§5 相对论电动力学

相对论电动力学

① 电动力学：麦克斯韦方程和洛仑兹力在任何惯性参照系中均成立;

② 相对论电动力学并不是改变这些定律或规则,而是把*原先似乎任意、没有关联的一些电磁规律*，以相对论特征的形式表示出来；

③ 这样做的目的，为的是使得我们对电动力学的相关定律有更深的理解。

Lorentz证明：

- 麦克斯韦电磁规律无需修改，*就满足Lorentz协变性*；
- 只不过需要把电磁量和电磁方程改写成四维形式。

$$x_4 = \mathrm{i}ct$$

空时坐标满足Lorentz 变换：

$$a = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

已经验证，*a*是**正交矩阵**，满足：$a^{-1} = \tilde{a}$

$$a_{\mu\alpha} a_{\mu\beta} = \delta_{\alpha\beta}.$$

求和标记

四维矢量

① 在洛仑兹变换下，其变换关系与四维坐标的变换关系相同，则称为四维矢量。

② 四维矢量的变换关系为

$$U' = aU \quad \text{或者} \quad U_\mu' = a_{\mu\nu} U_\nu$$

四维张量

在洛仑兹变换下，满足以下变换关系的物理量称为四维张量

$$T_{\mu\nu}' = a_{\mu\lambda} a_{\nu\delta} T_{\lambda\delta}$$

- 在四维空时坐标中，从一个惯性系变换到另一个惯性系时，物理量按照一定的方式变换；
- 由物理量构成的物理定律，应保持其方程的形式不变——**协变性。**

爱因斯坦相对性原理：

① 一切惯性系在物理上都是等价的。或者说：物理规律在不同的惯性系中应具有相同形式；

② 根据爱因斯坦相对性原理，物理规律应当写成以下一般形式：

$$\left[F_{\mu}\right]=\left[G_{\mu\nu}\right]\left[A_{\nu}\right]+\left[B_{\mu}\right].$$

F、A、B为四维矢量，而G则是二阶张量。

$$F_\mu = G_{\mu\nu} A_\nu + B_\mu .$$

可以验证：经Lorentz 变换, 上述形式保持不变

$$F'_\mu = a_{\mu\alpha} F_\alpha .$$

$$= a_{\mu\alpha} (G_{\alpha\beta} A_\beta + B_\alpha)$$

$$a_{\mu\alpha} a_{\mu\beta} = \delta_{\alpha\beta} .$$

$$= a_{\mu\alpha} \delta_{\beta\lambda} G_{\alpha\beta} A_\lambda + a_{\mu\alpha} B_\alpha$$

$$= a_{\mu\alpha} a_{\nu\beta} G_{\alpha\beta} a_{\nu\lambda} A_\lambda + a_{\mu\alpha} B_\alpha$$

$$F'_\mu = G'_{\mu\nu} A'_\nu + B'_\mu$$

本节主要内容：

1. 电荷守恒定律、四维电流密度矢量
2. 矢势和标势统一为四维势矢量
3. 电场和磁场统一为四维张量

1、电荷守恒　四维电流密度

- 封闭系统内的总电荷守恒；
- 系统的总电荷量与物体的运动的速度无关;
- 在Lorentz变换下，系统的总电荷是一个不变量。

1）电荷守恒的数学表达式：

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

2）四维电流密度：

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

第四维坐标：$x_4 = \mathrm{i}ct$

$$\frac{\partial J_1}{\partial x_1} + \frac{\partial J_2}{\partial x_2} + \frac{\partial J_3}{\partial x_3} + \frac{\partial(\mathrm{i}c\rho)}{\partial x_4} = 0$$

定义：$J_4 = \mathrm{i}c\rho$

电流、电荷密度统一为四维电流密度矢量:

$$J_\mu = (J_1,\ J_2,\ J_3,\ \mathrm{i}c\rho)$$

$$\frac{\partial J_\mu}{\partial x_\mu} = 0$$

$$x_4 = \mathrm{i}ct$$

$$J_4 = \mathrm{i}c\rho$$

3）电荷守恒定律的四维形式

$$J_\mu = (J_1,\ J_2,\ J_3,\ J_4)$$

$$\frac{\partial J_\mu}{\partial x_\mu} = 0$$

① 等式左边为四维空间(Lorentz)标量；

② 电荷守恒定律的四维形式对任何惯性参照系均成立。

2、四维势矢量

1）矢势和标势描述电磁场：

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t}$$

2）规范性

电磁场本身对 A 的散度没有任何的限制——**规范自由度；**

对 $\nabla \cdot \vec{A}$ 的每一种选择称为一种规范。

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla \varphi - \frac{\partial \vec{A}}{\partial t}$$

