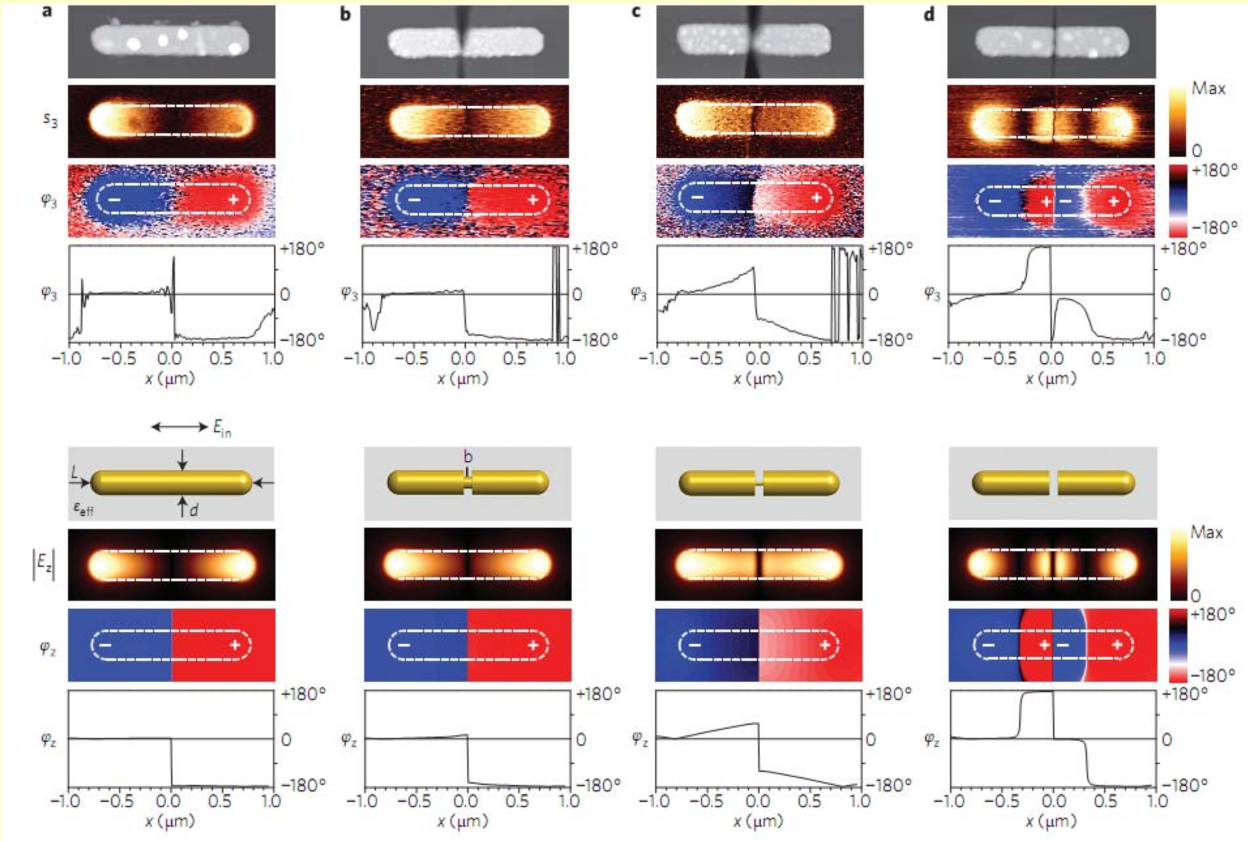
Nanorods Pair

nature photonics
LETTERS
PUBLISHED ONLINE: 19 APRIL 2009 | DOI: 10.1038/NPHOTON.2009.46

Controlling the near-field oscillations of loaded plasmonic nanoantennas

M. Schnell[1], A. García-Etxarri[2], A. J. Huber[1,3], K. Crozier[4], J. Aizpurua[2] and R. Hillenbrand[1,3]*





Bowtie

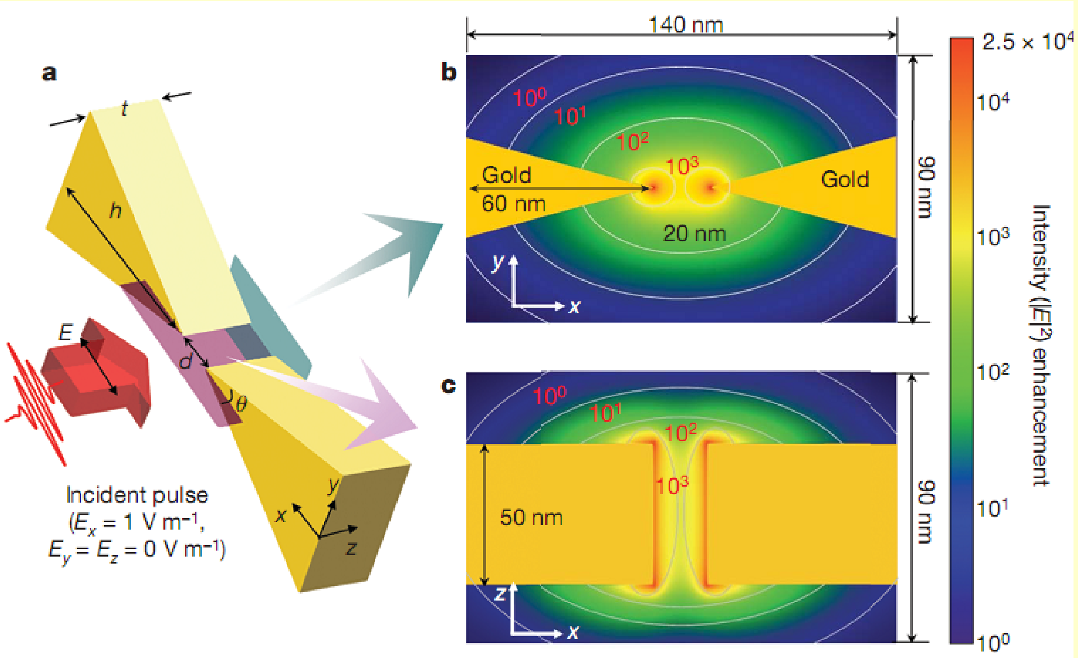
Vol 453 | 5 June 2008 | doi:10.1038/nature07012

