第三章

傅里叶变换

3.1 引言

- 时域分析－＞变换域分析，要讨论的变换
  - 傅氏变换 (FT)
  - 复频域分析 (LT)
  - 离散信号的Z域变换
- 信号的分解－正交基底函数
- FT的发展（1965年FFT）
- FT的内容
  - 周期的模拟信号FS
  - 非周期的模拟信号FT
  - 离散的非周期序列(今后讨论)

3.2 周期信号的傅氏级数分析

- 狭利赫利条件
  - 一个周期内，周期信号绝对可积
  - 一个周期内，周期信号的极值数目有限
  - 一个周期内，周期信号只有有限个间断点
- 周期信号（周期T_1）可展成傅氏级数
  - 三角函数形式
  - 复指数形式

- 三角形式的傅氏级数

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos(n\omega_1 t) + b_n \sin(n\omega_1 t)]$$

$$a_n = \frac{\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} f(t)\cos n\omega_1 t dt}{\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} \cos^2 n\omega_1 t dt} = \frac{2}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} f(t)\cos n\omega_1 t dt$$

$$\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} \cos n\omega_1 t \cos m\omega_1 t dt = \begin{cases} 0 & m \neq n \\ \frac{T_1}{2} & m = n \end{cases}$$

$$a_0 = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} f(t) dt$$

$$a_0 = \frac{1}{T_1} \int_{t_0}^{t_0 + T_1} f(t) \mathrm{d}t$$

$$a_n = \frac{2}{T_1} \int_{t_0}^{t_0 + T_1} f(t) \cos(n\omega_1 t) \mathrm{d}t$$

$$b_n = \frac{2}{T_1} \int_{t_0}^{t_0 + T_1} f(t) \sin(n\omega_1 t) \mathrm{d}t$$

$$f(t)=c_0+\sum_{n=1}^{\infty}c_n\cos(n\omega_1 t+\varphi_n)$$

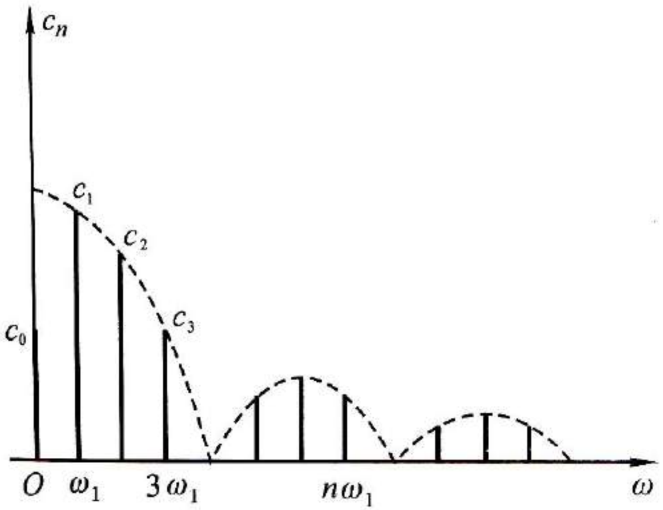
$$f(t)=d_0+\sum_{n=1}^{\infty}d_n\sin(n\omega_1 t+\theta_n)$$

$$c_n=\sqrt{a_n^2+b_n^2}$$

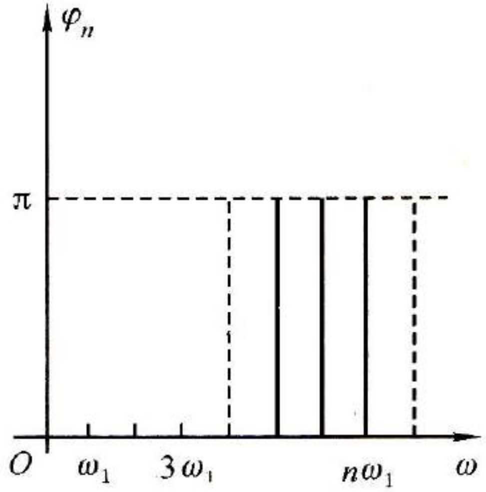
$$a_n=c_n\cos\varphi_n$$

$$b_n=c_n\sin\varphi_n$$

任何周期信号在满足Dirchlet条件下，均可分解为直流分量基波分量和谐波分量



(a) 幅度谱



(b) 相位谱

周期信号的谱为离散谱

$$a_0 = c_0 = d_0$$

$$c_n = d_n = \sqrt{a_n^2 + b_n^2}$$

$$a_n = c_n \cos\varphi_n = d_n \sin\theta_n$$

$$b_n = -c_n \sin\varphi_n = d_n \cos\theta_n$$

$$\tan\theta_n = \frac{a_n}{b_n}$$

$$\tan\varphi_n = -\frac{b_n}{a_n}$$

$$(n = 1, 2, \cdots)$$

- 指数形式的傅氏级数

$$f(t)=\sum_{n=-\infty}^{\infty}F(n\omega_1)e^{jn\omega_1 t}$$

$$F_n=\frac{1}{T_1}\int_{t_0}^{t_0+T_1}f(t)e^{-jn\omega_1 t}dt$$

$$\cos n\omega_1 t=\frac{1}{2}(e^{jn\omega_1 t}+e^{-jn\omega_1 t})$$

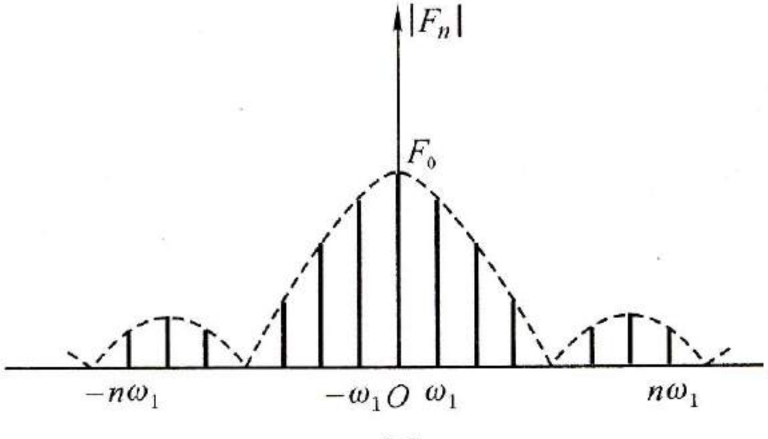
$$\sin n\omega_1 t=\frac{1}{2j}(e^{jn\omega_1 t}-e^{-jn\omega_1 t})$$

欧拉公式

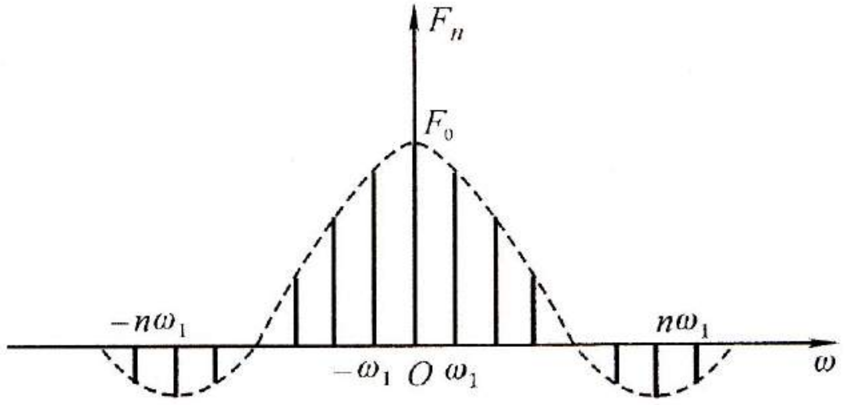
$$f(t)=a_0+\sum_{n=1}^{\infty}a_n\cos n\omega_1 t+b_n\sin n\omega_1 t$$

$$F_n=\frac{1}{2}(a_n-jb_n)$$

$$F_{-n}=\frac{1}{2}(a_n+jb_n)$$



(a)



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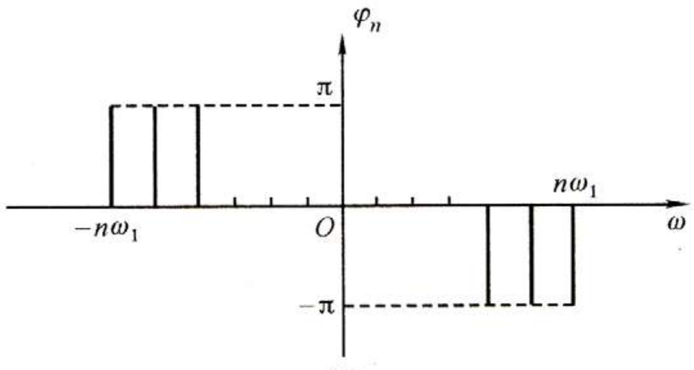


(b)

$$c_0 = d_0 = a_0 = F_0$$

$$|F_n| = |F_{-n}| = \frac{1}{2}c_n = \frac{1}{2}d_n = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

$$|F_n| + |F_{-n}| = c_n$$

- 例：某周期信号基频$\omega_1=2\pi$，由有限项谐波组成的表达式为：

$$f(t)=\sum_{n=-3}^{3}F_n e^{jn2\pi t}$$

$$F_0=1/2$$
$$F_1=F_{-1}=1/8$$
$$F_2=F_{-2}=1/4$$
$$F_3=F_{-3}=1/6$$

设系数Fn均为实数，求f(t)的三角表示形式

$$f(t) = \frac{1}{2} + \frac{1}{8}(e^{j2\pi t} + e^{-j2\pi t})$$

$$+ \frac{1}{4}(e^{j4\pi t} + e^{-j4\pi t}) + \frac{1}{6}(e^{j6\pi t} + e^{-j6\pi t})$$

$$= \frac{1}{2} + \frac{1}{4}\cos 2\pi t + \frac{1}{2}\cos 4\pi t + \frac{1}{3}\cos 6\pi t$$

- 函数的对称性与傅氏级数的关系
  - 波形对称性对被积函数积分区间的奇偶性的判断可简化计算

$$[\text{奇函数}]\times\left[\text{偶函数}\right]=\left[\text{奇函数}\right]$$

$$[\text{奇函数}]\times\left[\text{奇函数}\right]=\left[\text{偶函数}\right]$$

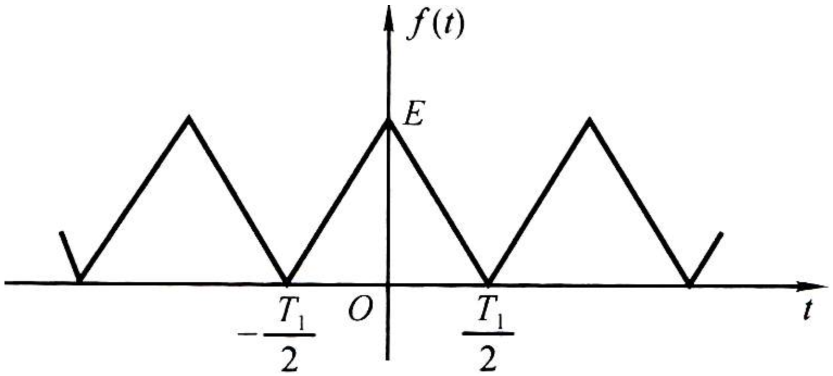
$$[\text{偶函数}]\times\left[\text{偶函数}\right]=\left[\text{偶函数}\right]$$

  - 奇函数展开时 $a_0, a_n = 0$
  - 偶函数展开时 $b_n = 0$

– 偶函数 f(t)=f(-t)

$$a_n = \frac{4}{T_1}\int_0^{\frac{T_1}{2}} f(t)\cos(n\omega_1 t)\mathrm{d}t$$

$$b_n = 0$$



$$a_0 = \frac{2}{T_1}\int_0^{T_1/2}\frac{2E}{T_1}(\frac{T_1}{2} - t)dt = \frac{E}{2}$$

$$a_n = \frac{4}{T_1}\int_0^{T_1/2}\frac{2E}{T_1}(\frac{T_1}{2} - t)\cos(n\omega_1 t)dt$$

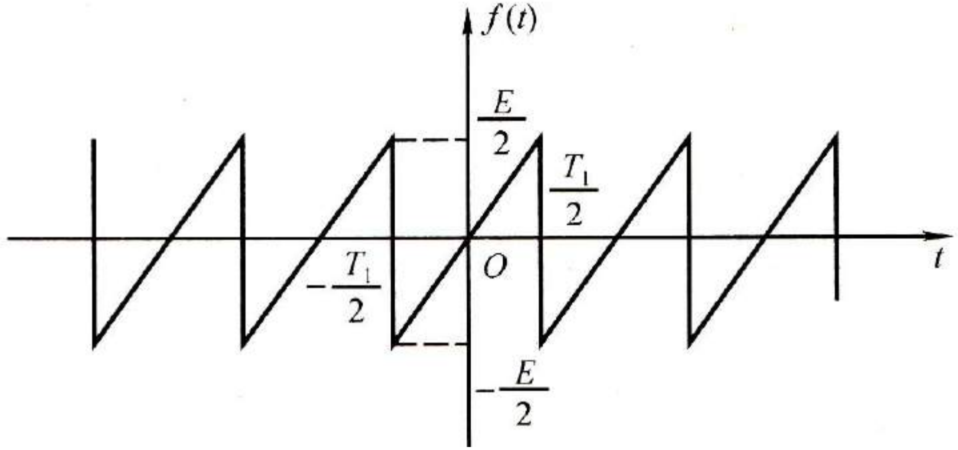
$$= \frac{4E}{\pi^2}\frac{1}{n^2}\sin^2(\frac{n\pi}{2})$$

$$f(t) = \frac{E}{2} + \frac{4E}{\pi^2}\left[\cos(\omega_1 t) + \frac{1}{9}\cos(3\omega_1 t) + \frac{1}{25}\cos(5\omega_1 t) + \cdots\right]$$

– 奇函数 f(t)=-f(-t)

$$a_0=0, \quad a_n=0$$

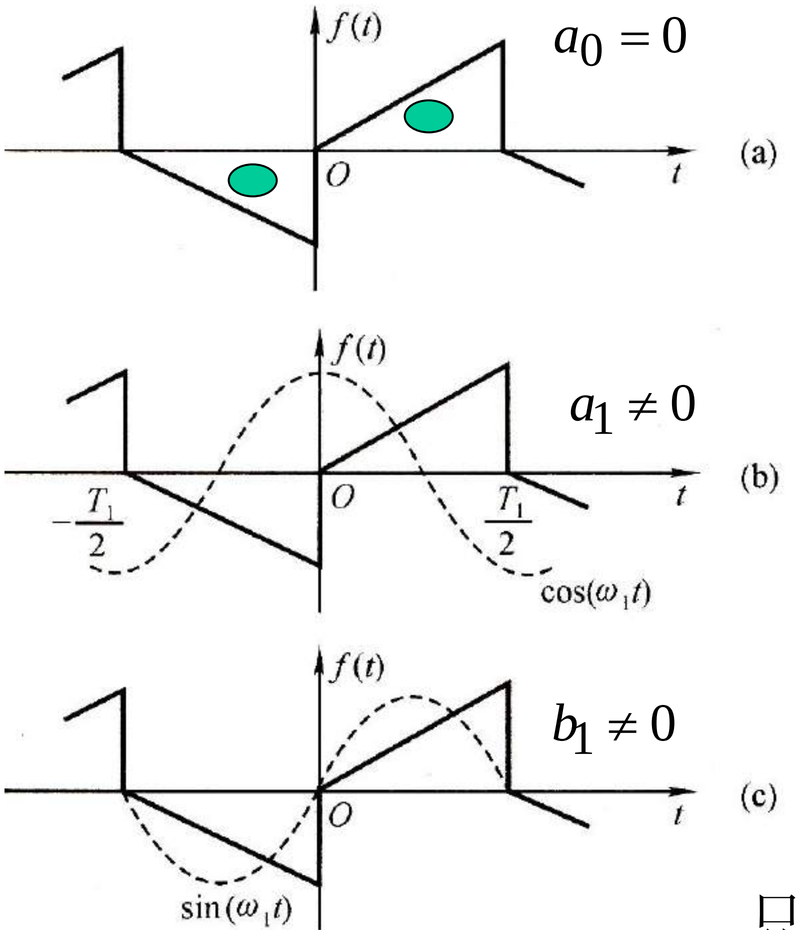
$$b_n=\frac{4}{T_1}\int_0^{\frac{T_1}{2}} f(t)\sin(n\omega_1 t)\mathrm{d}t$$