3）Lorenz规范（注意不是Lorentz）

① 规范条件：

$$\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0$$

$$\nabla \cdot \vec{A} + \frac{\partial}{\partial(\mathrm{i}ct)}\left(\mathrm{i}\frac{\varphi}{c}\right) = 0$$

$$x_4 = \mathrm{i}ct$$

② 定义四维势的四个分量：

$$A_4 = \mathrm{i}\frac{\varphi}{c}, \quad A_\mu = \left(A_1,\ A_2,\ A_3,\ \mathrm{i}\frac{\phi}{c}\right)$$

$$\nabla \cdot \vec{A} + \frac{\partial}{\partial(\mathrm{i}ct)}\left(\mathrm{i}\frac{\varphi}{c}\right) = 0$$

$$x_4 = \mathrm{i}ct$$

$$A_4 = \mathrm{i}\frac{\varphi}{c}$$

③ **Lorenz规范的四维形式：**

$$\Rightarrow \quad \frac{\partial A_\mu}{\partial x_\mu} = 0$$

假设： Σ系中有一个点电荷 q 沿 x 方向作匀速运动，求产生的势。

解： 在点电荷上建立 Σ' 坐标系。在Σ'系中电荷静止，因此有

$$\vec{A}' = 0.$$

$$\phi'(r') = \frac{q}{4\pi\varepsilon_0 r'},$$

$$A_4' = \mathrm{i}\frac{\varphi'}{c} = \mathrm{i}\frac{q}{4\pi\varepsilon_0 c r'}, \quad \left[A_\mu'\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{\mathrm{i}q}{4\pi\varepsilon_0 c r'} \end{bmatrix}$$

$$\tilde{a} = \begin{bmatrix} \gamma & 0 & 0 & -\mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

$$\left[A'_\mu\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{\mathrm{i}q}{4\pi\varepsilon_0 c r'} \end{bmatrix}$$

$$A = \tilde{a} A'$$

变换到 Σ系，有：

$$A_1 = -\mathrm{i}\beta\gamma A_4' = \gamma v \frac{q}{4\pi\varepsilon_0 c^2 r'}$$

$$\tilde{a} = \begin{bmatrix} \gamma & 0 & 0 & -\mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

$$\left[A'_{\mu} \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{\mathrm{i}q}{4\pi\varepsilon_0 c r'} \end{bmatrix}$$

$$A = \tilde{a} A'$$

$$A_2 = 0, A_3 = 0,$$

$$A_4 = \gamma A'_4, \quad \mathrm{i}\frac{\phi}{c} = \gamma\left(\mathrm{i}\frac{\phi'}{c}\right)$$

$$\varphi = \gamma\varphi' = \gamma\frac{q}{4\pi\varepsilon_0 r'},$$

$$A_1 = \gamma v \frac{q}{4\pi\varepsilon_0 c^2 r'} = \frac{v}{c^2}\phi$$

$$\phi = \gamma \frac{q}{4\pi\varepsilon_0 r'}$$

还需要将 r' 用 Σ系中的坐标 r 表示：

$$r' = \sqrt{x'^2 + y'^2 + z'^2}$$

$$= \sqrt{\gamma^2 (x - vt)^2 + y^2 + z^2}$$

$$x' = \gamma(x - vt)$$

代入上式后得

$$\phi = \frac{\gamma q}{4\pi\varepsilon_0 r'} = \frac{\gamma q}{4\pi\varepsilon_0\sqrt{\gamma^2(x-vt)^2+y^2+z^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{(x-vt)^2+\gamma^{-2}(y^2+z^2)}}.$$

$$\vec{A} = \frac{\vec{v}}{c^2}\phi = \frac{\mu_0}{4\pi}\frac{q\vec{v}}{\sqrt{(x-vt)^2+\gamma^{-2}(y^2+z^2)}}.$$

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$$

5）达朗贝尔方程协变形式

在Lorenz 规范下，矢势和标势满足的方程：

$$\left.\begin{aligned}\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}&=-\mu_0\vec{J}\\ \nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}&=-\frac{\rho}{\varepsilon_0}\end{aligned}\right\}$$

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

$$x_4 = \mathrm{i}ct$$

注意到：

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \equiv \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\mu} = \Box^2$$

达朗贝尔方程方程可以改写成：

$$\Box^2 \vec{A} = -\mu_0 \vec{J},$$

$$\Box^2 \varphi = -\mu_0 c^2 \rho$$

$$\Box^2 \vec{A} = -\mu_0 \vec{J},$$
$$\Box^2 \varphi = -\mu_0 c^2 \rho$$

⇒

$$\Box^2 \vec{A} = -\mu_0 \vec{J},$$
$$\Box^2 \frac{\mathrm{i}\varphi}{c} = -\mu_0 \left(\mathrm{i} c \rho\right)$$

J_μ

$$\left[A_\mu\right] = \left(A_1,\ \ A_2,\ \ A_3,\ \ \frac{\mathrm{i}}{c}\varphi\right)$$