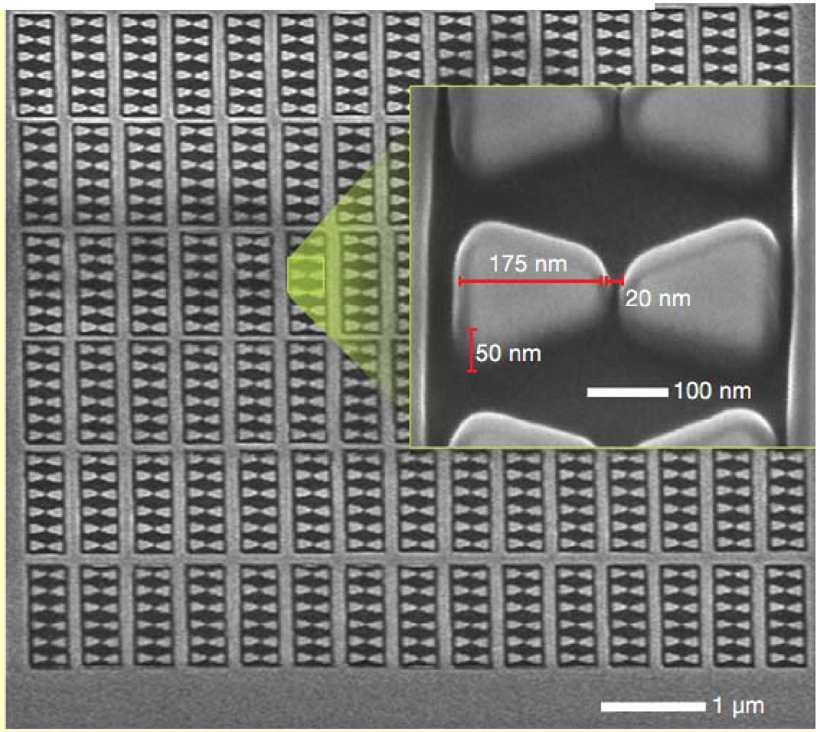
nature

LETTERS

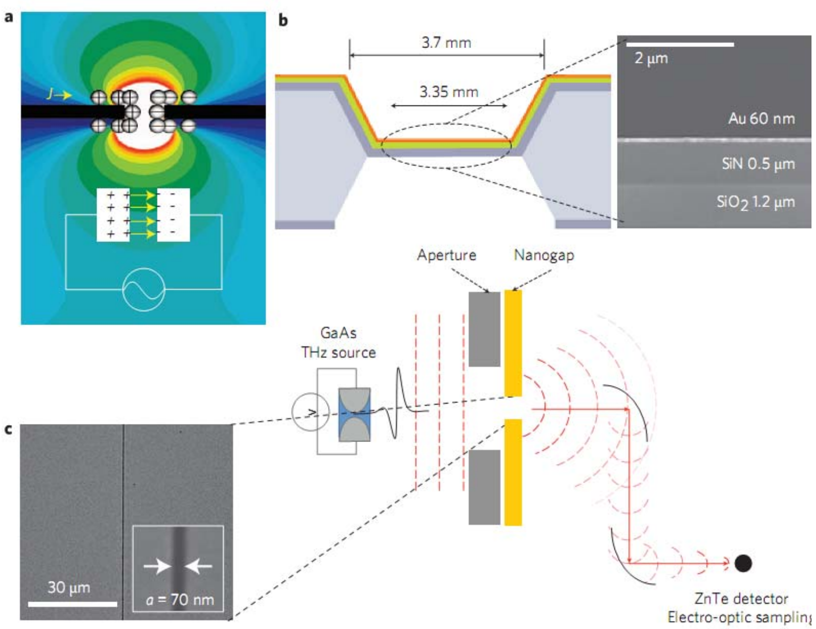
High-harmonic generation by resonant plasmon field enhancement

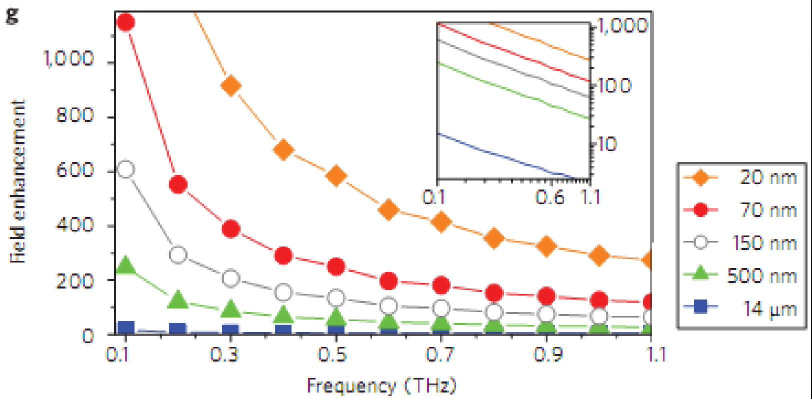
Seungchul Kim[1]*, Jonghan Jin[1]*, Young-Jin Kim[1], In-Yong Park[1], Yunseok Kim[1] & Seung-Woo Kim[1]