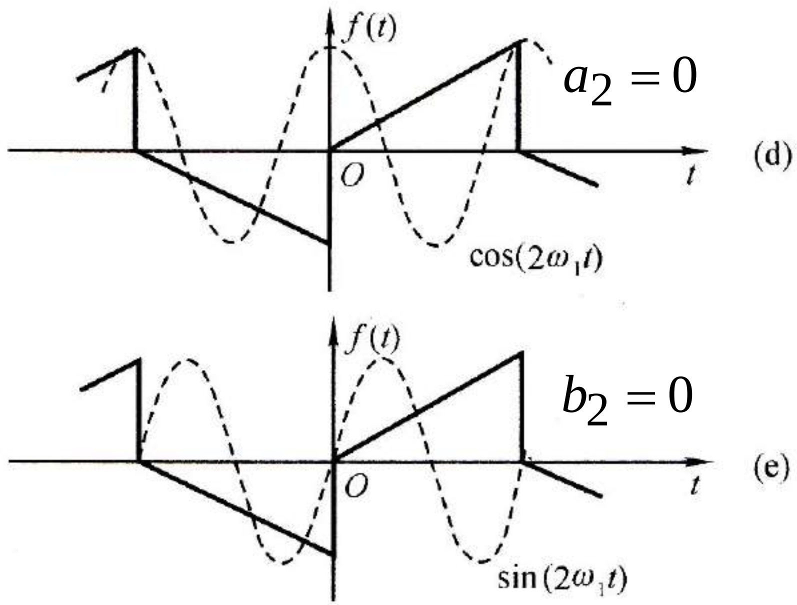


$$f(t)=\frac{E}{\pi}\left[\sin(\omega_1 t)-\frac{1}{2}\sin(2\omega_1 t)+\frac{1}{3}\sin(3\omega_1 t)-\cdots\right]$$

– 奇谐函数 f(t)=-f(-t±T_1/2)

半周期镜像信号





只含有基波及奇次谐波

- 傅氏级数与最小方均误差
  - 周期信号的傅氏级数

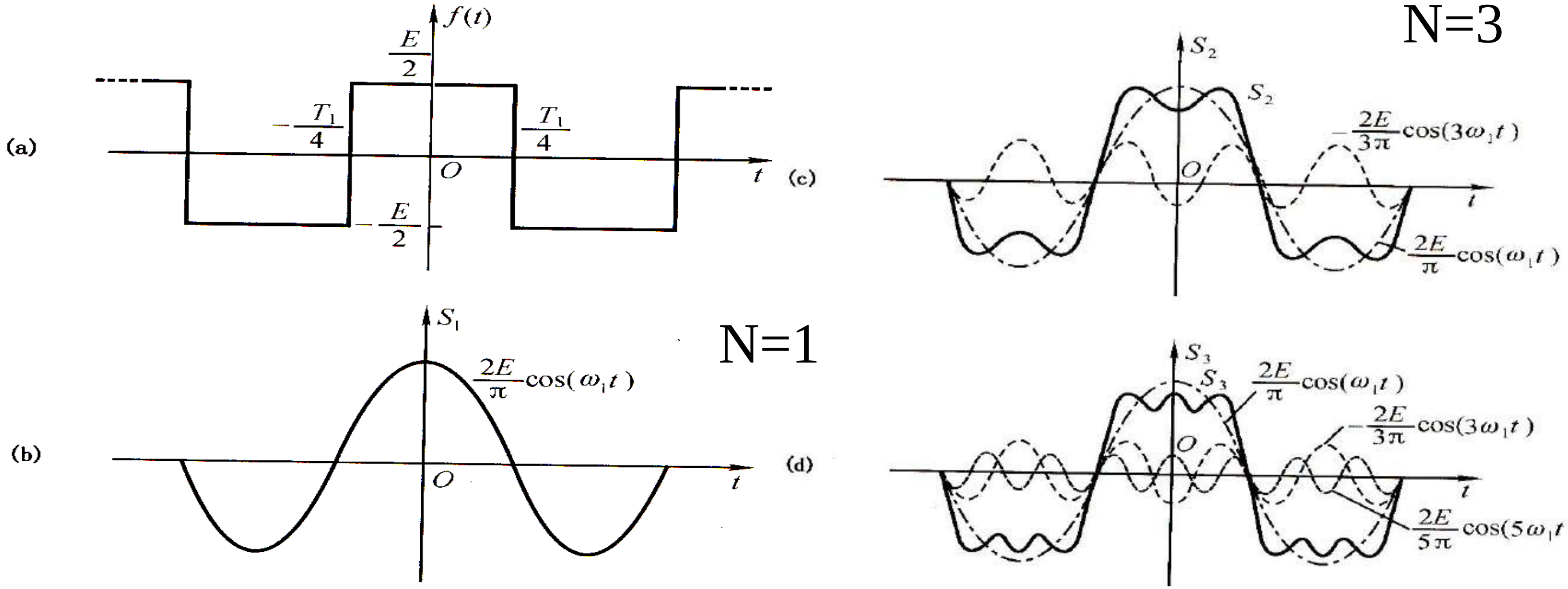
$$f(t)=a_0+\sum_{n=1}^{\infty}[a_n\cos(n\omega_1 t)+b_n\sin(n\omega_1 t)]$$

$$f(t)=a_0+\sum_{n=1}^{N}[a_n\cos(n\omega_1 t)+b_n\sin(n\omega_1 t)]$$

  - 误差函数

$$\overline{\varepsilon^2}=\frac{1}{T_1}[\int_0^{T_1} f^2(t)dt-\sum_{r=1}^{N}C_r^2k_r]$$ ← 完备的正交函数集

$$\overline{\varepsilon^2}=\frac{1}{T_1}\int_0^{T_1} f^2(t)dt-[a_0^2+\frac{1}{2}\sum_{n=1}^{N}(a_n^2+b_n^2)]$$



N=5

既是偶函数，又是奇谐函数

$$a_n = \frac{2E}{n\pi}\sin(\frac{n\pi}{2}) \quad (n = 1,3,5...)$$

$$\overline{\varepsilon^2} = \frac{1}{T_1}\int_0^{T_1} f^2(t)dt - [a_0^2 + \frac{1}{2}\sum_{n=1}^{N}(a_n^2 + b_n^2)]$$

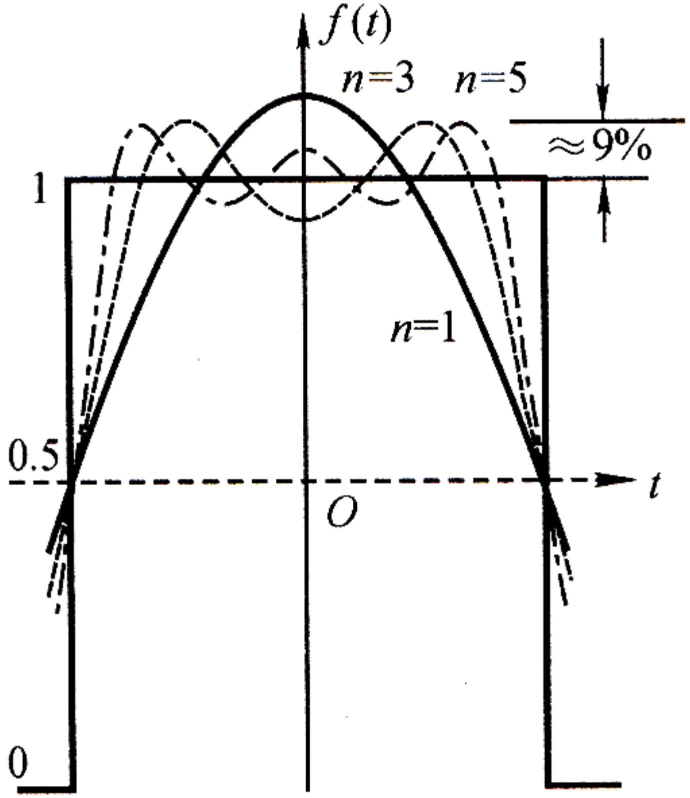
$$N = 1$$

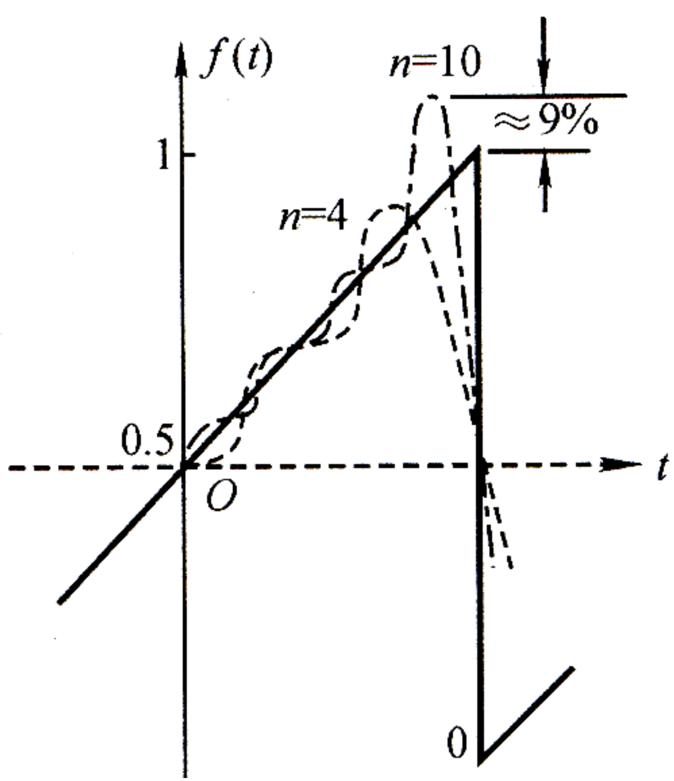
$$\overline{\varepsilon^2} = \frac{1}{T_1}\int_{-T_1/4}^{T_1/4} \frac{E^2}{4}dt - \frac{1}{2}(\frac{2E}{\pi})^2 \approx 0.05E^2$$

$$N = 3$$

$$\overline{\varepsilon^2} = \frac{1}{T_1}\int_{-T_1/4}^{T_1/4} \frac{E^2}{4}dt - \frac{1}{2}(\frac{2E}{\pi})^2 - \frac{1}{2}(\frac{2E}{3\pi})^2 \approx 0.02E^2$$

$$E_1 \approx 0.05E^2, E_3 \approx 0.02E^2, E_5 \approx 0.015E^2$$



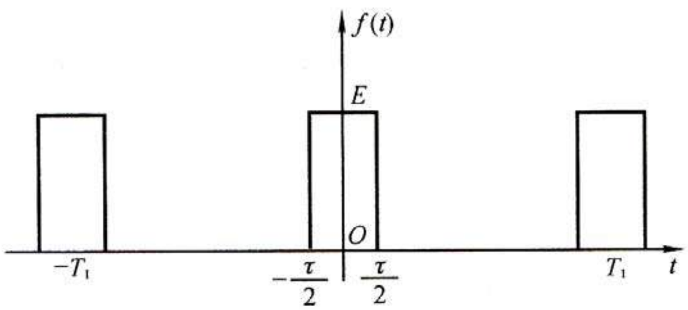


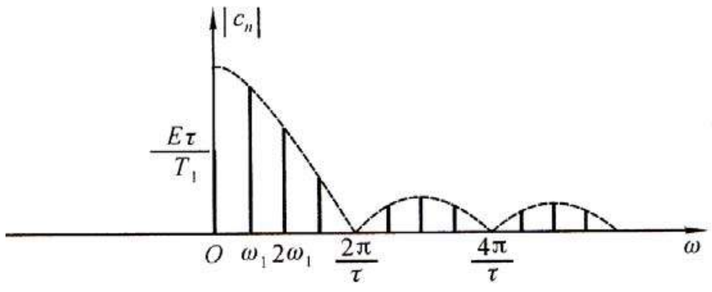
- N越大，相加后的波形越接近f(t),误差越小
- 高频分量对应跳变，低频分量影响脉冲的顶部
- Gibbs现象—不连续点的幅度为1，不论N多大，峰值为1.09

3.3 典型周期信号的傅氏级数

- 周期矩形脉冲信号

$$f(t)=\frac{E\tau}{T_1}+\frac{2E\tau}{T_1}\sum_{n=1}^{\infty}Sa(\frac{n\pi\tau}{T_1})\cos(n\omega_1 t)$$



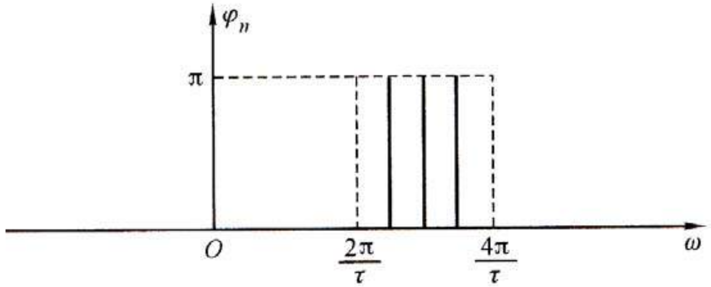


(a)

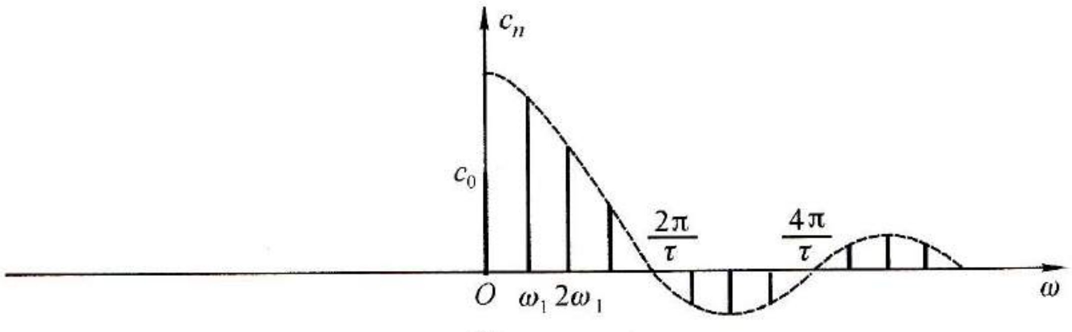
$$f(t)=E[u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})]$$

$$a_0=\frac{E\tau}{T_1}$$

$$a_n=\frac{2E}{n\pi}\sin(\frac{n\pi\tau}{T_1})=\frac{2E\tau}{T_1}Sa(\frac{n\pi\tau}{T_1})$$