达朗贝尔方程的协变形式：

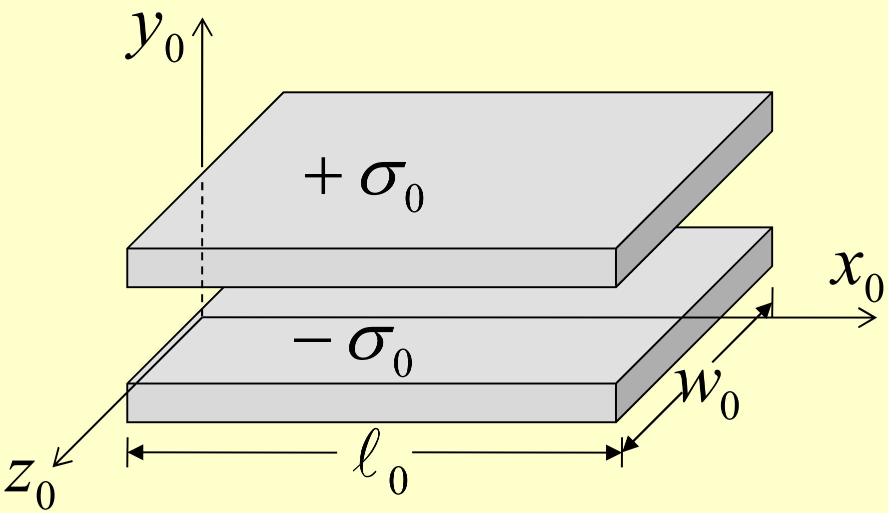
$$\Box^2 A_\mu = -\mu_0 J_\mu$$
$$(\mu = 1,2,3,4)$$

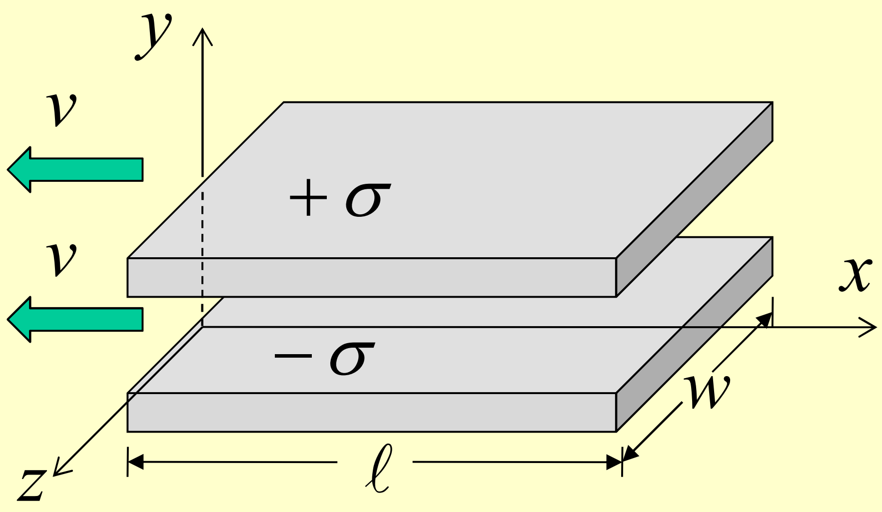
$$\Box^2 = \frac{\partial}{x_\mu}\frac{\partial}{x_\mu} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

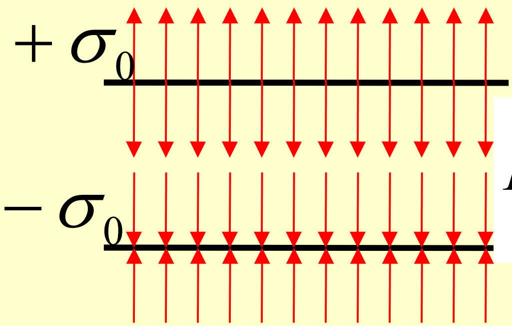
3、电场和磁场统一为四维张量

1）电磁场如何变换?

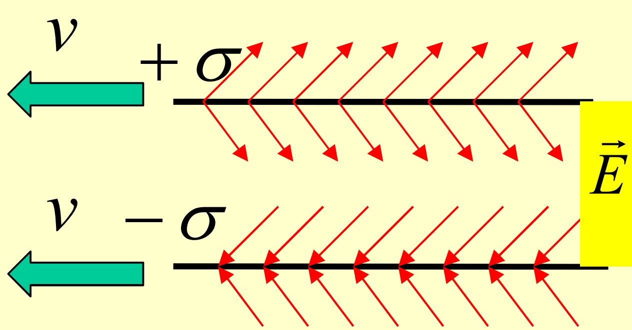
- 电场和磁场本身不是某个四维矢量的分量;
- 电场、磁场如何变换?







$$\vec{E} = \frac{\sigma_0}{\varepsilon_0}\vec{e}_y$$



$$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{e}_y$$

$$\vec{E} = \frac{\sigma_0}{\varepsilon_0}\vec{e}_y \Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0}\vec{e}_y$$