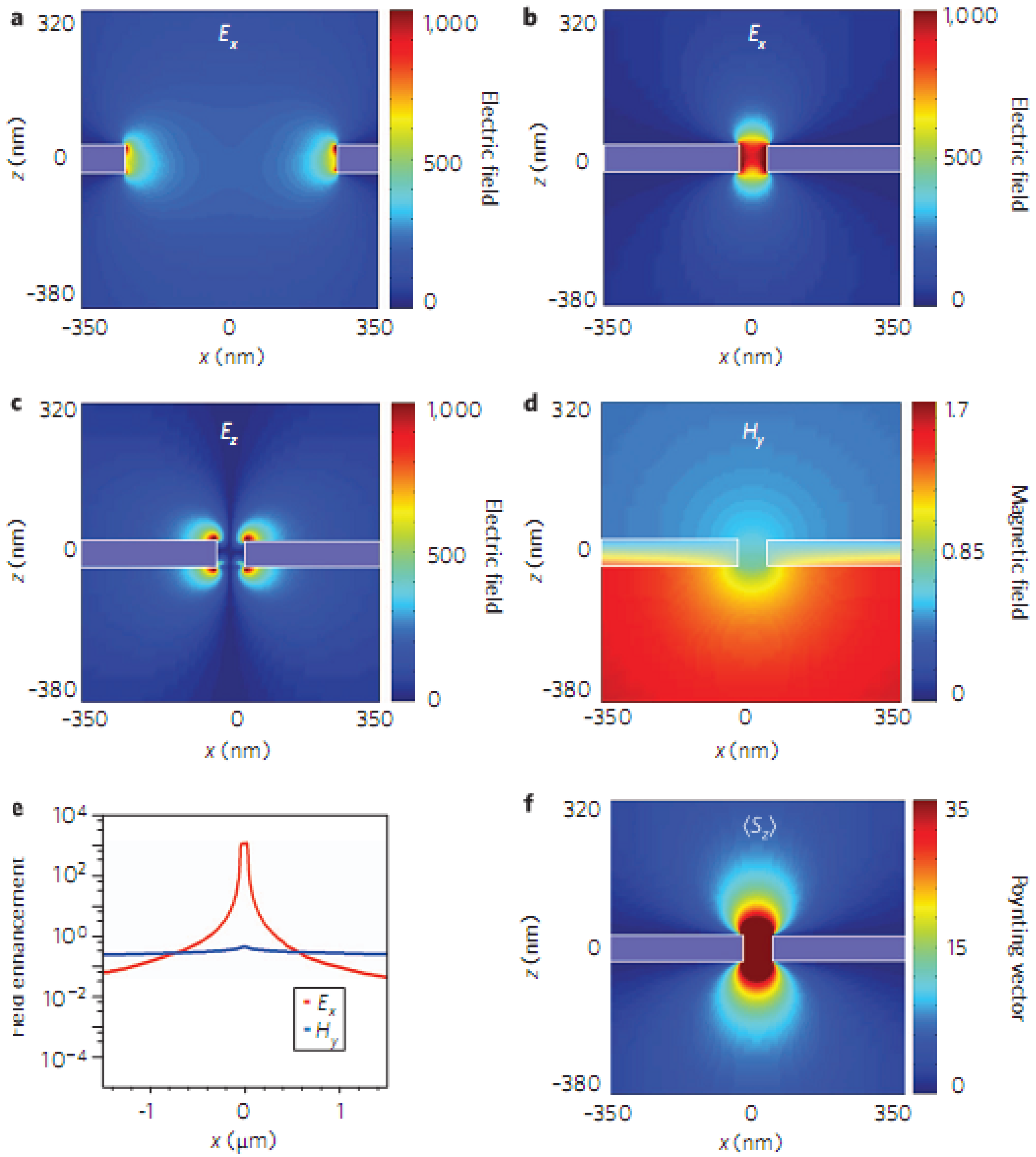




Nanoslit





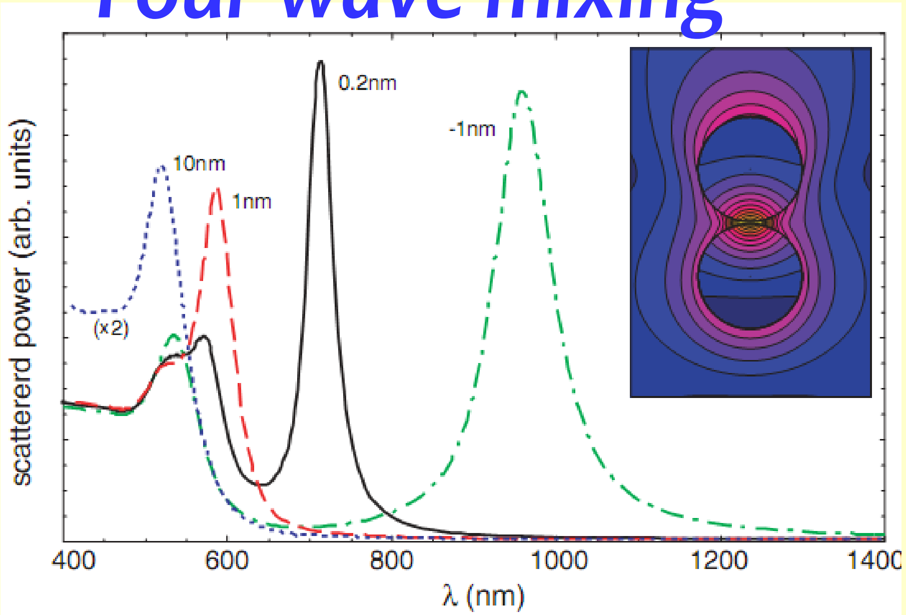


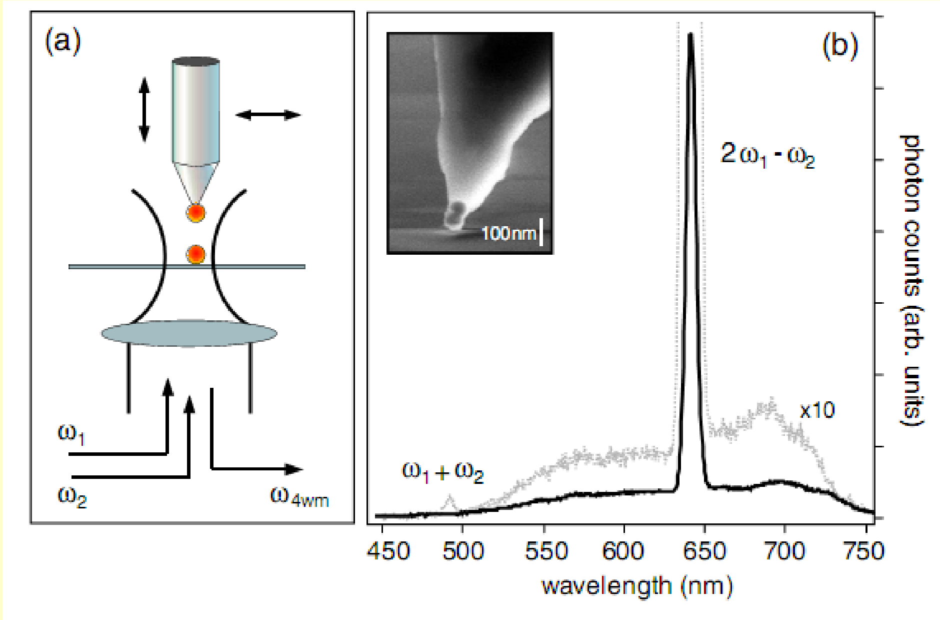
NPHOTON.2009.22

Nonlinear Antenna behavior

the size of the structures become comparable with the electron mean free path, such as close to tips,corners,gaps

Four wave mixing



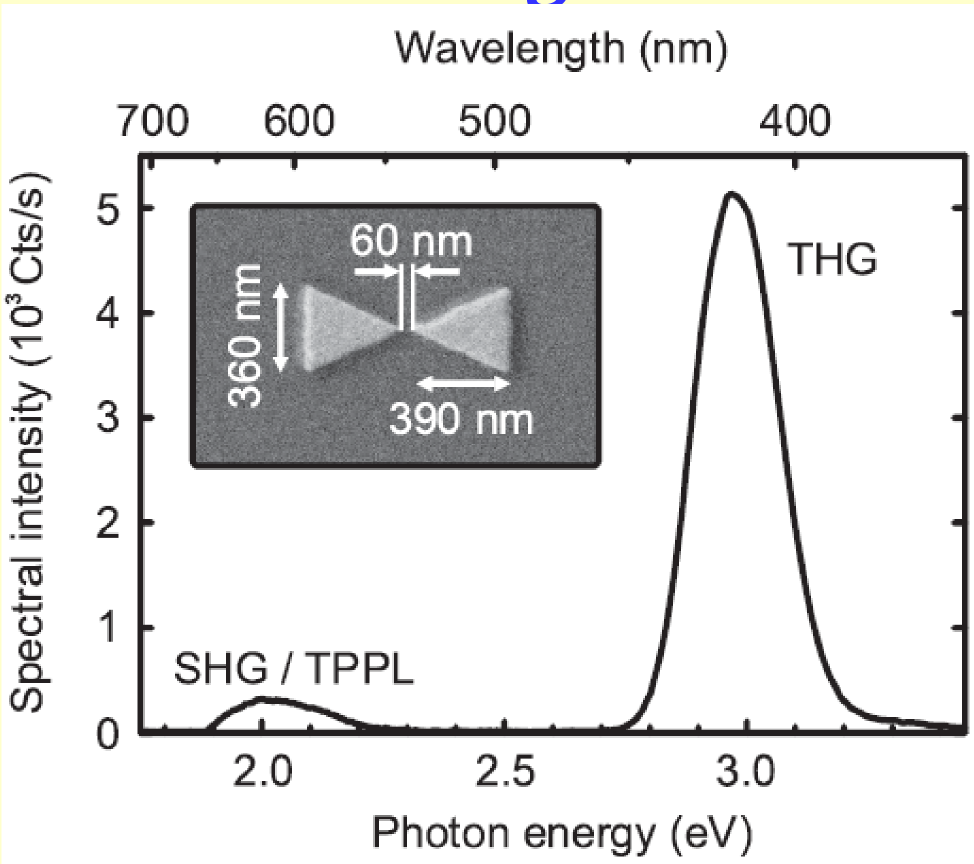


Laser pulses of wavelength w1,w2

PRL 98, 026104 (2007)

second-harmonic generation

third-harmonic generation

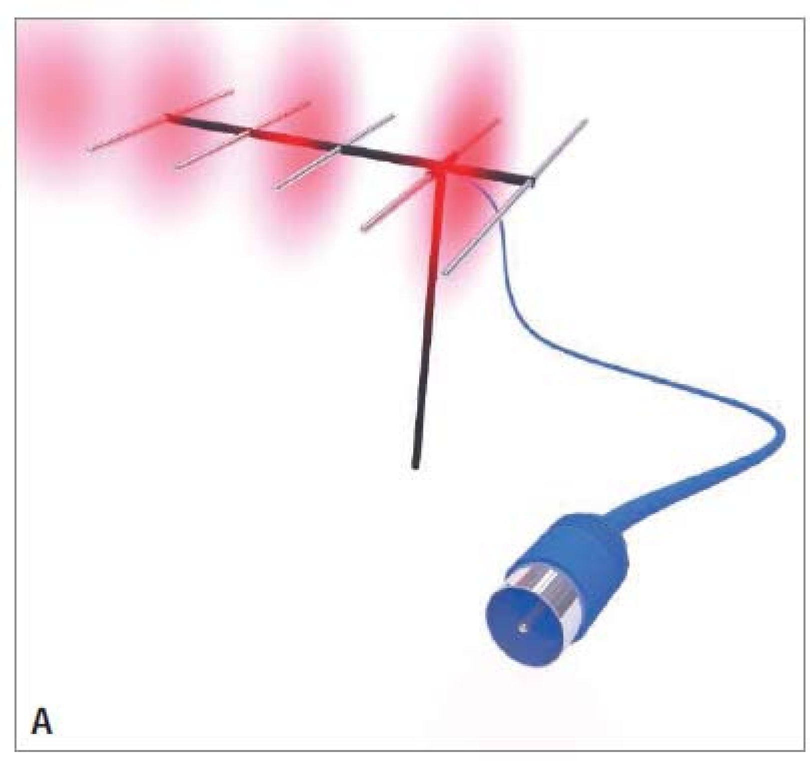


nonlinear emission spectrum from a single gold optical antenna excited with the total bandwidth of the 7.8-fs pulse