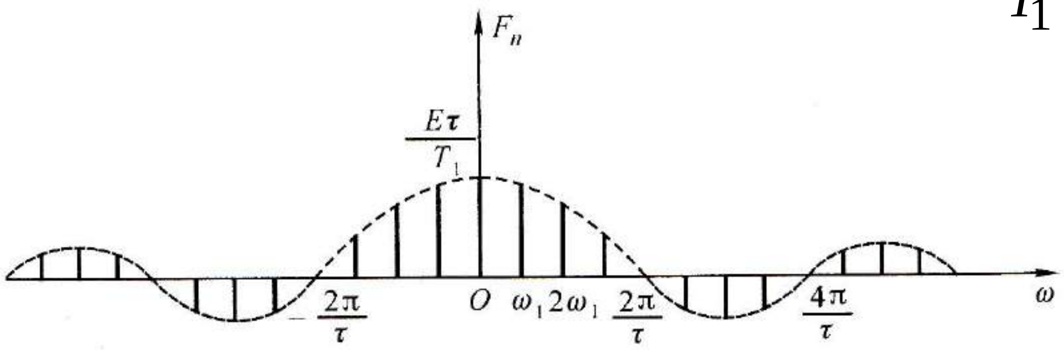


(b)



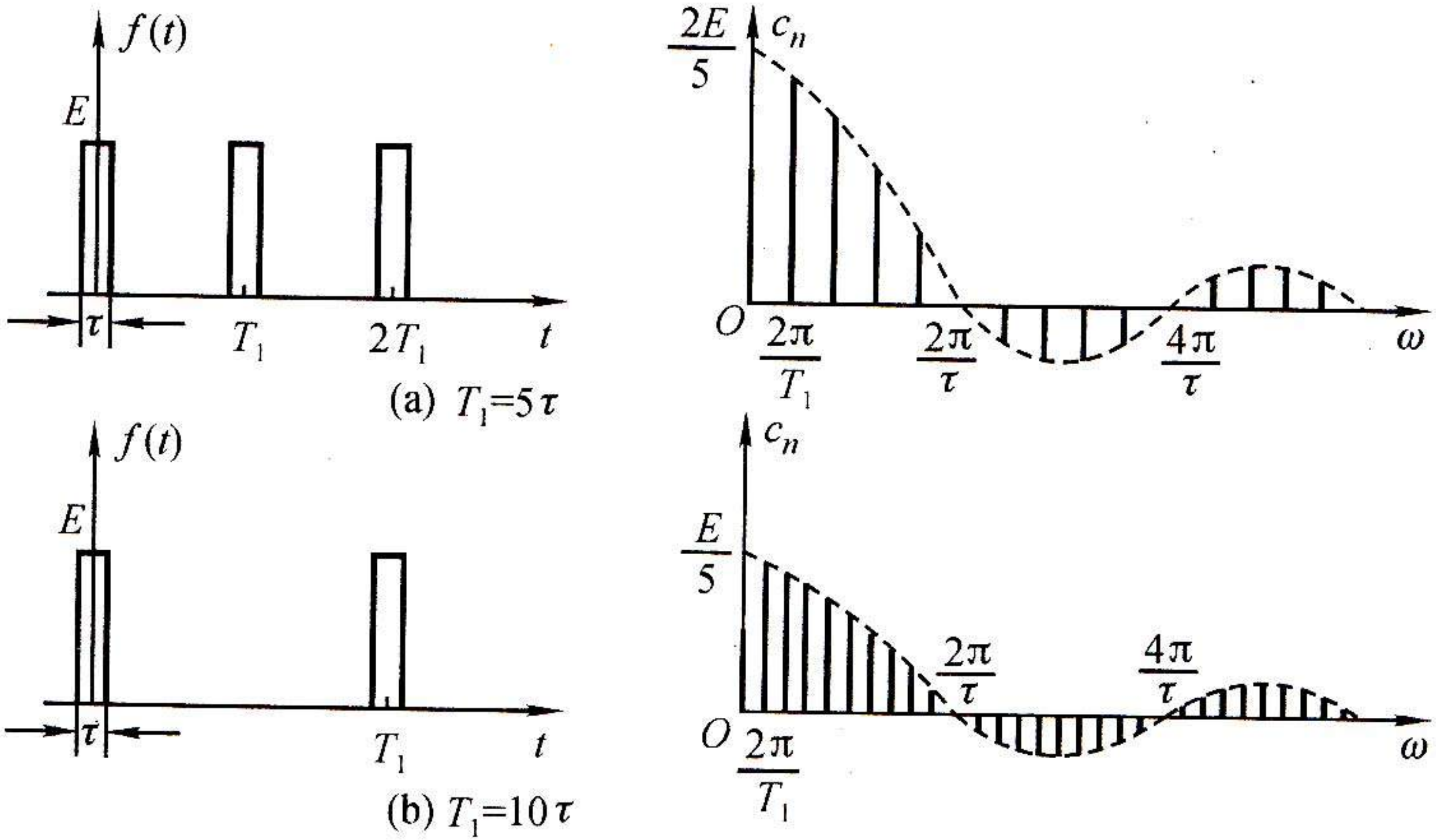
(c)

$$f(t)=\frac{E\tau}{T_1}\sum_{n=-\infty}^{\infty}Sa\left(\frac{n\omega_1\tau}{2}\right)e^{jn\omega_1 t}$$

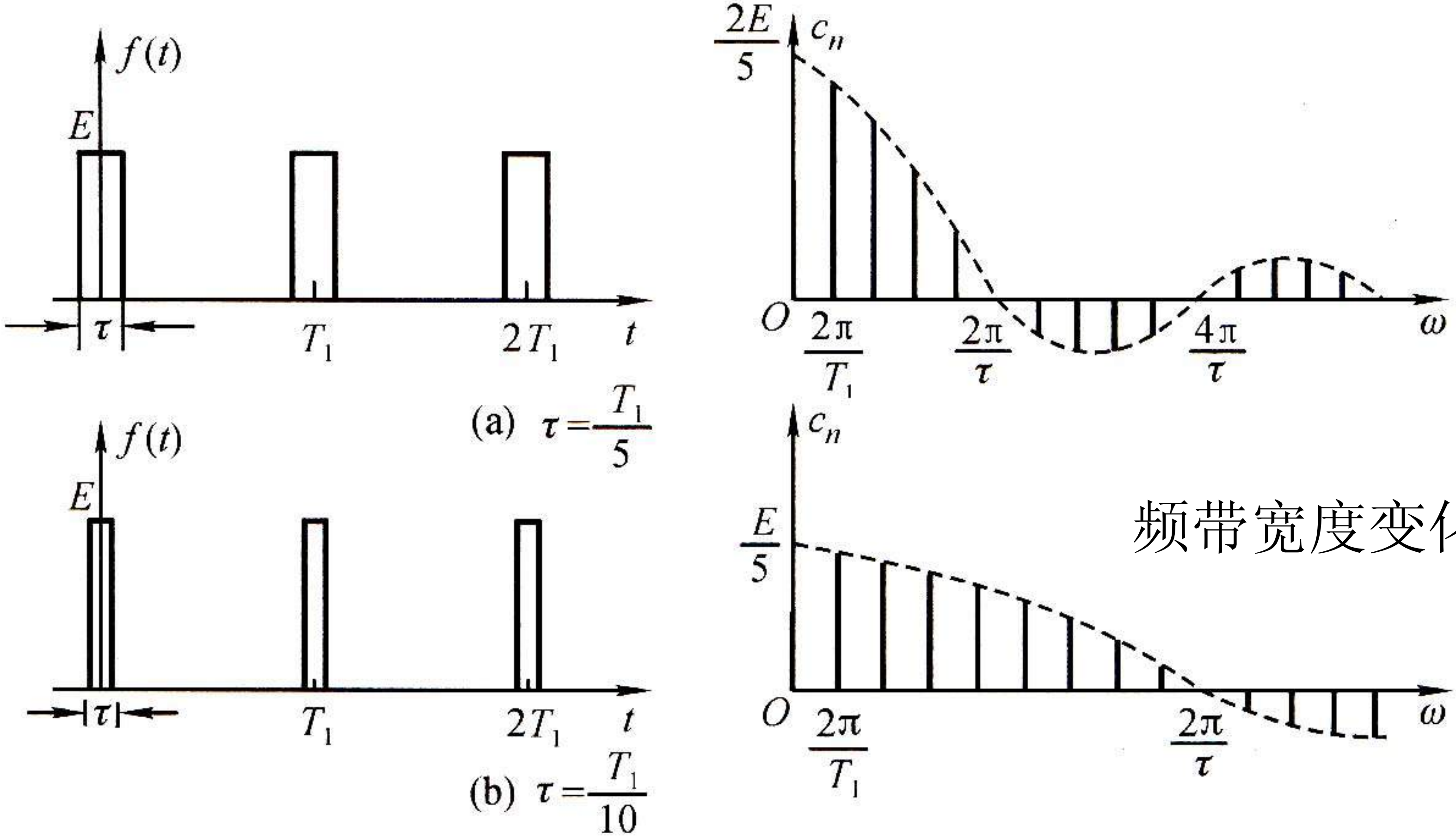


(d)

- 离散谱线，间隔为ω_1, 幅度正比于E、τ，反比于T_1
- 谱线包络为Sa函数，$\omega=2m\pi/\tau$为零点
- 信号的主要能量宽度—第一个零点以内(频带宽度)，　$B_\omega=2\pi/\tau$



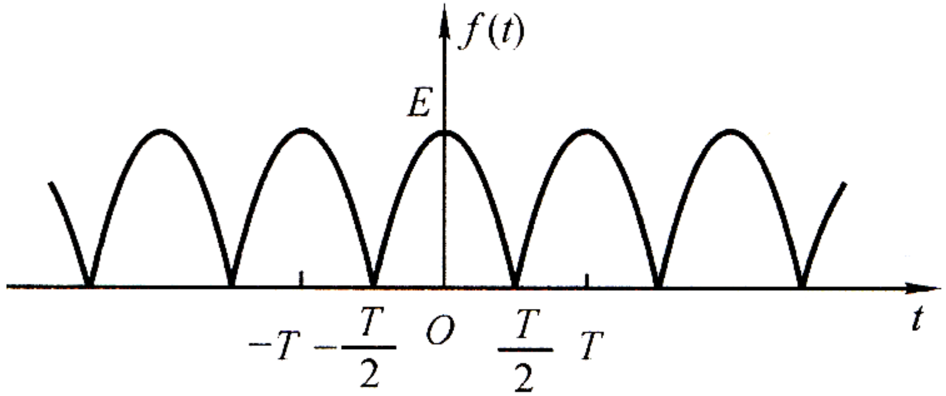
τ保持不变，$T_1=10\tau$， $T_1=5\tau$ 时的频谱



频带宽度变化

T_1保持不变， $\tau = T_1/10$， $\tau = T_1/5$ 时的频谱

- 周期全波整流信号



$$f_1(t) = E\cos\omega_0 t$$

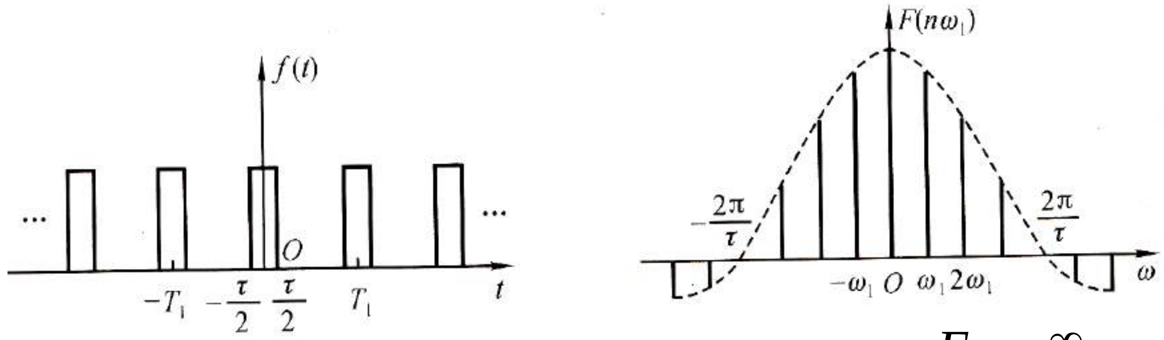
$$f(t) = E\,|\cos\omega_0 t\,|$$

$$\omega_1 = 2\omega_0$$

$$f(t) = \frac{2E}{\pi} + \frac{4E}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{4n^2-1}\cos(2n\omega_0 t)$$

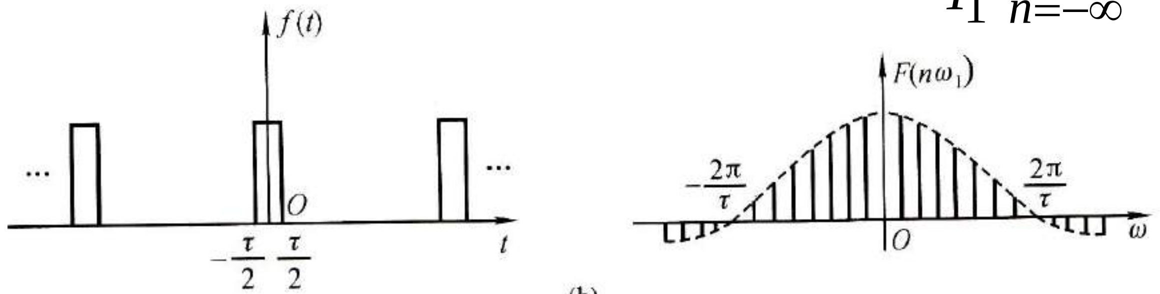
3.4 傅里叶变换

- 周期信号的离散谱到非周期信号的连续谱

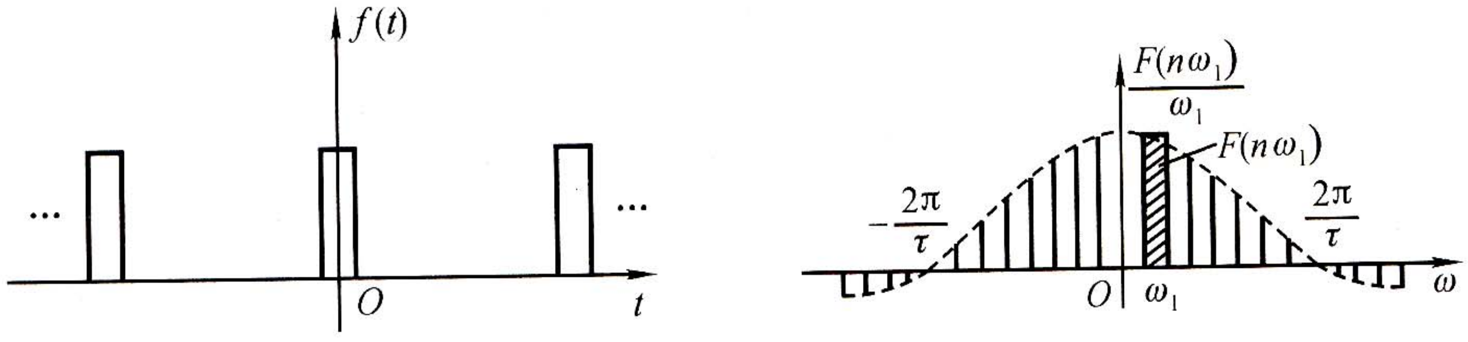


(a)