- 电量是一个不变量，因此运动下极板上的电量应该不变；
- 考虑到沿着运动方向的Lorentz收缩，极板的长度将减小

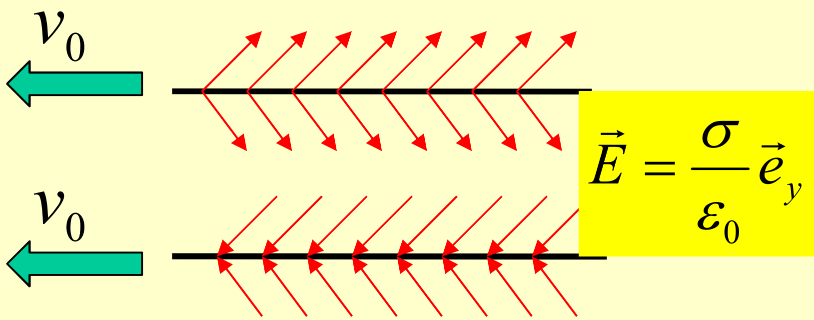
$$\gamma^{-1} = \sqrt{1-\beta^2} = \sqrt{1-\frac{v^2}{c^2}}(\text{倍})$$

- 因此极板上的电荷面密度将增加至：

$$\sigma = \gamma\sigma_0 = \frac{1}{\sqrt{1-\beta^2}}\sigma_0$$

A）与运动方向相垂直（极板间）电场的变换关系：

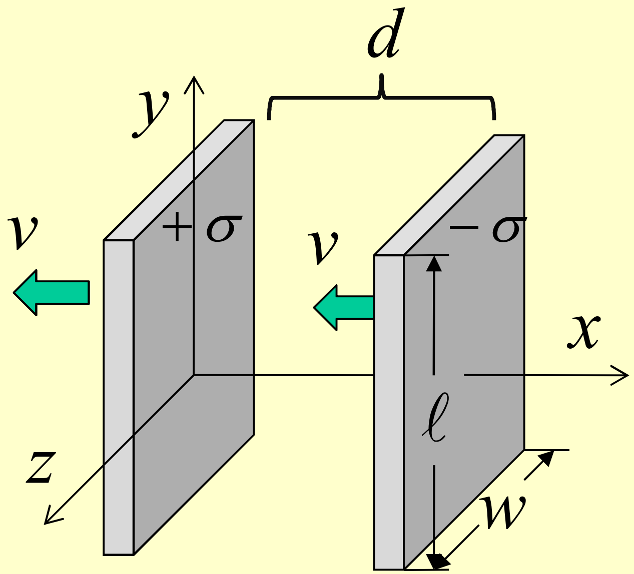
$$\vec{E} = \vec{E}_{\perp} = \gamma\vec{E}_{0\perp}$$



加上“垂直”号，表示这样的变换规则适用于与运动方向垂直的电场分量。

B）与运动方向平行的电场的变换关系：

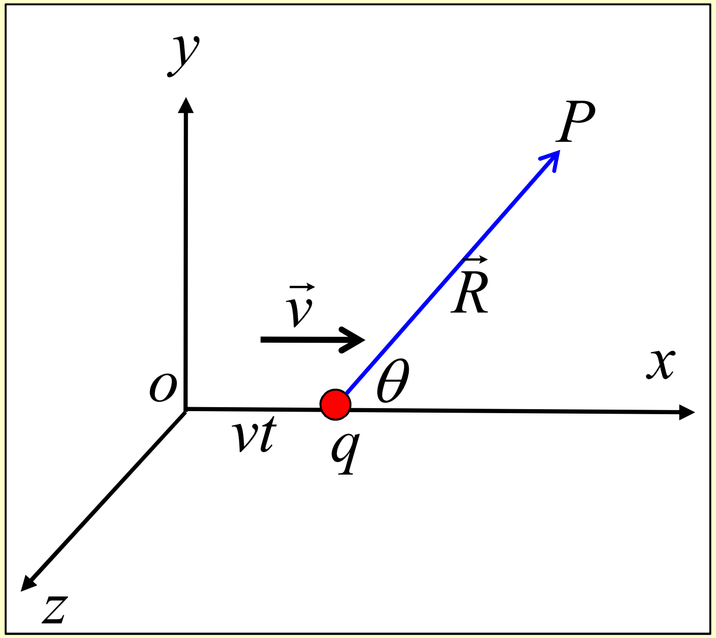
$$\vec{E}_{//} = \vec{E}_{0//}$$



C）相对于参照系做匀速运动的点电荷的电场

在Σ' 参照系中，点电荷的电场为 $\vec{E}'=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r'^3}\vec{r}'$