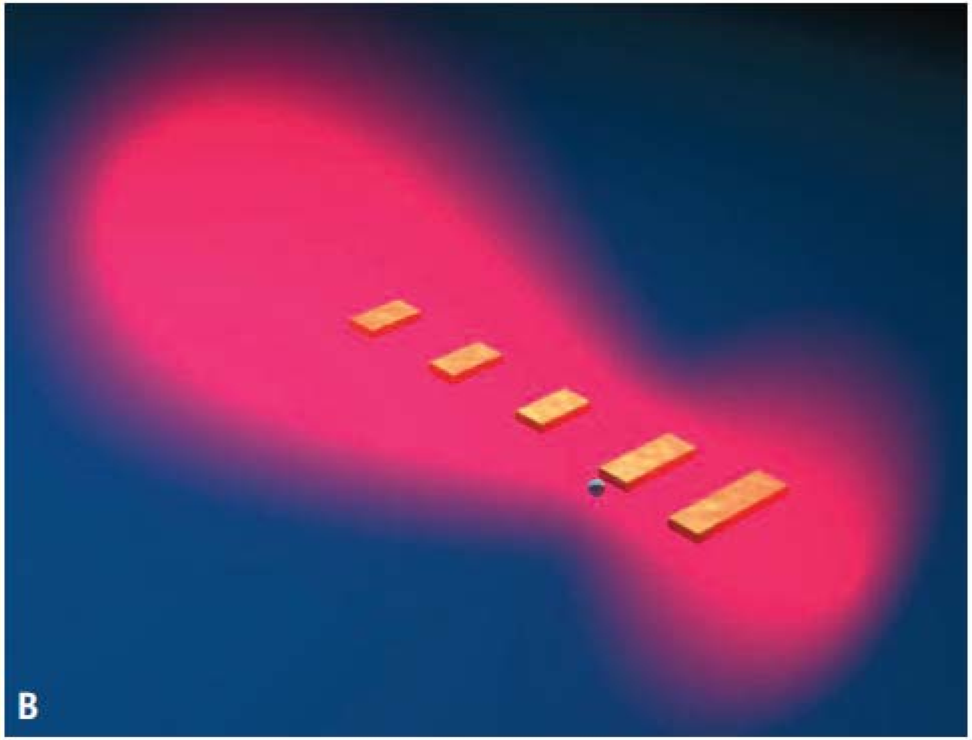
the plasmon resonance of the metal nano-structure in the near infrared

PRL 103, 257404 (2009)

directional control of radiation
（天线辐射的方向控制）



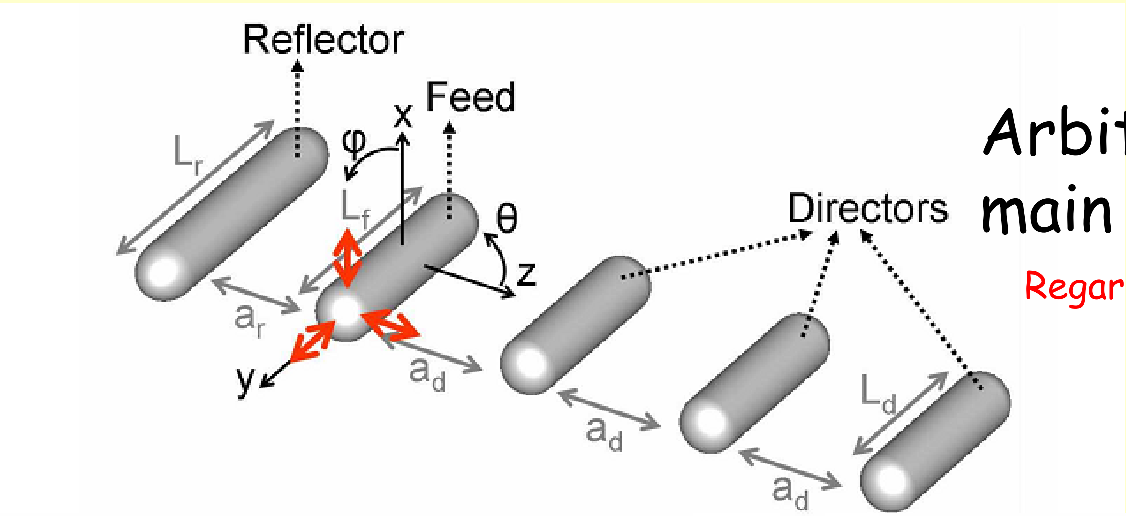
RF Yagi-Uda Antenna



Optical Yagi-Uda Antenna

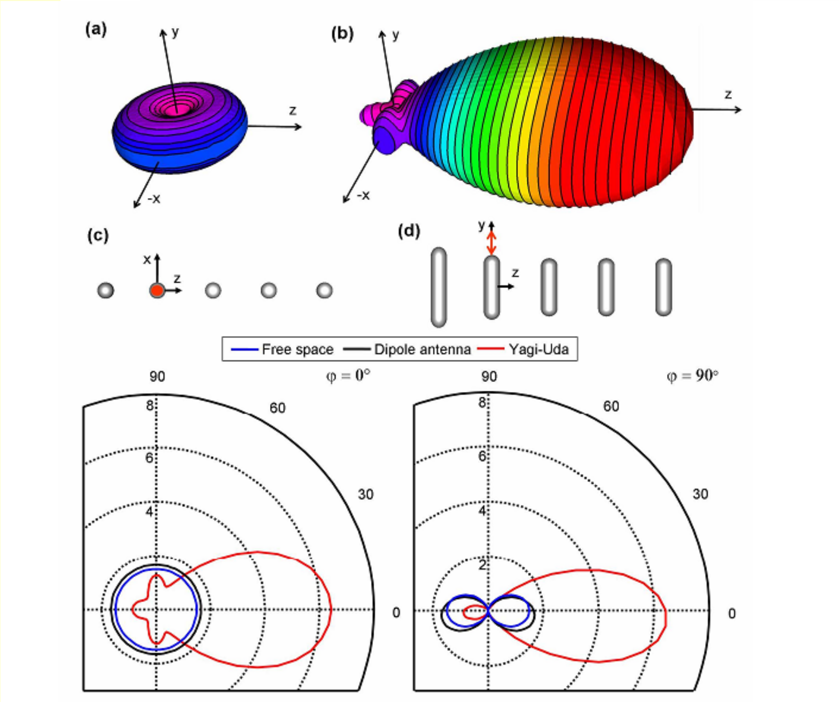
Science Vol. 329 no. 5994 pp. 910-911

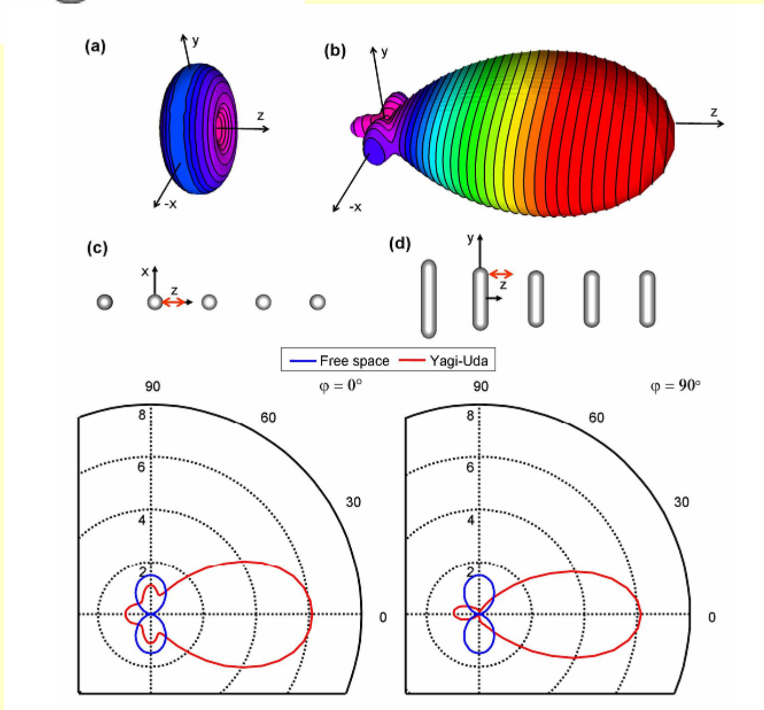
Yagi-Uda Antenna(八木宇田天线)