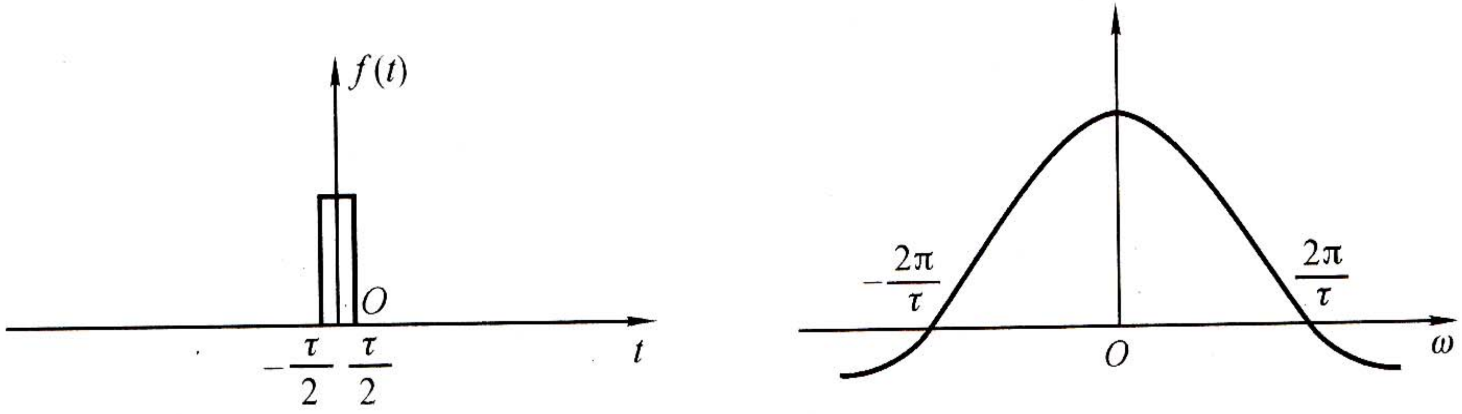
$$f(t)=\frac{E\tau}{T_1}\sum_{n=-\infty}^{\infty}Sa\left(\frac{n\omega_1\tau}{2}\right)e^{jn\omega_1 t}$$



(b)



(c)



(d)

周期信号→非周期信号

谱线间隔愈来愈密

离散谱→连续谱

- 周期信号的FS

$$f(t)=\sum_{n=-\infty}^{\infty}F(n\omega_1)\mathrm{e}^{\mathrm{j}n\omega_1 t}$$

$$F(n\omega_1)=\frac{1}{T_1}\int_0^{T_1}f(t)e^{-jn\omega_1 t}dt$$

- 非周期信号FT

$$F(j\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(j\omega)e^{j\omega t}d\omega$$

$$T_1 \to \infty, \omega_1 \to 0, \Delta(n\omega_1) \to d\omega$$

离散频率$n\omega_1 \to$连续频率ω

$$F(n\omega_1) \to 0, F(n\omega_1)T_1 = \frac{2\pi F(n\omega_1)}{\omega_1} \neq 0 \to \text{频谱密度函数F}(j\omega)$$

$$F(j\omega) = \int_{-T_1/2}^{T_1/2} f(t)e^{-jn\omega_1 t}dt$$

$$= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \longleftarrow \text{FT}$$

FT存在的充分条件：

$\int_{-\infty}^{\infty} |f(t)|dt < \infty$绝对可积

$$\text{设}f_\sigma(t)=\frac{1}{2\pi}\int_{-\sigma}^{\sigma}F(j\omega)e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-\sigma}^{\sigma}[\int_{-\infty}^{\infty}f(\tau)e^{-j\omega\tau}d\tau]e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(\tau)[\int_{-\sigma}^{\sigma}e^{-j\omega(t-\tau)}d\omega]d\tau$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}f(\tau)\frac{2\sin[\sigma(t-\tau)]}{t-\tau}d\tau$$

$$\lim_{\sigma\to\infty}\frac{\sin[\sigma(t-\tau)]}{\pi(t-\tau)}=\lim_{\sigma\to\infty}\frac{\sigma}{\pi}Sa[\sigma(t-\tau)]=\delta(t-\tau)$$

$$\lim_{\sigma\to\infty}f_\sigma(t)=\int_{-\infty}^{\infty}f(\tau)\delta(t-\tau)d\tau=f(t)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(j\omega)e^{j\omega t}d\omega$$ ← IFT

$$f(t)\rightleftharpoons F(j\omega)$$

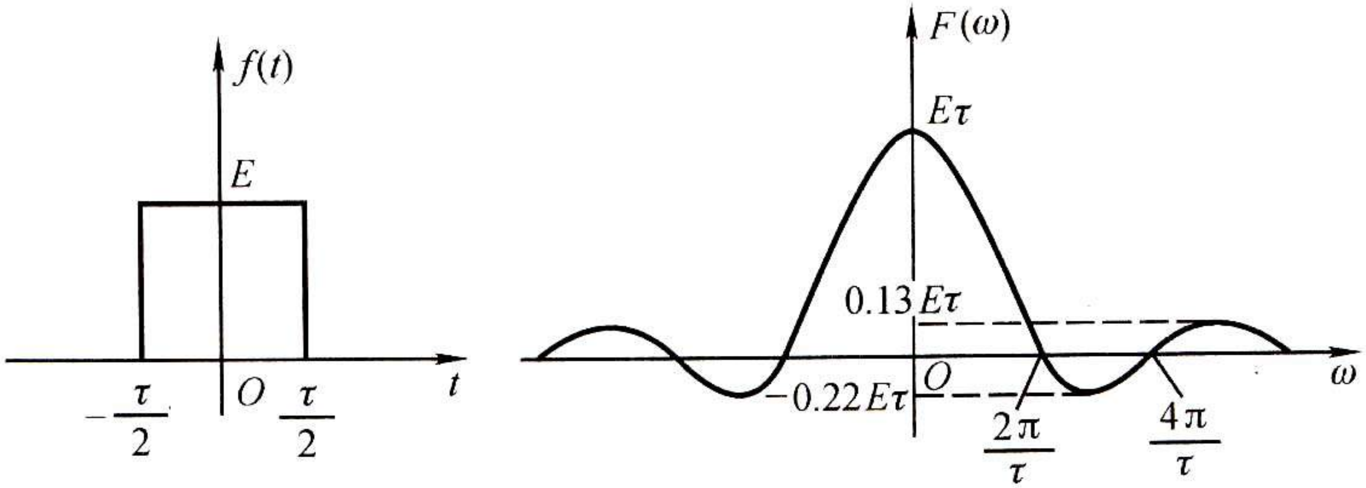
$$F(j\omega)=F[f(t)],\ f(t)=F^{-1}[F(j\omega)]$$

例：求图中矩形脉冲的FT

$$F(j\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$=E\int_{-\tau/2}^{\tau/2} e^{-j\omega t}dt=-\frac{E}{j\omega}(e^{-j\omega\tau/2}-e^{j\omega\tau/2})$$

$$=E\tau Sa(\frac{\omega\tau}{2})$$



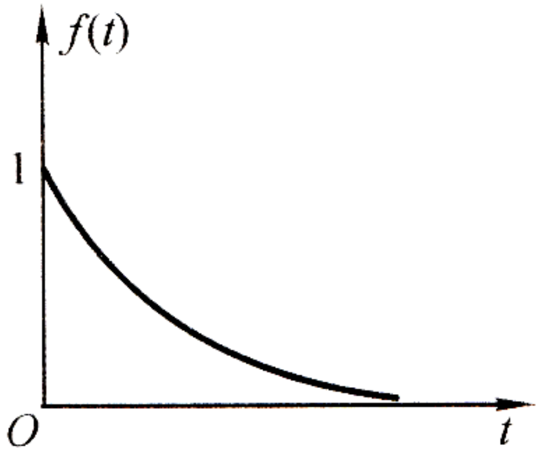
- 连续谱
- 具有收敛性（峰值比较）

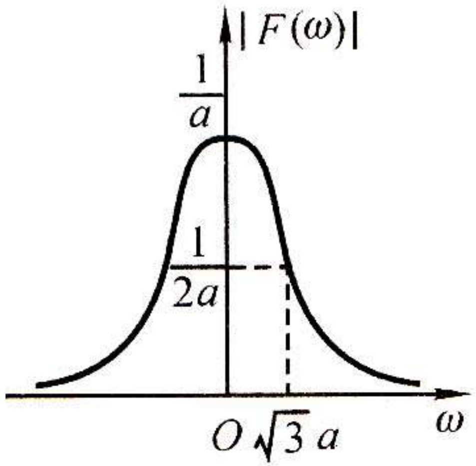
3.5 常见信号的傅里叶变换

- 单边指数信号

$f(t) = e^{-at}U(t) \quad (a > 0) \qquad F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{0}^{\infty} e^{-at}e^{-j\omega t}dt$

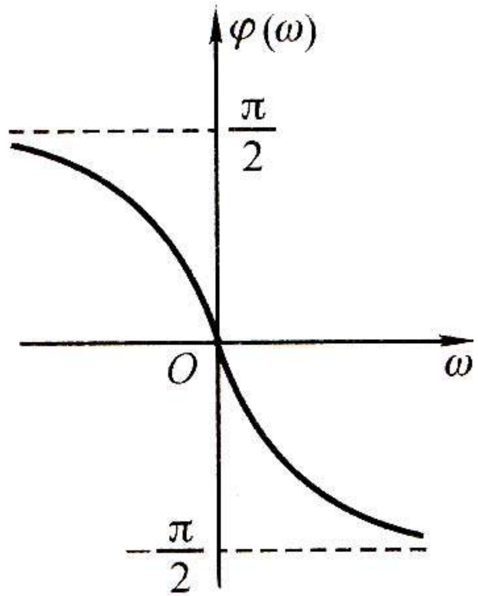
$$\mathrm{F(j\omega)} = \frac{1}{\mathrm{a + j\omega}}$$





$$|\mathrm{F(j\omega)}| = \frac{1}{\sqrt{\mathrm{a}^2 + \omega^2}}$$

$$\varphi(\omega) = -\tan^{-1}\omega / a$$



- **符号函数**

$$Sgn(t) = U(t) - U(-t)$$

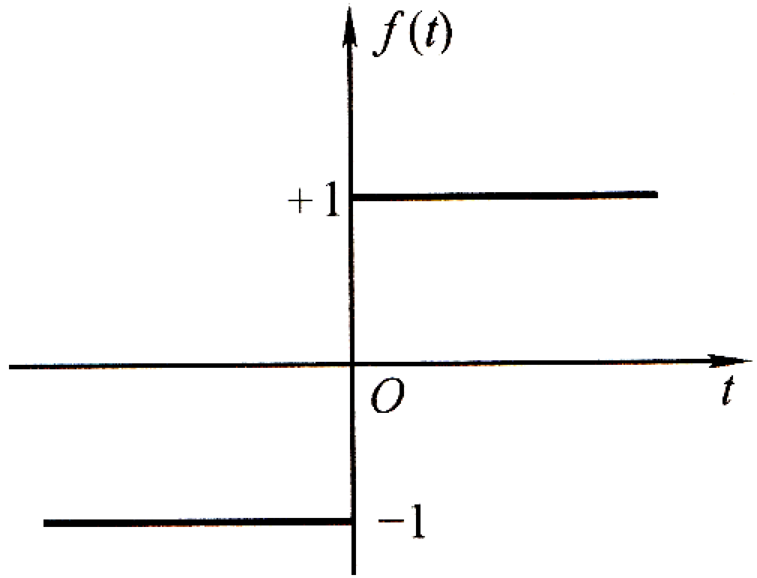
不满足绝对可积条件

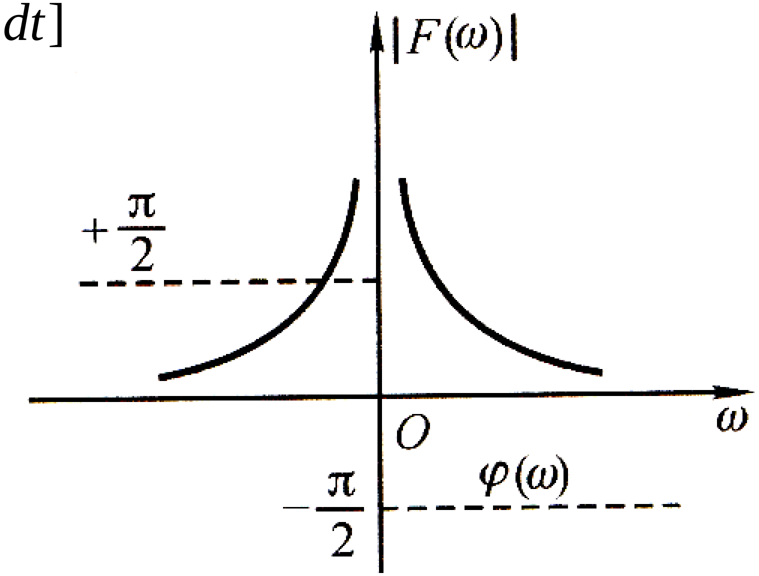
$$Sgn(t) = \lim_{a \to 0} [e^{-at}U(t) - e^{at}U(-t)]$$

$$F(j\omega) = \lim_{a \to 0} [\int_0^{\infty} e^{-at} e^{-j\omega t} dt - \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt]$$

$$= \lim_{a \to 0} [\frac{e^{-(a+j\omega)}}{-(a+j\omega)} |_0^{\infty} - \frac{e^{(a-j\omega)}}{(a-j\omega)} |_{-\infty}^{0}]$$

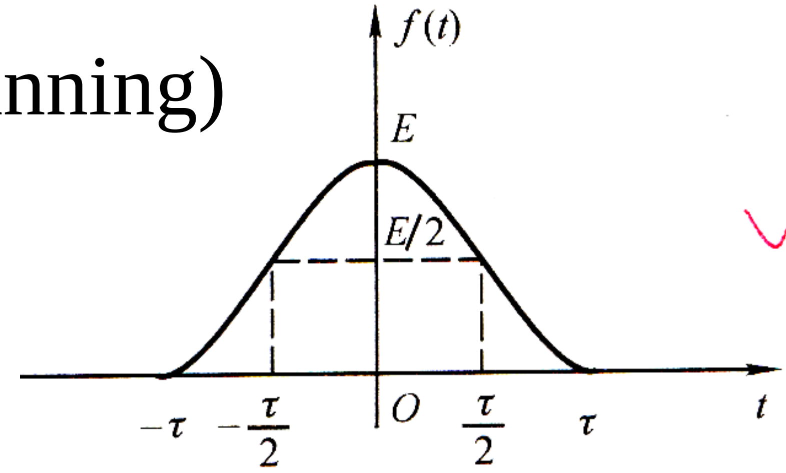
$$= \lim_{a \to 0} \frac{-2j\omega}{a^2+\omega^2} = \frac{2}{j\omega}$$





奇函数→FT只有虚部

- 升余弦脉冲函数（Hanning)