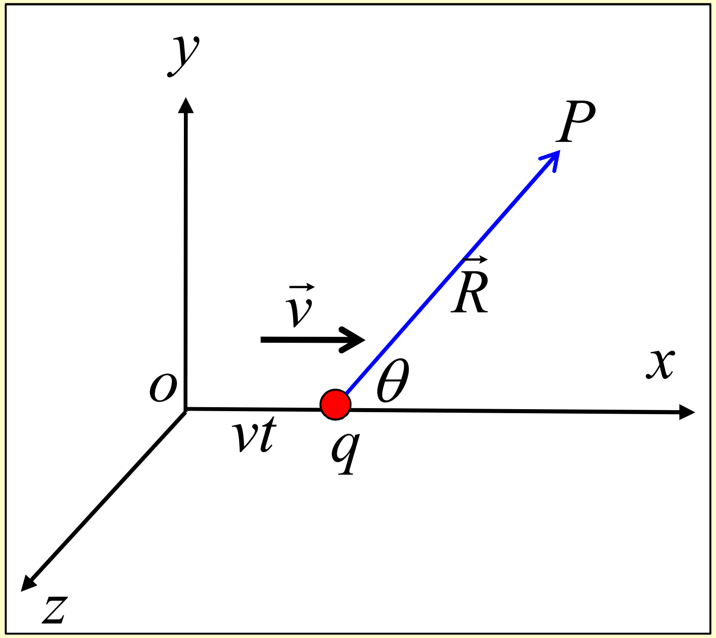
$$\begin{cases} E_x'=\dfrac{1}{4\pi\varepsilon_0}\dfrac{qx'}{(x'^2+y'^2+z'^2)^{3/2}} \\ E_y'=\dfrac{1}{4\pi\varepsilon_0}\dfrac{qy'}{(x'^2+y'^2+z'^2)^{3/2}} \\ E_z'=\dfrac{1}{4\pi\varepsilon_0}\dfrac{qz'}{(x'^2+y'^2+z'^2)^{3/2}} \end{cases}$$



根据上面的结论，在Σ参照系中点电荷的电场为

$$E_x=E_x',\ E_y=\gamma E_y',\ E_z=\gamma E_z'$$

Σ参照系中，点电荷的电场分量



$$\begin{cases} E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qx'}{(x'^2+y'^2+z'^2)^{3/2}} \\ E_y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\gamma qy'}{(x'^2+y'^2+z'^2)^{3/2}} \\ E_z = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\gamma qz'}{(x'^2+y'^2+z'^2)^{3/2}} \end{cases}$$

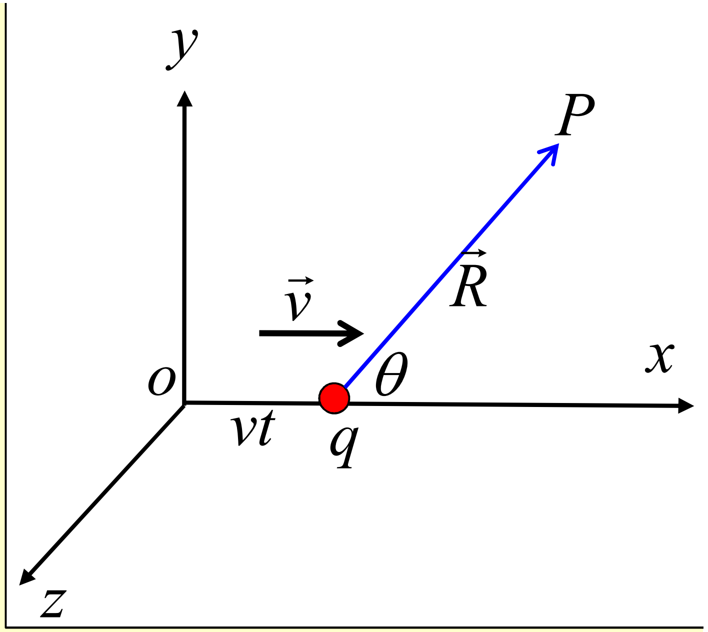
将 r' 用 Σ系中的坐标 r 表示：

$$\begin{cases} E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\gamma(x-vt)}{[\gamma^2(x-vt)^2+y^2+z^2]^{3/2}} \\ E_y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\gamma qy}{[\gamma^2(x-vt)^2+y^2+z^2]^{3/2}} \\ E_z = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\gamma qz}{[\gamma^2(x-vt)^2+y^2+z^2]^{3/2}} \end{cases}$$

$$x' = \gamma(x - vt)$$

在Σ参照系中，点电荷的电场

$$
\begin{cases}
E_x = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q\gamma(x - vt)}{[\gamma^2(x - vt)^2 + y^2 + z^2]^{3/2}} \\
E_y = \dfrac{1}{4\pi\varepsilon_0} \dfrac{\gamma q y}{[\gamma^2(x - vt)^2 + y^2 + z^2]^{3/2}} \\
E_z = \dfrac{1}{4\pi\varepsilon_0} \dfrac{\gamma q z}{[\gamma^2(x - vt)^2 + y^2 + z^2]^{3/2}}
\end{cases}
$$