Arbitrary control over the main direction of emission

Regardless of the emission of the emitter



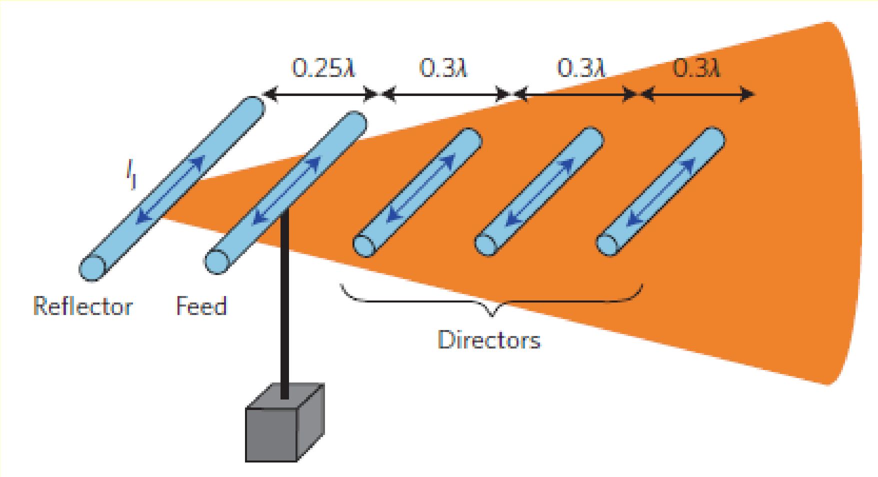


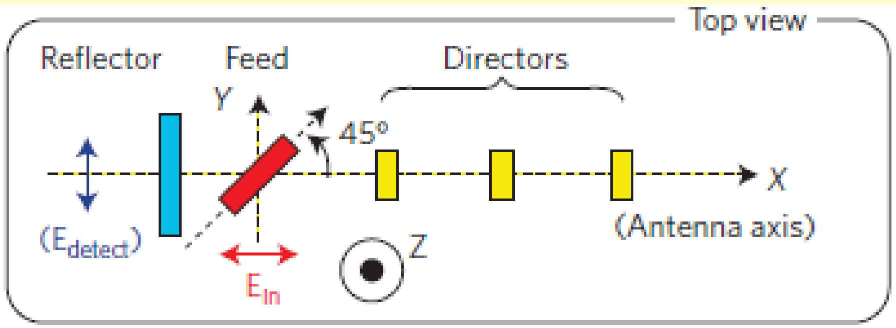
OE,16,10858(2008)

nature photonics
LETTERS
PUBLISHED ONLINE: 14 MARCH 2010 | DOI: 10.1038/NPHOTON.2010.34

Directional control of light by a nano-optical Yagi-Uda antenna

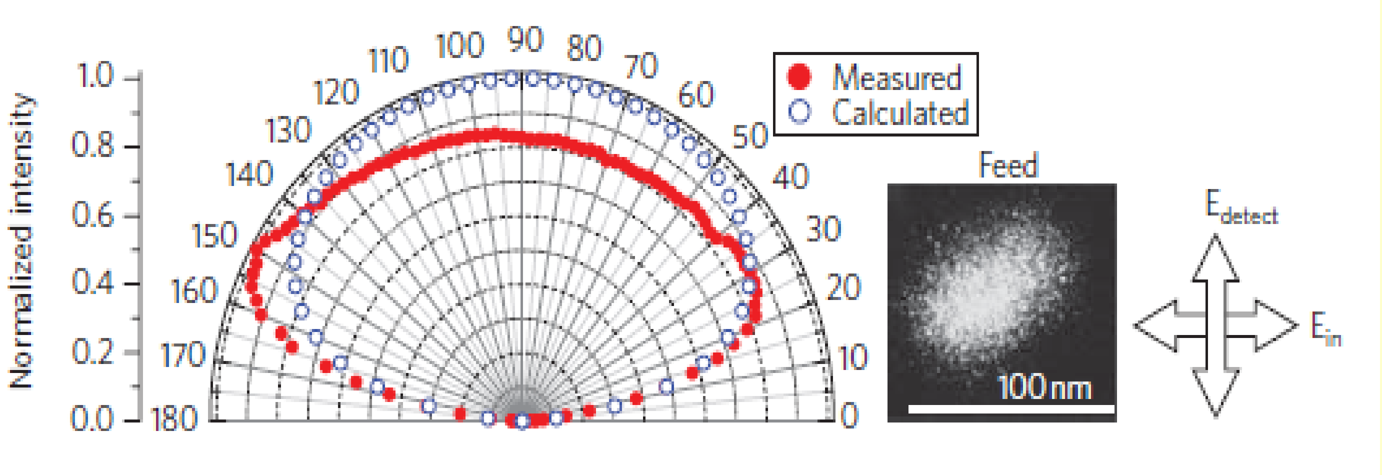
Terukazu Kosako, Yutaka Kadoya* and Holger F. Hofmann