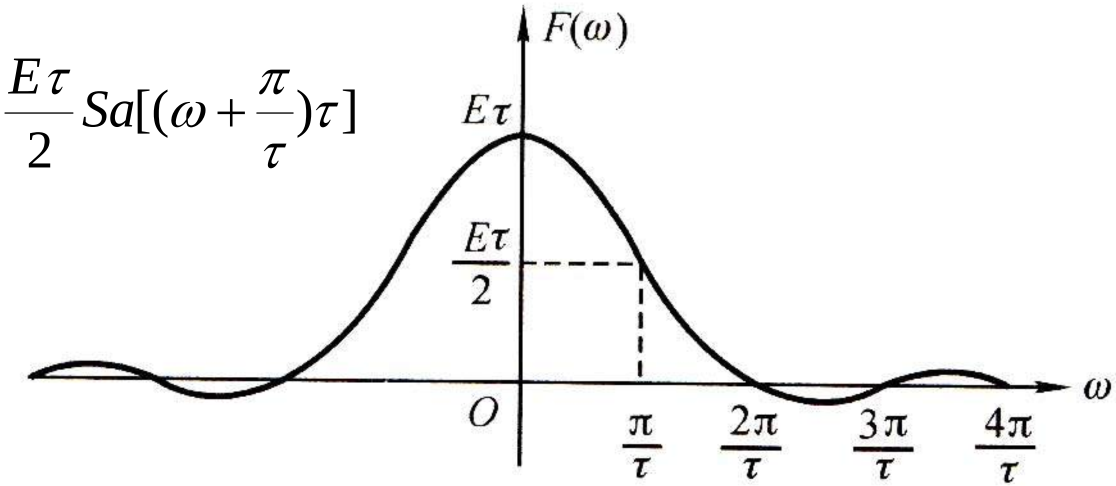
$$f(t)=\frac{E}{2}(1+\cos\frac{\pi t}{\tau})\qquad 0\le|t|\le\tau$$

$$F(j\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$$

$$=\int_{-\tau}^{\tau}\frac{E}{2}(1+\cos\frac{\pi t}{\tau})e^{-j\omega t}dt$$

$$=\frac{E}{2}\int_{-\tau}^{\tau}e^{-j\omega t}dt+\frac{E}{2}\int_{-\tau}^{\tau}\frac{e^{j\frac{\pi t}{\tau}}+e^{-j\frac{\pi t}{\tau}}}{2}e^{-j\omega t}dt$$

$$=E\tau Sa(\omega\tau)+\frac{E\tau}{2}Sa[(\omega-\frac{\pi}{\tau})\tau]+\frac{E\tau}{2}Sa[(\omega+\frac{\pi}{\tau})\tau]$$



主瓣加宽1倍
副瓣的幅度抑制

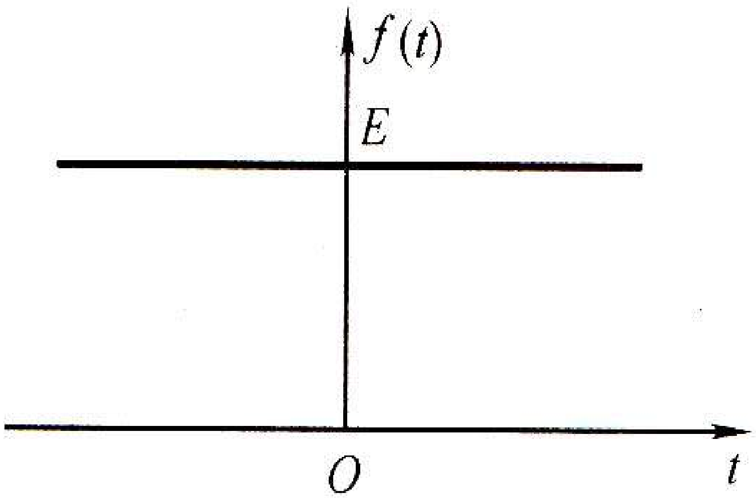
- 直流信号

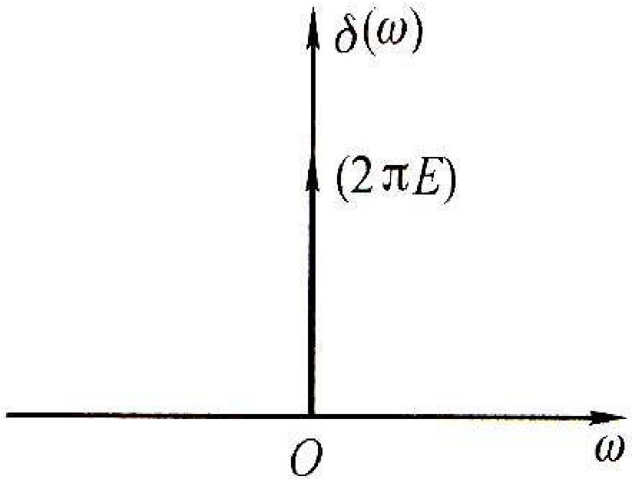
偶函数→FT只有实部

$$f(t) = E$$

$$F(j\omega) = \lim_{\tau\to\infty}\int_{-\tau}^{\tau} Ee^{-j\omega t}dt = E\lim_{\tau\to\infty}\frac{e^{-j\omega t}}{-j\omega}\Big|_{-\tau}^{\tau}$$

$$= E\lim_{\tau\to\infty}\frac{2\sin\omega\tau}{\omega} = 2\pi E\lim_{\tau\to\infty}\frac{\tau}{\pi}\frac{\sin\omega\tau}{\omega\tau} = 2\pi E\delta(\omega)$$





3.6 冲激信号及阶跃信号的FT

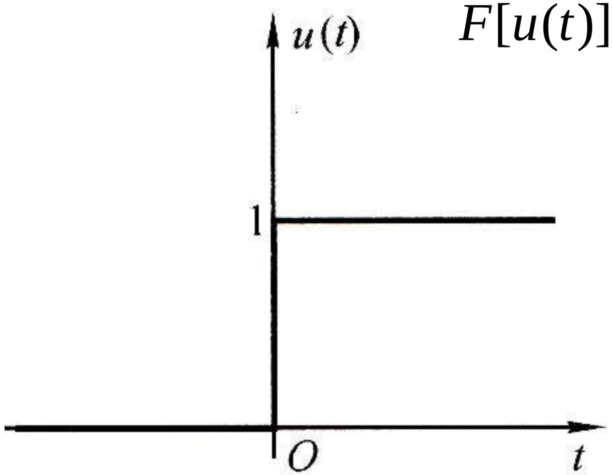
- 阶跃信号

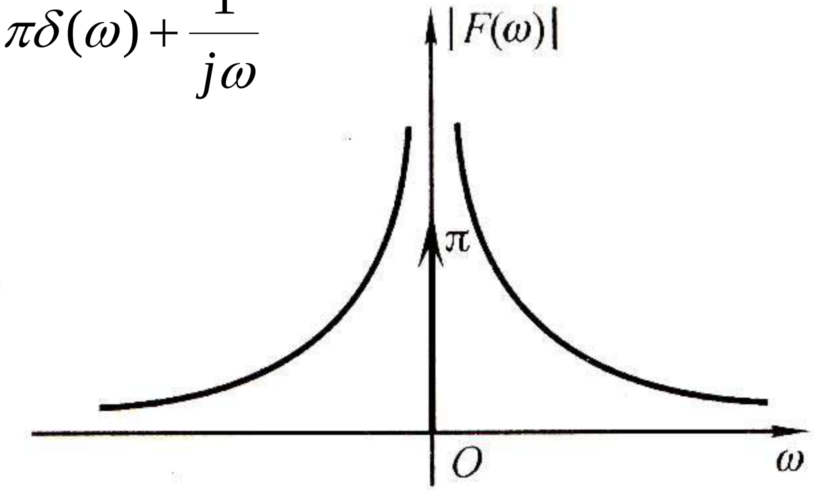
$$u(t) = \frac{1}{2} + \frac{1}{2} Sgn(t)$$

$$F[1/2] = \frac{1}{2} 2\pi\delta(\omega) = \pi\delta(\omega)$$

$$F[Sgn(t)] = \frac{2}{j\omega}$$

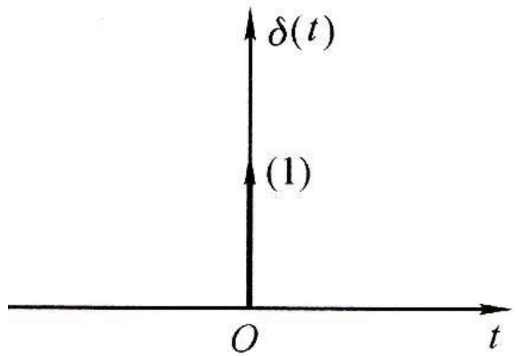
$$F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$$

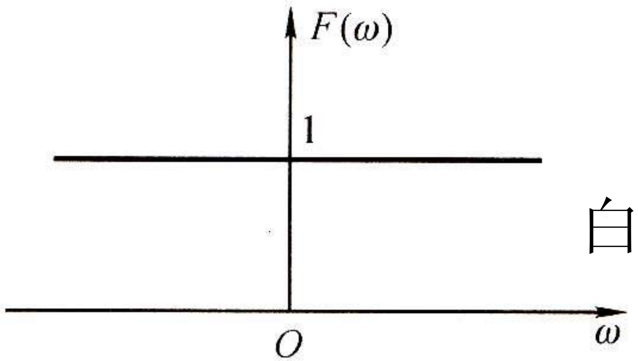




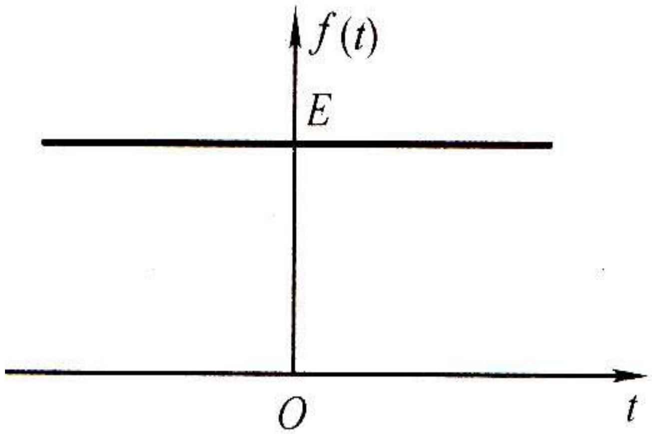
- 冲激信号

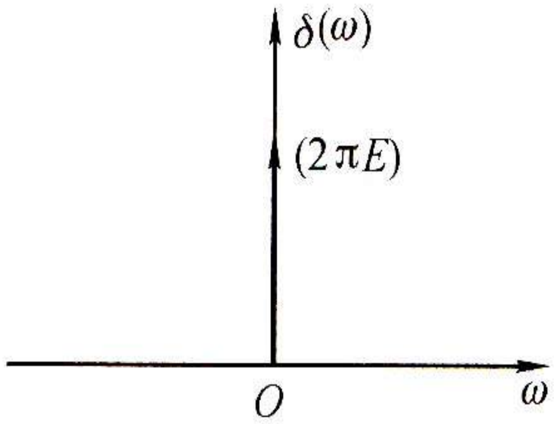
$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$$





白色频谱





- 冲激偶的FT

$$F[\delta(t)]=1$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(j\omega)e^{j\omega t}d\omega$$

$$\therefore \delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{j\omega t}d\omega$$

$$\therefore \delta'(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}j\omega e^{j\omega t}d\omega$$

$$\because\ f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(j\omega)e^{j\omega t}d\omega$$

$$\therefore F[\delta'(t)]=j\omega$$

$\longrightarrow$ $$F[\frac{d^n}{dt^n}\delta(t)]=(j\omega)^n$$

求证：$F(t^n) = 2\pi(j)^n \frac{d^n}{d\omega^n}[\delta(\omega)]$

证：

$\because 2\pi\delta(\omega) = F[1]$

$\therefore \delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j\omega t} dt$

$$\frac{d\delta(\omega)}{d\omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} - jte^{-j\omega t} dt$$

n次微分：

$$\frac{d^n\delta(\omega)}{d\omega^n} = \frac{1}{2\pi}\int_{-\infty}^{\infty} (\frac{t}{j})^n e^{-j\omega t} dt$$

$$\therefore F[t^n] = 2\pi(j)^n \frac{d^n\delta(\omega)}{d\omega^n}$$

3.7 傅里叶变换的基本性质

$$f(t) \Leftrightarrow F(\omega)$$

$$F[f(t)] = F(\omega)$$

唯一性

$$F^{-1}[F(\omega)] = f(t)$$

1、线性

若：

$$f_1(t) \Leftrightarrow F_1(\omega)$$

$$f_2(t) \Leftrightarrow F_2(\omega)$$

则：

$$c_1 f_1(t) + c_2 f_2(t) \Leftrightarrow c_1 F_1(\omega) + c_2 F_2(\omega)$$

2、对称性

若：$F(\omega)=F[f(t)]$

则：$F[F(t)]=2\pi f(-\omega)$

$$\frac{1}{2\pi}F(t)-->f(-\omega)$$

$$1=F[\delta(t)]$$

$$F[1]=2\pi\delta(-\omega)=2\pi\delta(\omega)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$$

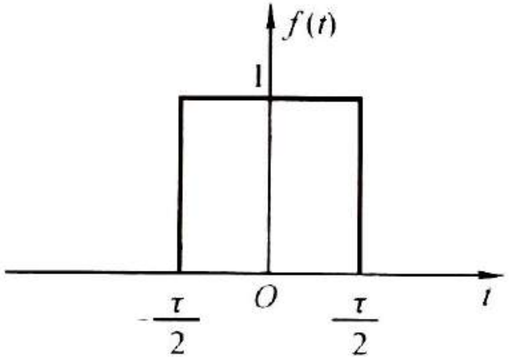
$$f(-t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{-j\omega t}d\omega$$

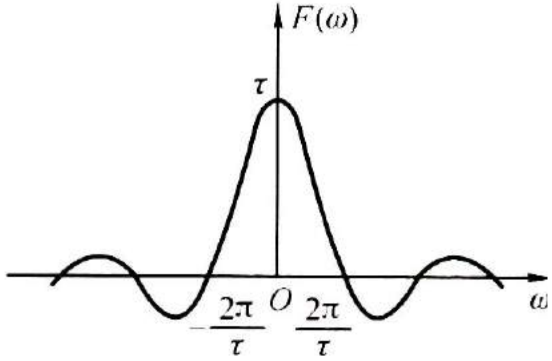
$$f(-t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(x)e^{-jxt}dx \qquad \omega\to x$$

$$f(-\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(x)e^{-jx\omega}dx \qquad t\to\omega$$

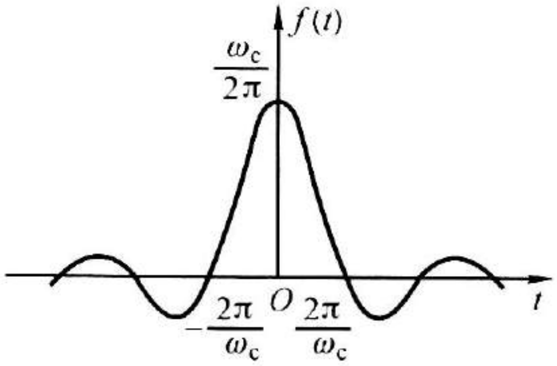
$$f(-\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(t)e^{-jt\omega}dt \qquad x\to t$$