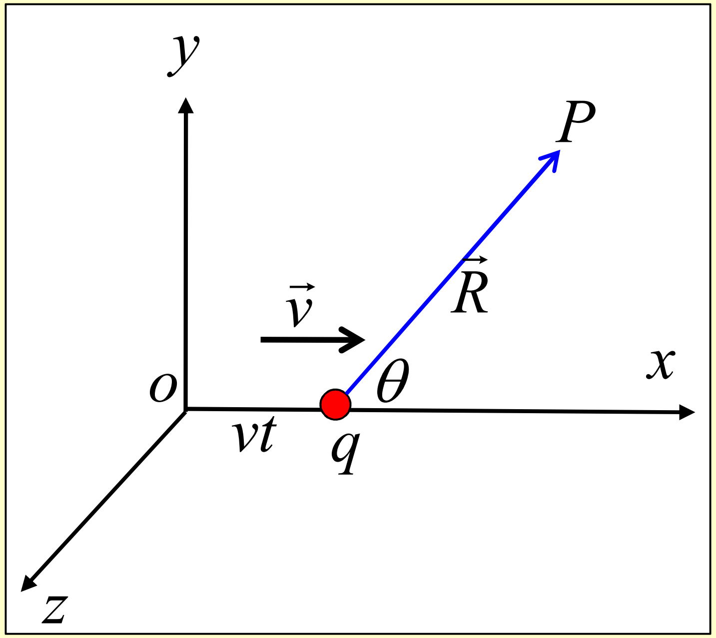
$$\vec{R} = (x - vt)\vec{e}_x + y\vec{e}_y + z\vec{e}_z$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q\gamma\vec{R}}{[\gamma^2(x - vt)^2 + y^2 + z^2]^{3/2}}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q\gamma\vec{R}}{[\gamma^2(x-vt)^2 + y^2 + z^2]^{3/2}}$$

$$x - vt = R\cos\theta$$

$$y^2 + z^2 = (R\sin\theta)^2$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q\gamma\vec{R}}{[\gamma^2(R\cos\theta)^2 + (R\sin\theta)^2]^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q\gamma}{\left(\gamma^2\cos^2\theta + \sin^2\theta\right)^{3/2}} \cdot \frac{\vec{R}}{R^3}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{\gamma^2\left(\cos^2\theta + \gamma^{-2}\sin^2\theta\right)^{3/2}} \cdot \frac{\vec{R}}{R^3}$$

说明:

① 借助平行板电容器的分析, 得到Σ的参照系中电场与在Σ' 参照系中的电场之间的变换关系只适用于Σ'参照系中的磁场为0的情形。

$$\left(E_x = E_x',\ \ E_y = \gamma E_y',\ \ E_z = \gamma E_z'\right)$$

② 在Σ'参照系中，如果磁场 B≠0，那两个参照系中的电、磁场之间的更一般的变换关系如何求得?

——此时需要借助电磁张量之变换求得。

2）电磁场与四维势的关系

$$\vec{B} = \nabla \times \vec{A}$$

$$B_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3},$$

$$B_2 = \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1},$$

$$B_3 = \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}$$

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} = \mathrm{i}c\left[\nabla\left(\mathrm{i}\frac{\varphi}{c}\right) - \frac{\partial\vec{A}}{\partial(\mathrm{i}ct)}\right] = \mathrm{i}c\left(\nabla A_4 - \frac{\partial\vec{A}}{\partial x_4}\right)$$

$$\begin{cases} E_1 = \mathrm{i}c\left(\dfrac{\partial A_4}{\partial x_1} - \dfrac{\partial A_1}{\partial x_4}\right), \\ E_2 = \mathrm{i}c\left(\dfrac{\partial A_4}{\partial x_2} - \dfrac{\partial A_2}{\partial x_4}\right), \\ E_3 = \mathrm{i}c\left(\dfrac{\partial A_4}{\partial x_3} - \dfrac{\partial A_3}{\partial x_4}\right) \end{cases}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$B_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3},$$

$$B_2 = \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1},$$

$$B_3 = \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}$$

$$E_1 = \mathrm{i}c\left(\frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4}\right),$$

$$E_2 = \mathrm{i}c\left(\frac{\partial A_4}{\partial x_2} - \frac{\partial A_2}{\partial x_4}\right),$$

$$E_3 = \mathrm{i}c\left(\frac{\partial A_4}{\partial x_3} - \frac{\partial A_3}{\partial x_4}\right)$$

电磁场可以表示成四维势的旋度的形式！

$$A_1 = A_x,\quad A_2 = A_y,\quad A_3 = A_z,\quad A_4 = i\frac{\varphi}{c}$$

3）在四维空间，定义反对称四维张量：

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}(\text{有16个元素})$$

$$F_{\mu\nu} = \begin{bmatrix} 0 & F_{12} & F_{13} & F_{14} \\ -F_{12} & 0 & F_{23} & F_{24} \\ -F_{13} & -F_{23} & 0 & F_{34} \\ -F_{14} & -F_{24} & -F_{34} & 0 \end{bmatrix}$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}$$