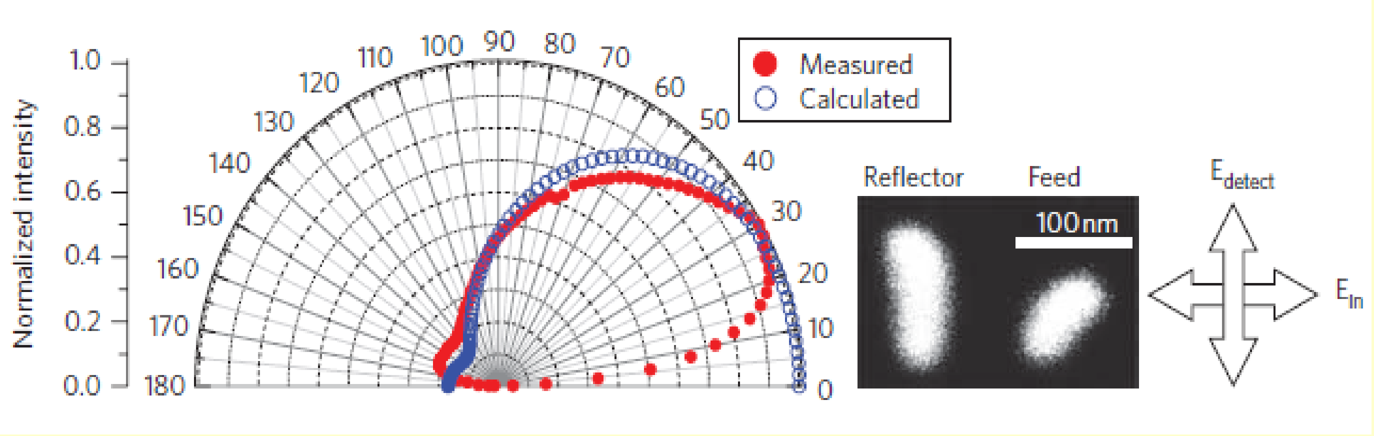


Typical geometry of a five-element RF Yagi–Uda antenna

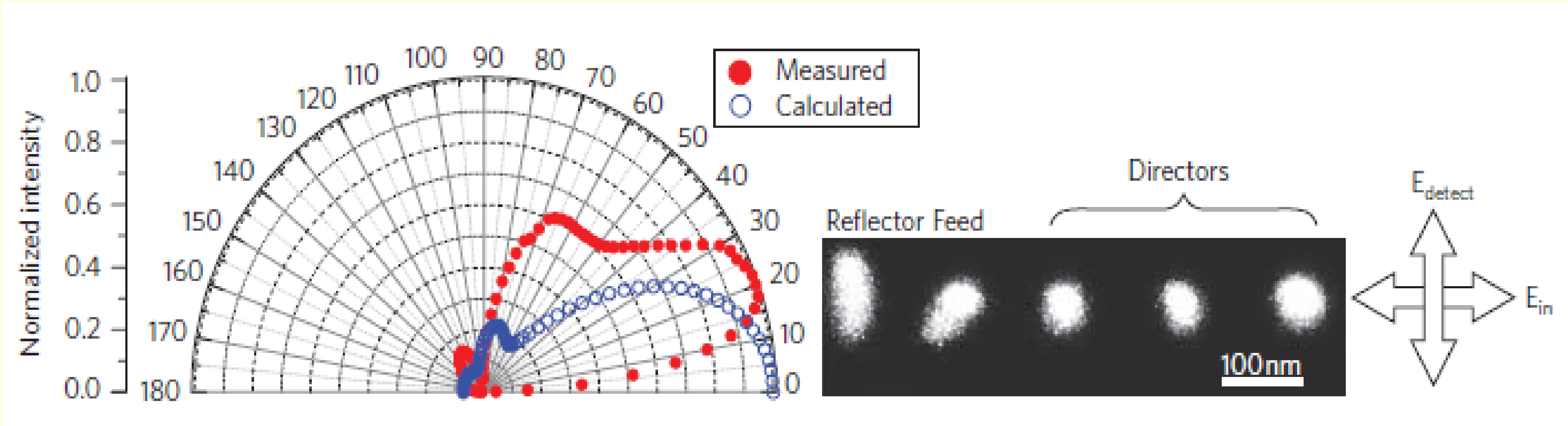
Top view of the antenna layout of the set-up



Emission pattern of a feed element



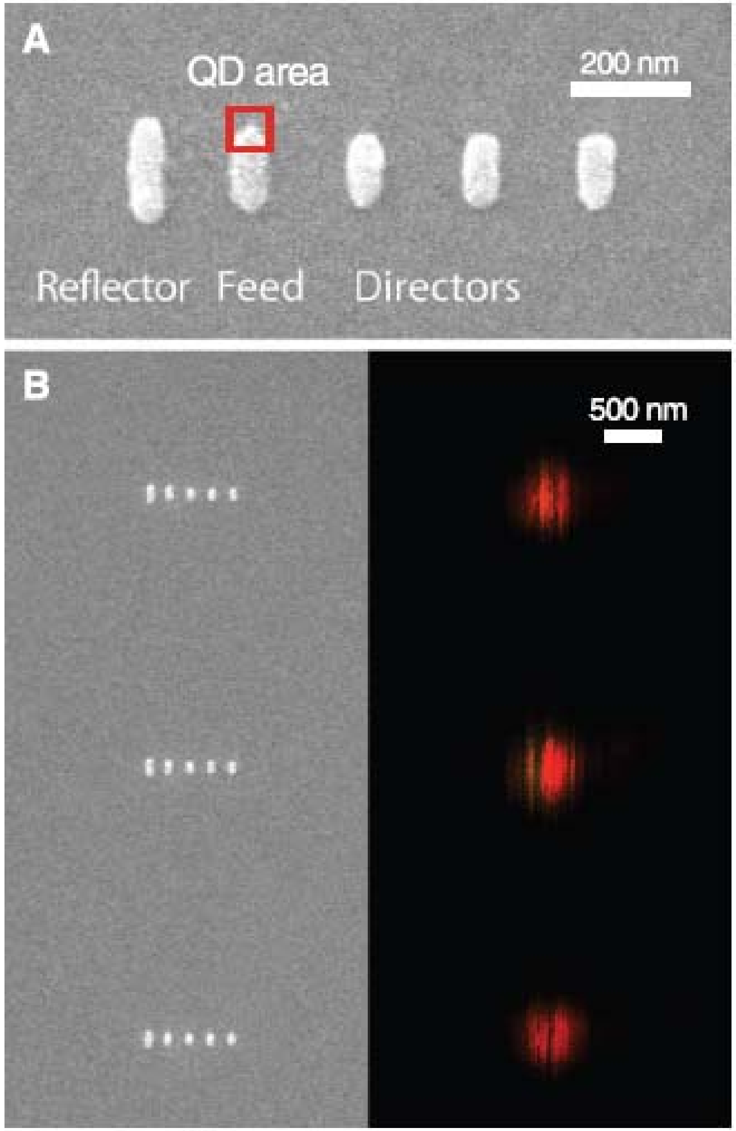
Feed-reflector antenna



radiation patterns of the five-element Yagi–Uda antenna

Yagi–Uda antenna can significantly improve the directionality of emission

Unidirectional Emission of a Quantum Dot Coupled to a Nanoantenna
Alberto G. Curto, *et al.*
Science **329**, 930 (2010);
DOI: 10.1126/science.1191922

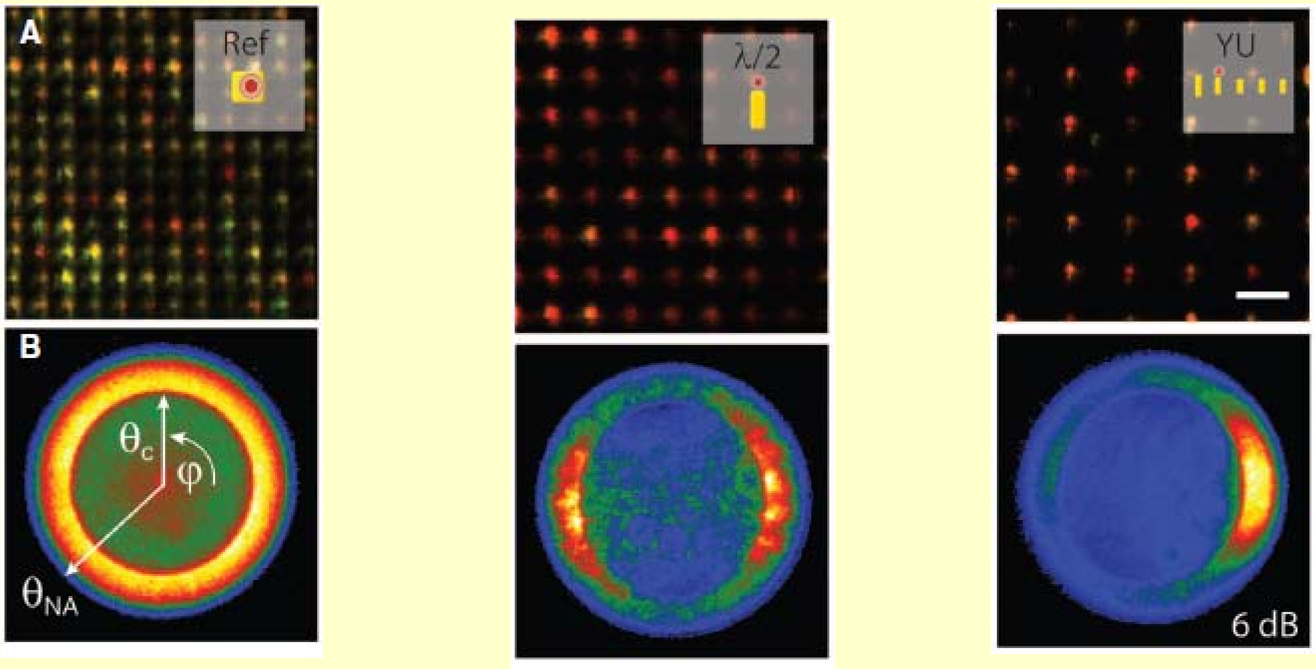


Optical Yagi-Uda antennas driven by quantum dots.