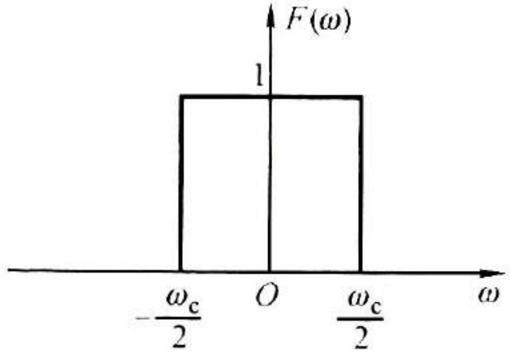
$$F[F(t)]=2\pi f(-\omega)$$





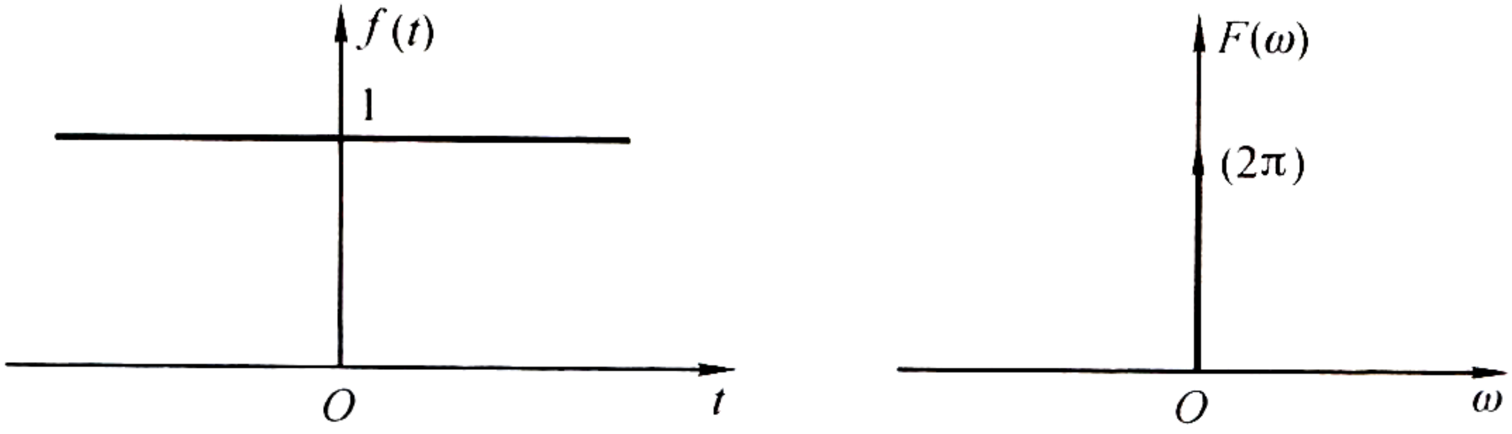
$$E\tau Sa(\frac{\omega\tau}{2})$$



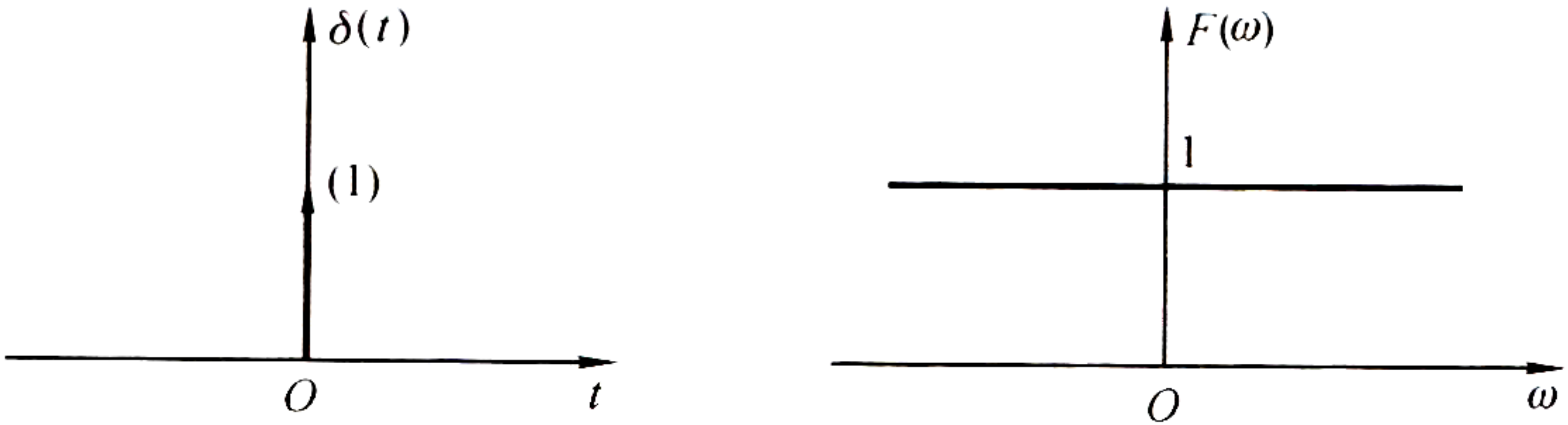


$$E\omega_c Sa(\frac{t\omega_c}{2})$$

$$2\pi E[U(\omega+\frac{\omega_c}{2})-U(\omega-\frac{\omega_c}{2})]$$



f(t)
1
O
t
F(ω)
(2π)
O
ω



δ(t)
(1)
O
t
F(ω)
1
O
ω

3、奇偶虚实性

– f(t)是实函数

$$F(\omega)=F^{*}(-\omega)$$

$$若F(\omega)=R(\omega)+jX(\omega)$$

$$F^{*}(\omega)=R(\omega)-jX(\omega)$$

$$F^{*}(-\omega)=R(-\omega)-jX(-\omega)$$

$\therefore R(\omega)=R(-\omega)$实部偶对称

$\therefore X(\omega)=-X(-\omega)$虚部奇对称

$$F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$$

$$=[\int_{-\infty}^{\infty}f^{*}(t)[e^{-j(-\omega)t}]^{*}dt]*$$

$$=[\int_{-\infty}^{\infty}f(t)e^{-j(-\omega)t}dt]^{*}$$

$$=F^{*}(-\omega)$$

$$F(j\omega)=\int_{\infty}^{\infty}f(t)e^{-j\omega t}dt$$

$$=\int_{\infty}^{\infty}f(t)\cos\omega t dt-j\int_{\infty}^{\infty}f(t)\sin\omega t dt$$

f(t)实偶 ⟶ F(ω)实偶

f(t)实奇 ⟶ F(ω)虚奇

– f(t)是虚函数

$$F(\omega) = -F^*(-\omega)$$

$$若F(\omega) = R(\omega) + jX(\omega)$$

$$F^*(\omega) = R(\omega) - jX(\omega)$$

$$-F^*(-\omega) = -R(-\omega) + jX(-\omega)$$

$$\therefore R(\omega) = -R(-\omega)实部奇对称$$

$$\therefore X(\omega) = X(-\omega)虚部偶对称$$

f(t)虚偶 ⟶ F(ω)实奇

f(t)虚奇 ⟶ F(ω)虚偶

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} - f^*(t)[e^{-j(-\omega)t}]^* dt$$

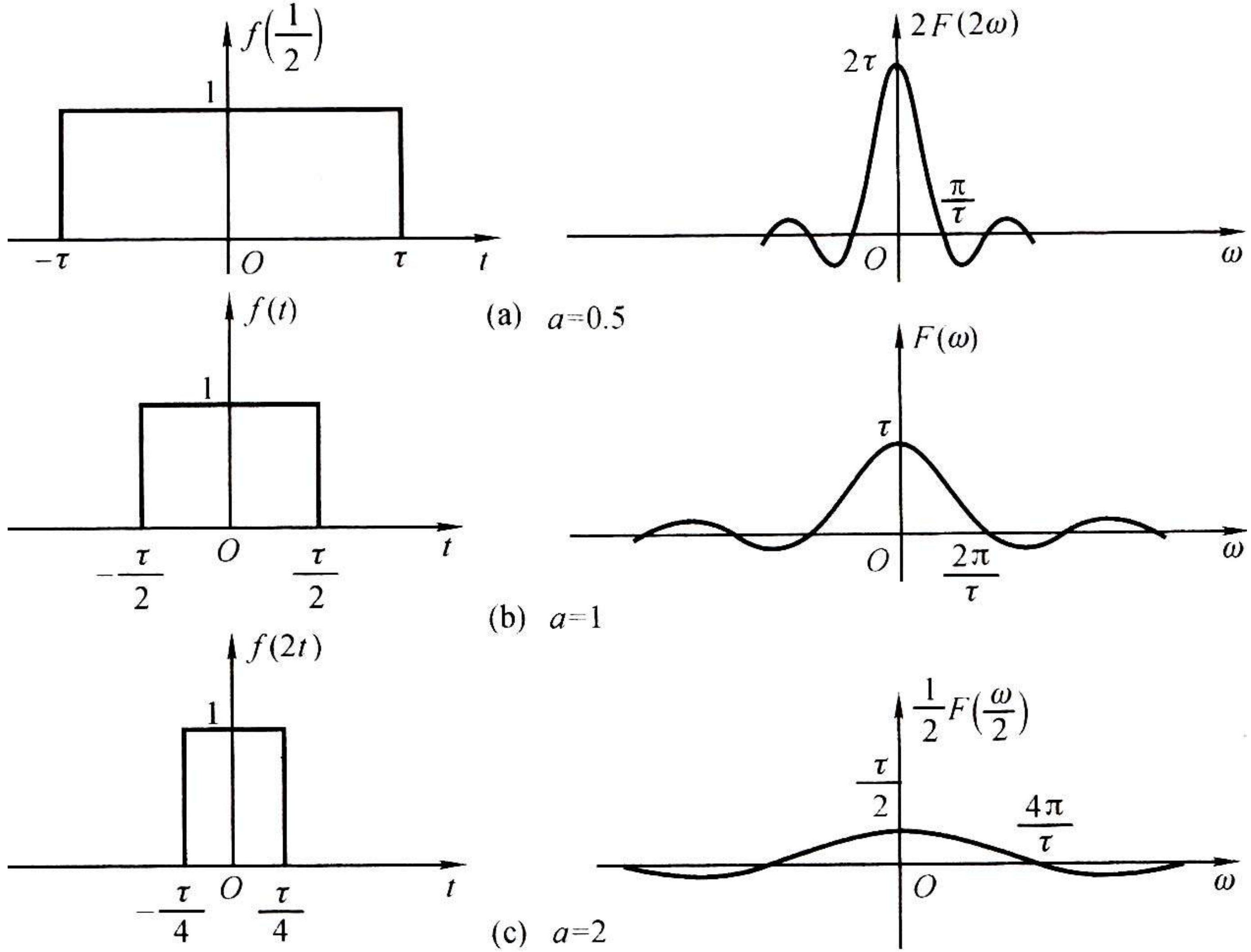
$$= -[\int_{-\infty}^{\infty} f(t) e^{-j(-\omega)t} dt]^*$$

$$= -F^*(-\omega)$$

4、尺度变换特性

$$F[f(t)] = F(\omega)$$

$$F[f(at)] = \frac{1}{|a|} F(\frac{\omega}{a})$$



5、时移特性

$$F[f(t-t_0)]=F(\omega)e^{-j\omega t_0}$$

$$\int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t}dt$$

$$t-t_0=x$$

$$\int_{-\infty}^{\infty} f(x)e^{-j\omega(x+t_0)}dx$$

$$=F(\omega)e^{-j\omega t_0}$$

幅度谱不变，有附加相移

例：求三脉冲的频谱

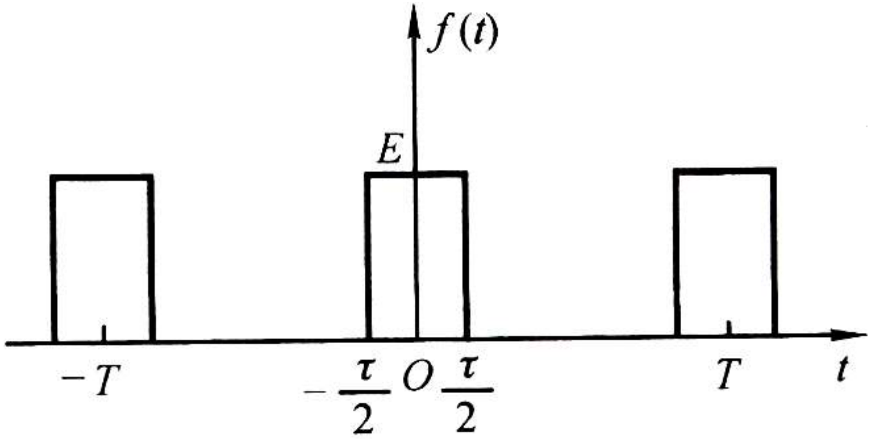
$$F[f_0(t)] = F_0(\omega) = E\tau Sa(\frac{\omega\tau}{2})$$

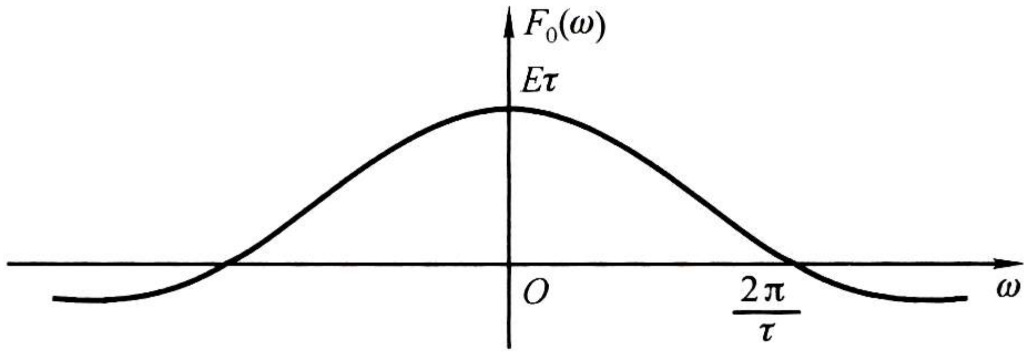
$$F[f_0(t-T) + f_0(t+T)]$$

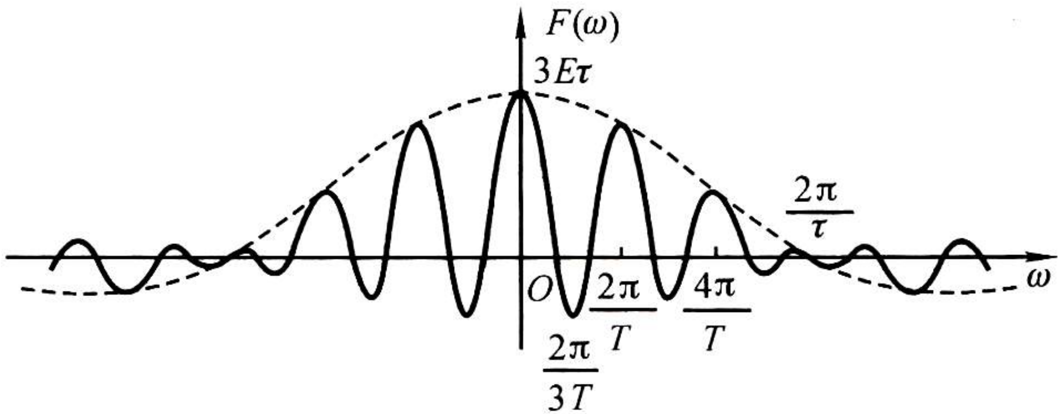
$$= F_0(\omega)(e^{j\omega T} + e^{-j\omega T})$$

$$= F_0(\omega)[2\cos\omega T]$$

$$F(\omega) = E\tau Sa(\frac{\omega\tau}{2})(1 + 2\cos\omega T)$$

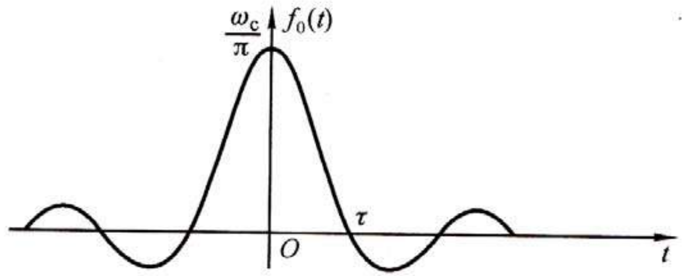






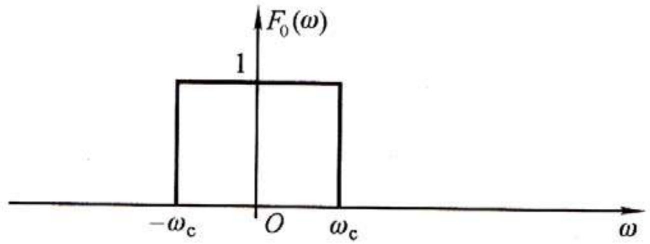
例：$f(t)=\frac{\omega_c}{\pi}\{Sa(\omega_c t)-Sa[\omega_c(t-2\tau)]\}$