4）电磁场构成一个反对称四维张量：

F_{12} F_{14}

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{bmatrix}$$

$$B_1 = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}, \quad E_1 = ic\left(\frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4}\right),$$

$$B_2 = \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}, \quad E_2 = ic\left(\frac{\partial A_4}{\partial x_2} - \frac{\partial A_2}{\partial x_4}\right),$$

$$B_3 = \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2} \quad E_3 = ic\left(\frac{\partial A_4}{\partial x_3} - \frac{\partial A_3}{\partial x_4}\right)$$

5）在不同的惯性系中电磁场的变换关系：

根据张量的变换关系： $F_{\mu\nu}'=a_{\mu\lambda}a_{\nu\delta}F_{\lambda\delta}$

可推导出电磁场的一般变换关系：

平行分量

$$\vec{E}_{//}'=\vec{E}_{//},$$
$$\vec{B}_{//}'=\vec{B}_{//},$$

垂直分量

$$\vec{E}_{\perp}'=\gamma\left(\vec{E}+\vec{v}\times\vec{B}\right)_{\perp},$$
$$\vec{B}_{\perp}'=\gamma\left(\vec{B}-\frac{1}{c^2}\vec{v}\times\vec{E}\right)_{\perp}$$

① //和⊥分别表示与相对速度平行和垂直的分量

② 在给定参照系中，电场和磁场表现出不同的性质；

③ 当参照系变换时，电场和磁场可以相互转化。

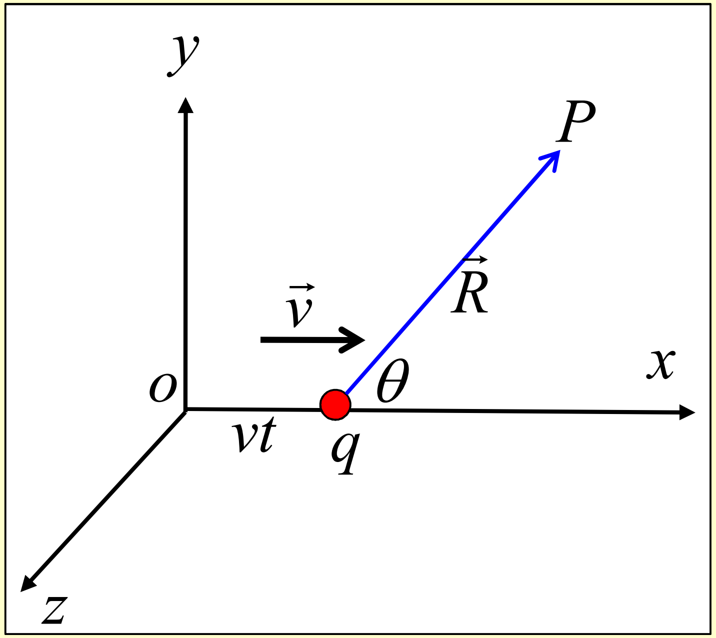
$$\vec{E}_{\perp}' = \gamma\left(\vec{E} + \vec{v} \times \vec{B}\right)_{\perp},$$
$$\vec{B}_{\perp}' = \gamma\left(\vec{B} - \frac{1}{c^2}\vec{v} \times \vec{E}\right)_{\perp}$$

④ 当速度远小于光速度时，过度到非相对论的电磁场变换关系：

$$\vec{E}_{\perp}' = \left(\vec{E} + \vec{v} \times \vec{B}\right)_{\perp},$$
$$\vec{B}_{\perp}' = \left(\vec{B} - \frac{1}{c^2}\vec{v} \times \vec{E}\right)_{\perp}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{\gamma^2 \left(\cos^2\theta + \gamma^{-2}\sin^2\theta\right)^{3/2}} \cdot \frac{\vec{R}}{R^3}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q(1-\beta^2)}{[1-(\beta\sin\theta)^2]^{3/2}} \cdot \frac{\vec{R}}{R^3}$$



讨论：相对于参照系 Σ 做匀速运动的点电荷的磁场=?

$$\vec{B}_{//} = ?, \vec{B}_{\perp} = ?$$

$$\vec{B}_{//}{}' = \vec{B}_{//}$$

$$\vec{B}_{//} = \vec{B}_{//}{}' = 0$$

$$\vec{B}_{\perp}{}' = \left(\vec{B} - \frac{1}{c^2}\vec{v}\times\vec{E}\right)_{\perp}$$