(A) Scanning electron microscopy (SEM) image

(B) Comparison of SEM and scanning confocal luminescence microscopy images of three antennas driven by QDs

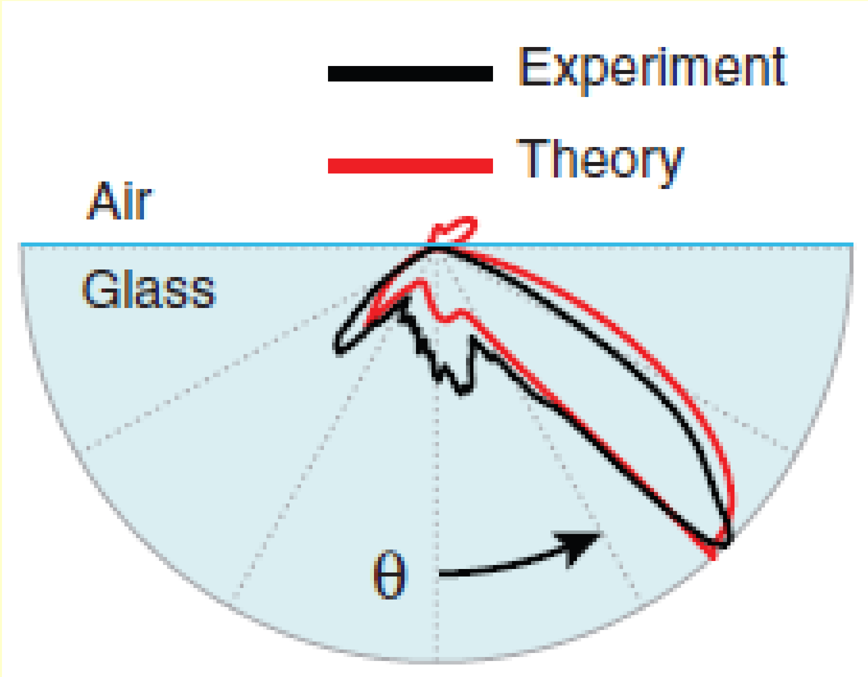
Unidirectional emission of a QD coupled to an optical Yagi-Uda antenna and comparison with other metal nanostructures



gold squares

half-wave dipole antennas

Yagi-Uda antennas



Angular radiation pattern in the polar angle for the Yagi-Uda antenna

NANO LETTERS
Letter
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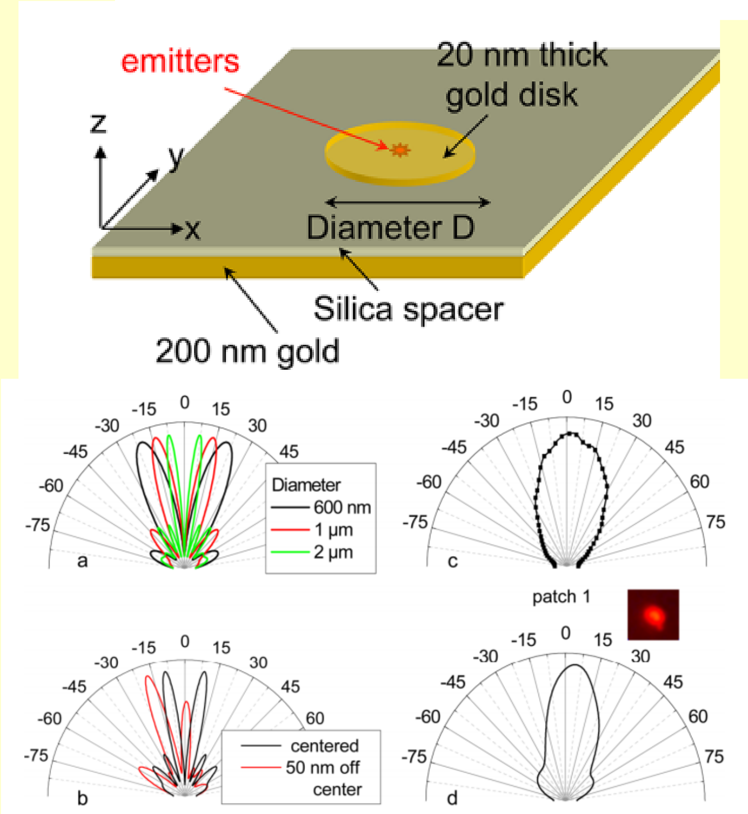
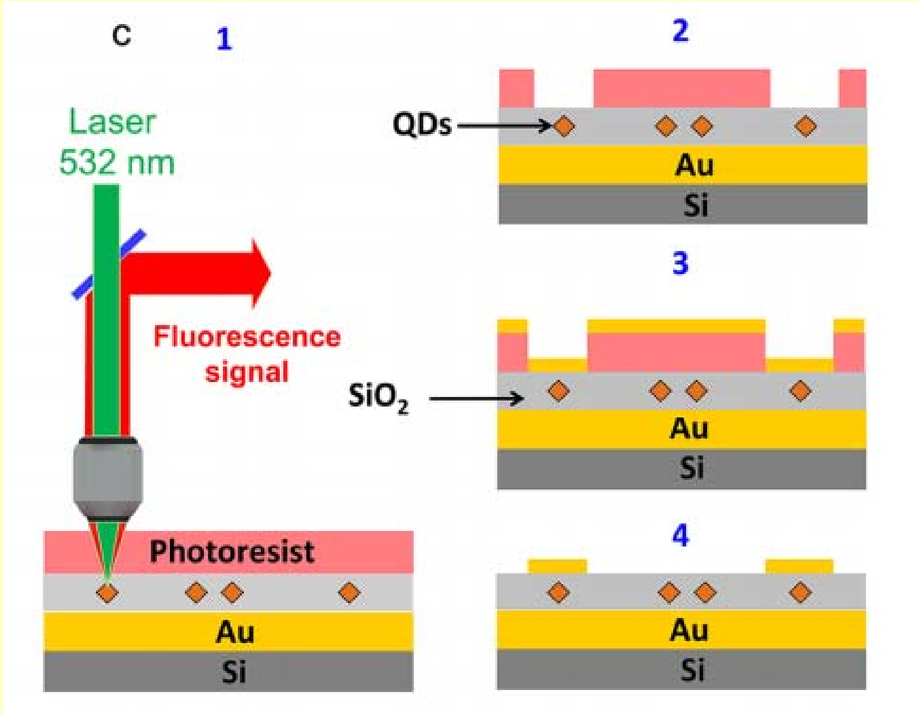
Controlling Spontaneous Emission with Plasmonic Optical Patch Antennas

C. Belacel,[†,‡,§] B. Habert,[‖] F. Bigourdan,[‖] F. Marquier,[‖] J.-P. Hugonin,[‖] S. Michaelis de Vasconcellos,[†] X. Lafosse,[†] L. Coolen,[‡,§] C. Schwob,[‡,§] C. Javaux,[⊥] B. Dubertret,[⊥] J.-J. Greffet,[‖] P. Senellart,[*,†] and A. Maitre[‡,§]





Shaping the Fluorescent Emission by Lattice Resonances in Plasmonic Crystals of Nanoantennas