求：$F(\omega)$



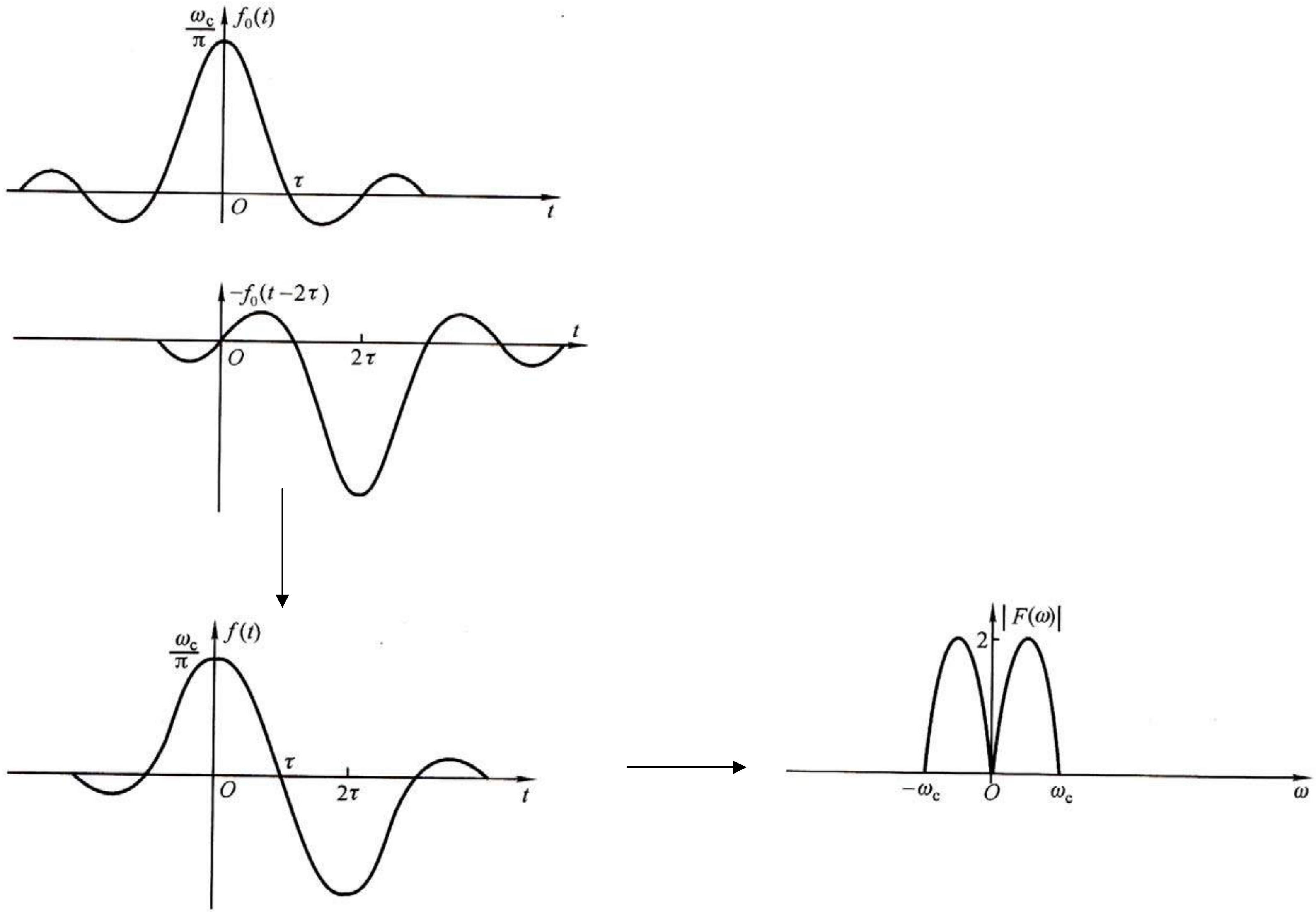
令$f_0(t)=\frac{\omega_c}{\pi}Sa(\omega_c t)$

$$F_0(\omega)=\begin{cases}1 & |\omega|<\omega_c \\ 0 & |\omega|>\omega_c\end{cases}$$



$$F[f_0(t-2\tau)]=\begin{cases}e^{-2j\omega\tau} & |\omega|<\omega_c \\ 0 & |\omega|>\omega_c\end{cases}$$

$$F(\omega)=\begin{cases}1-e^{-2j\omega\tau} & |\omega|<\omega_c \\ 0 & |\omega|>\omega_c\end{cases}$$



$\frac{\omega_c}{\pi}$
$f_0(t)$
τ
O
t
$-f_0(t-2\tau)$
t
O
2τ
$\frac{\omega_c}{\pi}$
$f(t)$
τ
O
2τ
t
$|F(\omega)|$
2
$-\omega_c$
O
ω_c
ω

6、频移特性

$$F[f(t-t_0)]=F(\omega)e^{-j\omega t_0}$$ ←—— 时移

$$F[f(t)e^{j\omega_0 t}]=F(\omega-\omega_0)$$ ←—— 频移

$$\cos\omega_0 t=\frac{1}{2}[e^{j\omega_0 t}+e^{-j\omega_0 t}]$$ ←—— 载频信号

$$\sin\omega_0 t=\frac{1}{2j}[e^{j\omega_0 t}-e^{-j\omega_0 t}]$$

$$F[f(t)\cos\omega_0 t]=\frac{1}{2}[F(\omega-\omega_0)+F(\omega+\omega_0)]$$

$$F[f(t)\sin\omega_0 t]=\frac{j}{2}[F(\omega+\omega_0)-F(\omega-\omega_0)]$$

例：已知$f(t)=e^{-at}\sin\omega_0 t \quad (a>0, t\geq 0)$

求$F(\omega)$

$\because t\geq 0$

$\therefore g(t)=e^{-at}u(t)$

$$G(\omega)=\frac{1}{a+j\omega}$$

$$F(\omega)=\frac{j}{2}\left[\frac{1}{a+j(\omega+\omega_0)}-\frac{1}{a+j(\omega-\omega_0)}\right]$$

$$=\frac{\omega_0}{(a+j\omega)^2+{\omega_0}^2}$$

7、微分特性

$$F[\frac{df(t)}{dt}] = j\omega F(\omega)$$

$$F[\frac{d^n f(t)}{dt^n}] = (j\omega)^n F(\omega)$$

← $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega$$

频域微分特性

$$F^{-1}[\frac{dF(\omega)}{d\omega}] = (-jt)f(t)$$

$$F^{-1}[\frac{d^n F(\omega)}{d\omega^n}] = (-jt)^n f(t)$$

← $$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

例：求图示三角脉冲信号的频谱

$$f(t)=E(1+\frac{2t}{\tau})[u(t+\frac{\tau}{2})-u(t)]+E(1-\frac{2t}{\tau})[u(t)-u(t-\frac{\tau}{2})]$$

$$f'(t)=\frac{2E}{\tau}[u(t+\frac{\tau}{2})-u(t)]-\frac{2E}{\tau}[u(t)-u(t-\frac{\tau}{2})]$$

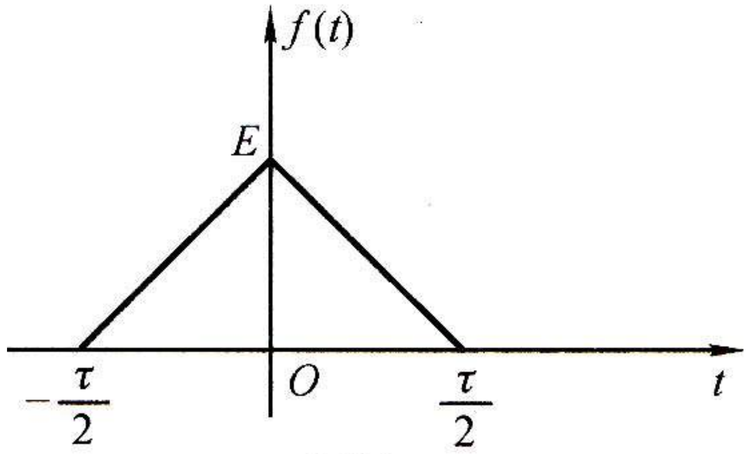
$$f''(t)=\frac{2E}{\tau}[\delta(t+\frac{\tau}{2})-2\delta(t)+\delta(t-\frac{\tau}{2})]$$

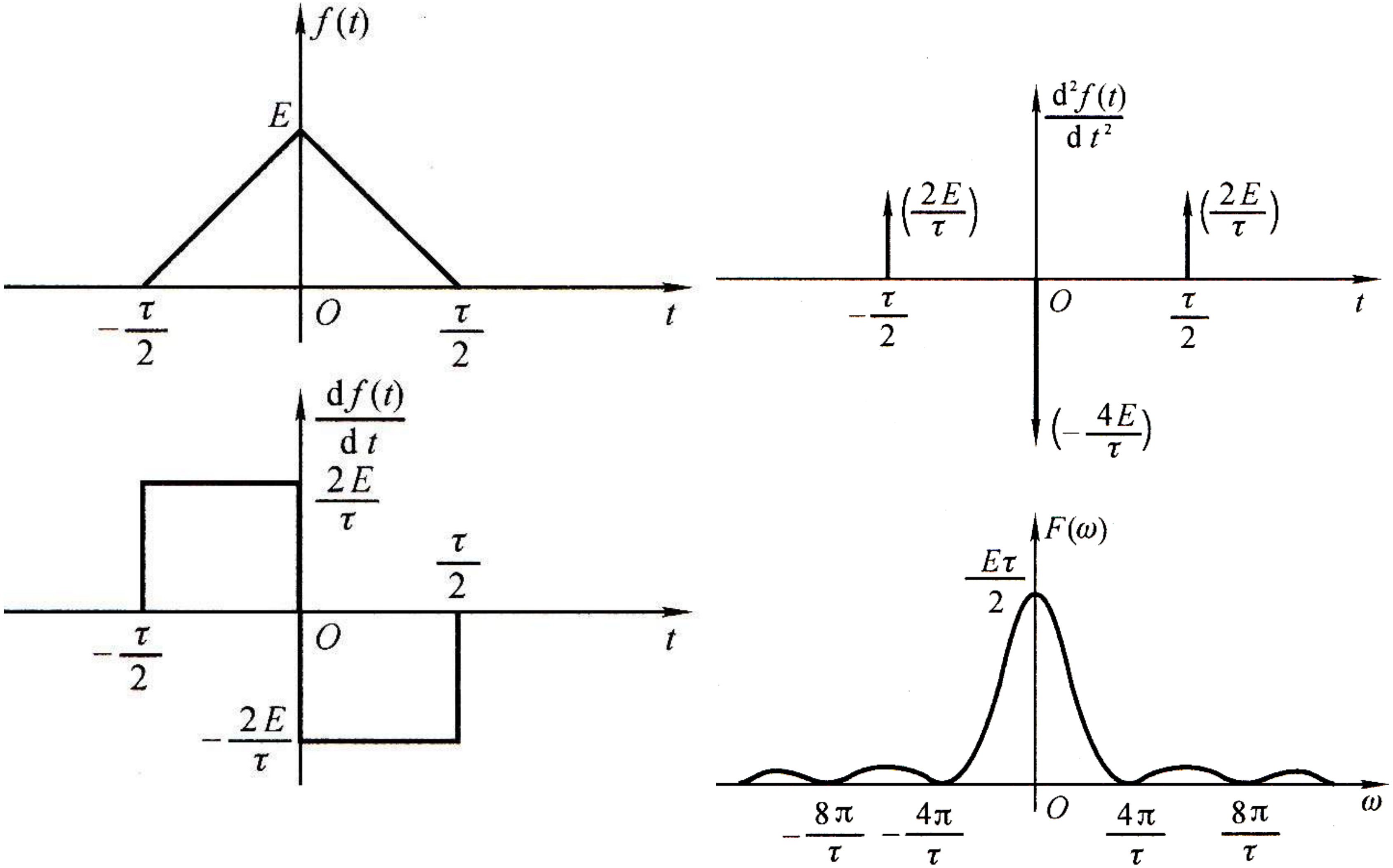
$$F[f''(t)]=\frac{2E}{\tau}[e^{j\frac{\omega\tau}{2}}+e^{-j\frac{\omega\tau}{2}}-2]$$

$$=\frac{4E}{\tau}(\cos\frac{\omega\tau}{2}-1)=\frac{-\omega^2E\tau}{2}Sa^2(\frac{\omega\tau}{4})$$

$$\therefore F(\omega)=\frac{1}{(j\omega)^2}[\frac{-\omega^2E\tau}{2}Sa^2(\frac{\omega\tau}{4})]$$

$$=\frac{E\tau}{2}Sa^2(\frac{\omega\tau}{4})$$





f(t)
E
−τ/2
O
τ/2
t
df(t)/dt
2E/τ
τ/2
−τ/2
O
t
−2E/τ
d²f(t)/dt²
(2E/τ)
(2E/τ)
−τ/2
O
τ/2
t
(−4E/τ)
F(ω)
Eτ/2
−8π/τ
−4π/τ
O
4π/τ
8π/τ
ω

8、积分特性

$$F[\frac{df(t)}{dt}] = j\omega F(\omega) \rightarrow F[\int_{-\infty}^{t} f(\tau)d\tau] = \frac{1}{j\omega}F(\omega)$$

积分特性

$$\rightarrow F[\int_{-\infty}^{t} f(\tau)d\tau] = \frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$$

积分引起的直流或平均值

$$F[u(t)] = \frac{1}{j\omega} + \pi\delta(\omega)$$

$$F[\delta(t)] = 1$$

$$F[\int_{-\infty}^{t} \delta(\tau)d\tau] = \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

$$F[\frac{du(t)}{dt}] = j\omega F[u(t)] = j\omega[\frac{1}{j\omega} + \pi\delta(\omega)]$$

$$= 1 + j\pi\omega\delta(\omega)$$

9、卷积特性

– 时域卷积

$$f_1(t) * f_2(t) \Leftrightarrow F_1(\omega) \cdot F_2(\omega)$$

$$F[f_1(t) * f_2(t)] = \int_{-\infty}^{\infty} [\int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} f_1(\tau) [\int_{-\infty}^{\infty} f_2(t-\tau) e^{-j\omega t} dt] d\tau$$

$$= \int_{-\infty}^{\infty} f_1(\tau) [\int_{-\infty}^{\infty} f_2(x) e^{-j\omega(x+\tau)} dx] d\tau$$

$$= \int_{-\infty}^{\infty} f_1(\tau) e^{-j\omega\tau)} [\int_{-\infty}^{\infty} f_2(x) e^{-j\omega x} dx] d\tau$$

$$= \int_{-\infty}^{\infty} f_1(\tau) e^{-j\omega\tau)} F_2(\omega) d\tau = F_1(\omega) F_2(\omega)$$

– 频域卷积

$$F[f_1(t)f_2(t)] = \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

$$F[f_1(t)f_2(t)] = \int_{-\infty}^{\infty} f_1(t)f_2(t)e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} f_2(t)[\frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(u)e^{-jut}du]e^{-j\omega t}dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)du\int_{-\infty}^{\infty} f_2(t)e^{-j\omega t}e^{jut}dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(u)F_2(\omega - u)du$$