$$\vec{B}_{\perp}{}' = 0$$

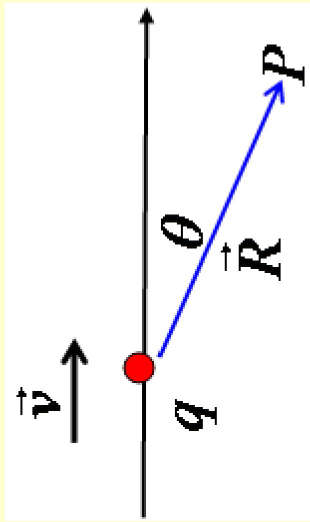
$$\vec{B}_{\perp} = \frac{1}{c^2}\vec{v}\times\vec{E}_{\perp} = \frac{1}{c^2}\vec{v}\times\vec{E}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\gamma^2\left(\cos^2\theta+\gamma^{-2}\sin^2\theta\right)^{3/2}}\cdot\frac{\vec{R}}{R^3}$$

$$\vec{B}_{\perp} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\gamma^2\left(\cos^2\theta+\gamma^{-2}\sin^2\theta\right)^{3/2}}\cdot\frac{\vec{v}\times\vec{R}}{c^2R^3}$$

$$= \frac{\mu_0}{4\pi}\frac{q}{\gamma^2\left(\cos^2\theta+\gamma^{-2}\sin^2\theta\right)^{3/2}}\cdot\frac{\vec{v}\times\vec{R}}{R^3}$$

$$= \frac{\mu_0}{4\pi}\frac{qv}{\gamma^2\left(\cos^2\theta+\gamma^{-2}\sin^2\theta\right)^{3/2}}\cdot\frac{1}{R^2}\vec{e}_{\phi}$$



4、麦克斯韦方程的协变形式

$$F=\begin{bmatrix} 0 & B_3 & -B_2 & -\frac{\mathrm{i}}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{\mathrm{i}}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{\mathrm{i}}{c}E_3 \\ \frac{\mathrm{i}}{c}E_1 & \frac{\mathrm{i}}{c}E_2 & \frac{\mathrm{i}}{c}E_3 & 0 \end{bmatrix}$$

$$\begin{cases} \nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}, \\ \nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \end{cases} \Rightarrow \frac{\partial F_{\mu\nu}}{\partial x_\nu}=\mu_0 J_\mu$$

$$\begin{cases} \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \\ \nabla\cdot\vec{B}=0 \end{cases} \Rightarrow \frac{\partial F_{\mu\nu}}{\partial x_\lambda}+\frac{\partial F_{\nu\lambda}}{\partial x_\mu}+\frac{\partial F_{\lambda\mu}}{\partial x_\nu}=0$$

ν 为求和脚标； $\mu=1, 2, 3, 4$

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}, \\ \nabla \cdot \vec{B} = 0 \end{cases} \Bigg\} \quad \frac{\partial F_{\mu\nu}}{\partial x_\lambda} + \frac{\partial F_{\nu\lambda}}{\partial x_\mu} + \frac{\partial F_{\lambda\mu}}{\partial x_\nu} = 0$$

$$F = \begin{bmatrix} 0 & B_3 & -B_2 & -\frac{\mathrm{i}}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{\mathrm{i}}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{\mathrm{i}}{c}E_3 \\ \frac{\mathrm{i}}{c}E_1 & \frac{\mathrm{i}}{c}E_2 & \frac{\mathrm{i}}{c}E_3 & 0 \end{bmatrix}$$

例如：$(\mu = 1, \nu = 2, \lambda = 3)$

$$\frac{\partial F_{12}}{\partial x_3} + \frac{\partial F_{23}}{\partial x_1} + \frac{\partial F_{31}}{\partial x_2} = 0 \;\Rightarrow\; \frac{\partial B_3}{\partial x_3} + \frac{\partial B_1}{\partial x_1} + \frac{\partial B_2}{\partial x_2} = 0 \;\Rightarrow\; \nabla \cdot \vec{B} = 0$$

再例如：$(\mu = 3, \nu = 4, \lambda = 2)$

$$\frac{\partial F_{34}}{\partial x_2} + \frac{\partial F_{42}}{\partial x_3} + \frac{\partial F_{23}}{\partial x_4} = 0 \;\Rightarrow\; -\frac{i}{c}\frac{\partial E_3}{\partial x_2} + \frac{i}{c}\frac{\partial E_2}{\partial x_3} + \frac{\partial B_1}{\partial x_4} = 0 \;\Rightarrow\; \frac{\partial E_3}{\partial x_2} - \frac{\partial E_2}{\partial x_3} = -\frac{\partial B_1}{\partial t}$$

$$\nabla \times \vec{E} = \left(\frac{\partial E_3}{\partial x_2} - \frac{\partial E_2}{\partial x_3}\right)\vec{e}_1 + \left(\frac{\partial E_1}{\partial x_3} - \frac{\partial E_3}{\partial x_1}\right)\vec{e}_2 + \left(\frac{\partial E_2}{\partial x_1} - \frac{\partial E_1}{\partial x_2}\right)\vec{e}_3$$

3）总结：电磁现象的参考系问题

① 电动力学的基本方程对任意惯性参照系中成立；

② 在坐标变换下，**势**按照四维矢量变换；**电磁场**按照四维张量变换。