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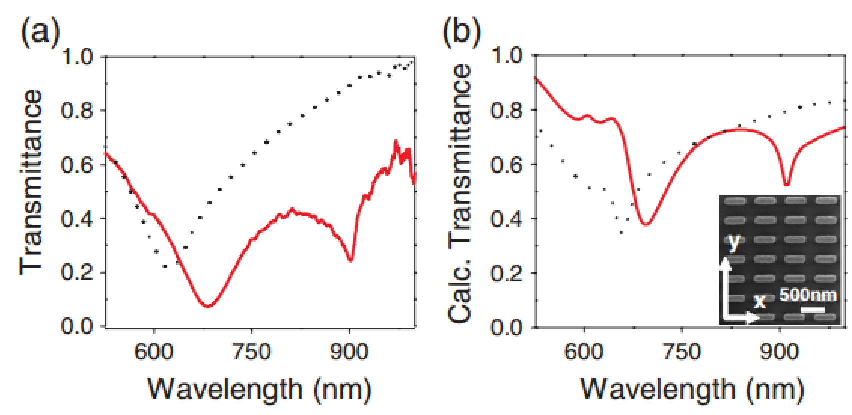
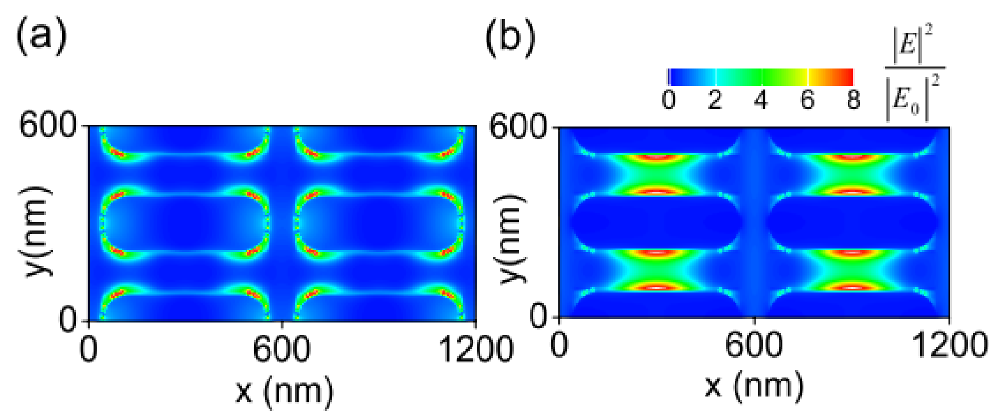
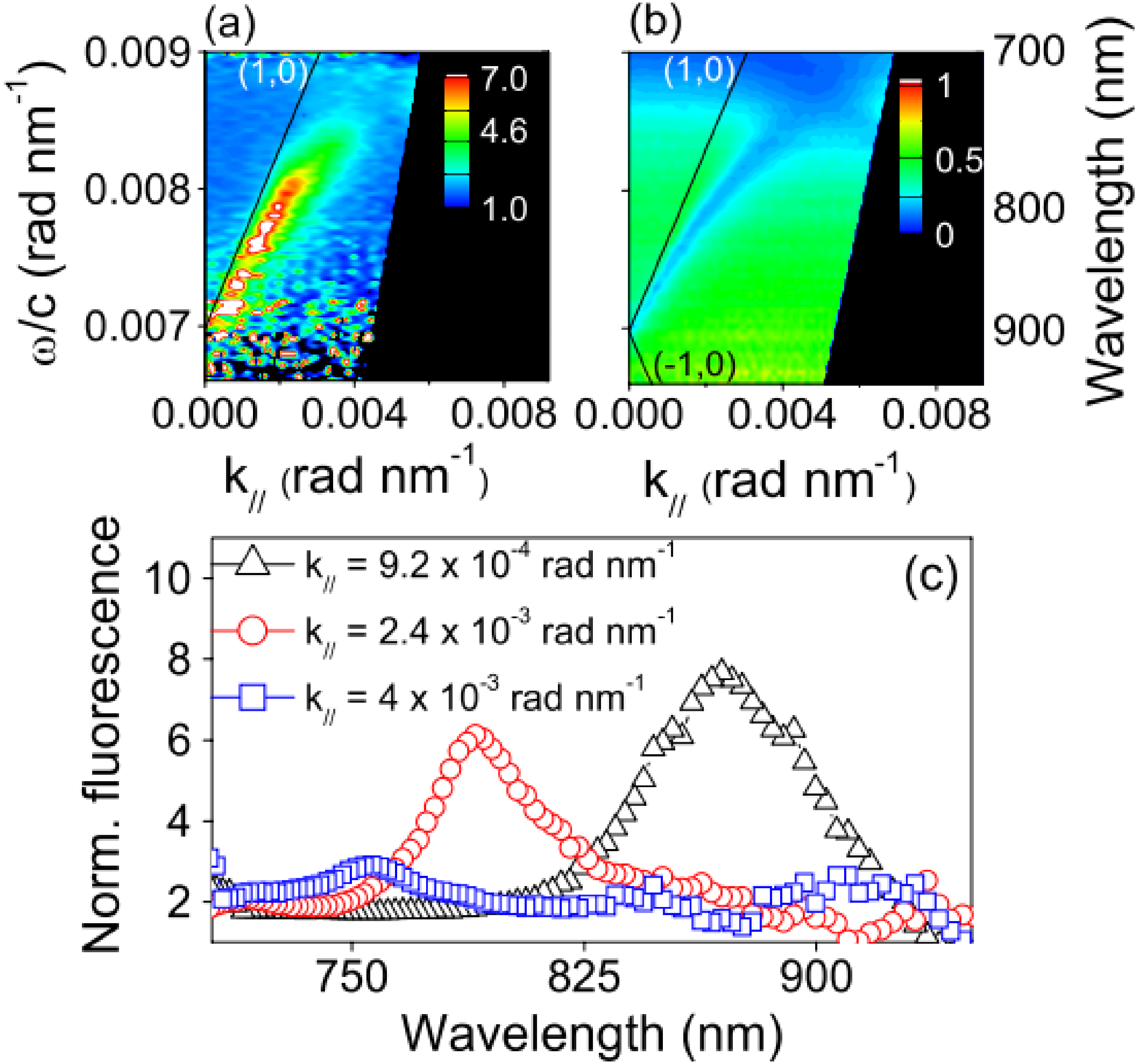
Center for Nanophotonics, FOM Institute AMOLF, c/o Philips Research Laboratories, High Tech Campus 4, 5656 AE Eindhoven, The Netherlands

(Received 23 December 2008; revised manuscript received 2 March 2009; published 10 April 2009)

We demonstrate that the emission of light by fluorescent molecules in the proximity of periodic arrays of nanoantennas or plasmonic crystals can be strongly modified when the arrays are covered by a dielectric film. The coupling between localized surface plasmon resonances and photonic states leads to surface modes which increase the density of optical states and improve light extraction. Excited dye molecules preferentially decay radiatively into these modes, exhibiting an enhanced and directional emission.






(a)
(b)
(c)
(1,0)
(-1,0)
7.0
4.6
1.0
1
0.5
0
ω/c (rad nm-1)
0.009
0.008
0.007
0.000
0.004
0.008
k// (rad nm-1)
700
800
900
Wavelength (nm)
k// = 9.2 x 10-4 rad nm-1
k// = 2.4 x 10-3 rad nm-1
k// = 4 x 10-3 rad nm-1
Norm. fluorescence
10
8
6
4
2
750
825
900
Wavelength (nm)

Summary

- *Nanoscale imaging and Spectroscopy*
- *Optical field Enhancement*

  Nonlinear behavior

  Photovoltaics

  light emission of quantum emitter
- *Ultrafast dynamics*

  nanoscale computing

  time-resolved Spectroscopy

  selective chemical bonding

- ***Quantum optics***

  single photons source

  coherent control

- ***Directional control***

Thank you！

Presentation Topics

1.超导AB效应，光子AB效应
2.AC效应
3.SPP波及SPP负折射
4.纳米激光器
5.金属和良导体的区别
6.光学天线和分子的相互作用（增强荧光和定向性）
7.广义的折射反射定义（metalsurface）
8.谐振腔：
1）. 介质颗粒（单个）上的全反射机制
2）. 透明介质的周期结构（光子晶体）
3）. 金属材料形成的表面等离激元微腔
…………

《电动力学》文献阅读口头报告

- 研究小组的成员不超过2－3人；
- 每个组口头报告时间为10-15分钟，包括报告和回答问题；
- 每位成员须参与报告和答辩过程；
- 评委将依据口头报告、问题答辩情况，进行打分；
- 希望同学们积极报名参加评委小组；
- 评分为百分制，陈述部分占70%，答辩部分占30%。
- 计入总评分数最多不超过3分。
- 口头报告分组、答辩时间、地点待定