- 例：用卷积定理证明积分特性

$$f(t) * u(t) = \int_{-\infty}^{\infty} f(\tau) u(t-\tau) d\tau$$

$$= \int_{-\infty}^{t} f(\tau) d\tau$$

$$F[\int_{-\infty}^{t} f(\tau) d\tau] = F[f(t) * u(t)]$$

$$= F(\omega)[\pi\delta(\omega) + \frac{1}{j\omega}]$$

$$= \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

例：已知

$$f(t)=\begin{cases} E\cos(\pi t/\tau) & |t|\le \tau/2 \\ 0 & |t|>\tau/2 \end{cases}$$

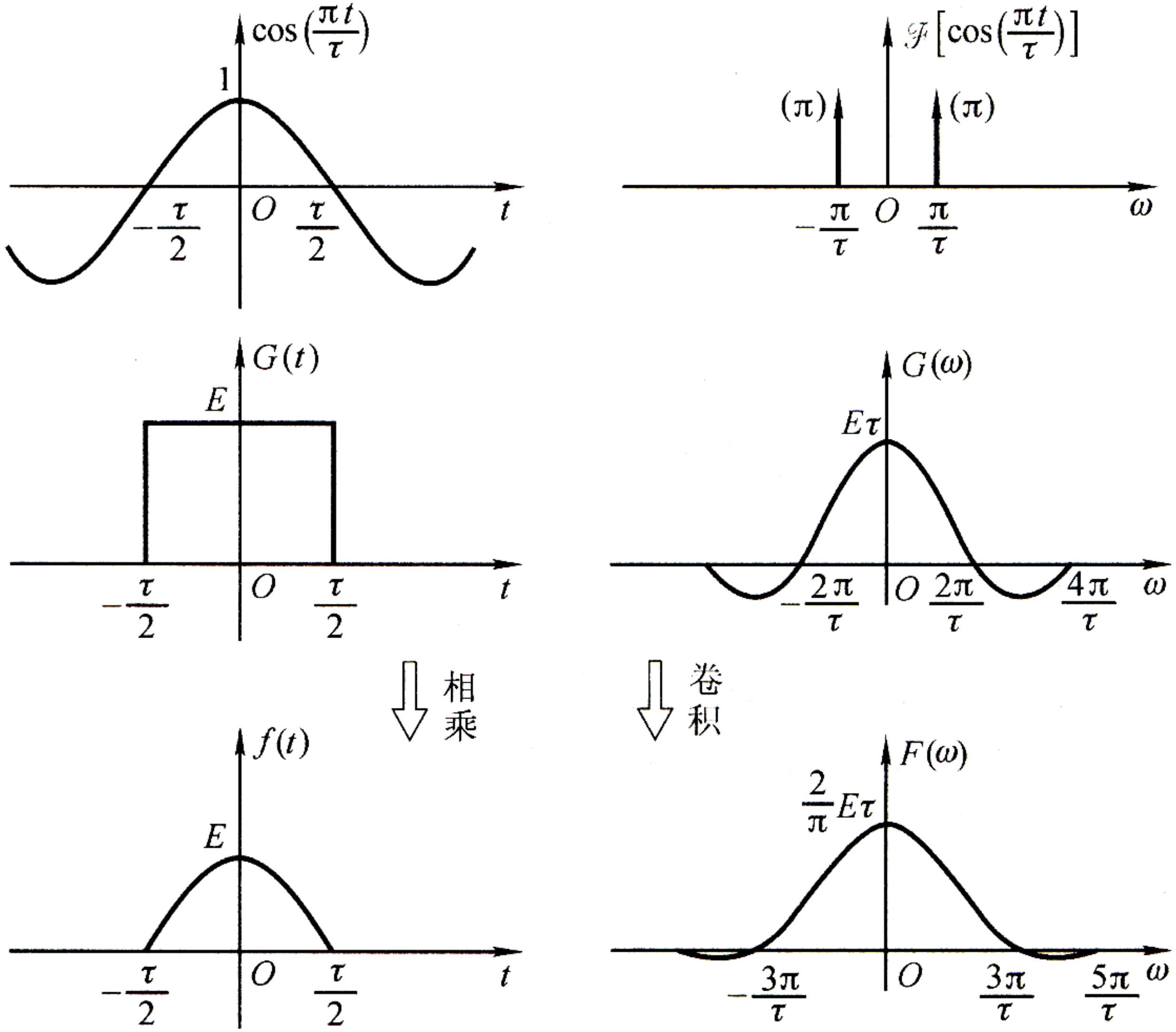
利用卷积定理求余弦脉冲的频谱

$$G(\omega)=E\tau Sa(\frac{\omega\tau}{2})$$

$$F[\cos(\frac{\pi t}{\tau})]=\pi\delta(\omega+\frac{\pi}{\tau})+\pi\delta(\omega-\frac{\pi}{\tau})$$

$$F(\omega)=\frac{1}{2\pi}E\tau Sa(\frac{\omega\tau}{2})*[\pi\delta(\omega+\frac{\pi}{\tau})+\pi\delta(\omega-\frac{\pi}{\tau})]$$

$$=\frac{E\tau}{2}Sa[(\omega+\frac{\pi}{\tau})\frac{\tau}{2}]+\frac{E\tau}{2}Sa[(\omega-\frac{\pi}{\tau})\frac{\tau}{2}]$$



$\cos\left(\frac{\pi t}{\tau}\right)$
1
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t
$\mathscr{F}\left[\cos\left(\frac{\pi t}{\tau}\right)\right]$
(π) (π)
$-\frac{\pi}{\tau}$ O $\frac{\pi}{\tau}$ ω
G(t)
E
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t
G(ω)
Eτ
$-\frac{2\pi}{\tau}$ O $\frac{2\pi}{\tau}$ $\frac{4\pi}{\tau}$ ω
相乘
卷积
f(t)
E
$-\frac{\tau}{2}$ O $\frac{\tau}{2}$ t
F(ω)
$\frac{2}{\pi}E\tau$
$-\frac{3\pi}{\tau}$ O $\frac{3\pi}{\tau}$ $\frac{5\pi}{\tau}$ ω

例：已知

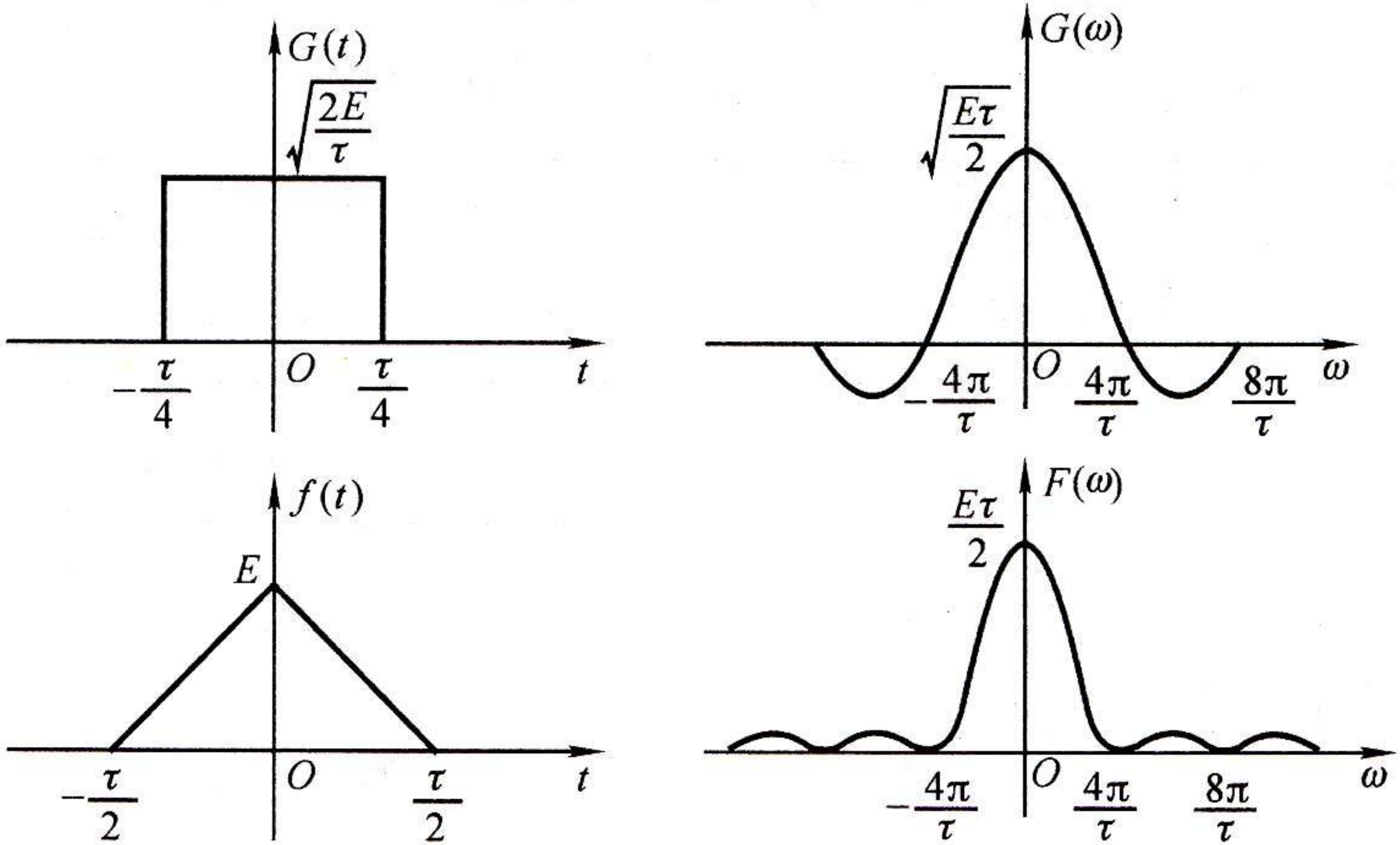
$$f(t)=\begin{cases}E(1-\frac{2|t|}{\tau} & |t|\le\frac{\tau}{2}\\ 0 & |t|>\frac{\tau}{2}\end{cases}$$

利用卷积定理求三角脉冲的频谱

$$G(\omega)=\sqrt{\frac{2E}{\tau}}\frac{\tau}{2}Sa(\frac{\omega\tau}{4})$$

$$F(\omega)=[\sqrt{\frac{2E}{\tau}}\frac{\tau}{2}Sa(\frac{\omega\tau}{4})]^2$$

$$=\frac{E\tau}{2}Sa^2(\frac{\omega\tau}{4})$$



$G(t)$
$\sqrt{\frac{2E}{\tau}}$
$-\frac{\tau}{4}$
O
$\frac{\tau}{4}$
t
$G(\omega)$
$\sqrt{\frac{E\tau}{2}}$
$-\frac{4\pi}{\tau}$
O
$\frac{4\pi}{\tau}$
$\frac{8\pi}{\tau}$
ω
$f(t)$
E
$-\frac{\tau}{2}$
O
$\frac{\tau}{2}$
t
$F(\omega)$
$\frac{E\tau}{2}$
$-\frac{4\pi}{\tau}$
O
$\frac{4\pi}{\tau}$
$\frac{8\pi}{\tau}$
ω

性　质	时域$f(t)$	频域$F(\omega)$	时域频域对应关系
1. 线性	$\sum_{i=1}^{n} a_i f_i(t)$	$\sum_{i=1}^{n} a_i F_i(\omega)$	线性叠加
2. 对称性	$F(t)$	$2\pi f(-\omega)$	对称
3. 尺度变换	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$	压缩与扩展
	$f(-t)$	$F(-\omega)$	反褶

4. 时移	$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$	时移与相移
	$f(at-t_0)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)e^{-j\frac{\omega t_0}{a}}$	
5. 频移	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$	调制与频移
	$f(t)\cos(\omega_0 t)$	$\frac{1}{2}[F(\omega+\omega_0)+F(\omega-\omega_0)]$	
	$f(t)\sin(\omega_0 t)$	$\frac{j}{2}[F(\omega+\omega_0)-F(\omega-\omega_0)]$	
6. 时域微分	$\frac{df(t)}{dt}$	$j\omega F(\omega)$	
	$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$	

性　质	时域$f(t)$	频域 $F(\omega)$	时域频域对应关系
7. 频域微分	$-\mathrm{j}tf(t)$	$\dfrac{\mathrm{d}F(\omega)}{\mathrm{d}\omega}$	
	$(-\mathrm{j}t)^n f(t)$	$\dfrac{\mathrm{d}^n F(\omega)}{\mathrm{d}\omega^n}$	
8. 时域积分	$\int_{-\infty}^{t} f(\tau)\mathrm{d}\tau$	$\dfrac{1}{\mathrm{j}\omega}F(\omega)+\pi F(0)\delta(\omega)$	
9. 时域卷积	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$	
10. 频域卷积	$f_1(t)f_2(t)$	$\dfrac{1}{2\pi}F_1(\omega) * F_2(\omega)$	乘积与卷积

3.9 周期信号的傅里叶变换

- 周期信号－FS
  非周期信号－FT
- 研究的问题：
  - 如何确定周期信号的FT？
  - 它与FS的谱系数的关系如何？

1、正弦信号及余弦信号的FT

$$\cos\omega_1 t = \frac{1}{2}(e^{j\omega_1 t} + e^{-j\omega_1 t})$$

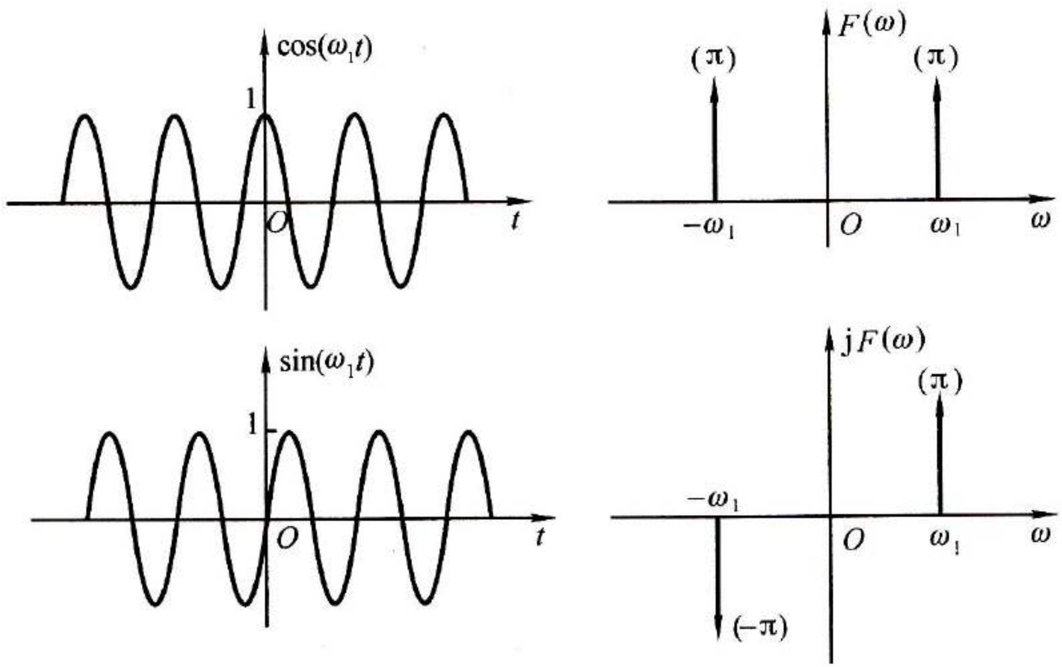
$$e^{j\omega_1 t} \Leftrightarrow 2\pi\delta(\omega-\omega_1), e^{-j\omega_1 t} \Leftrightarrow 2\pi\delta(\omega+\omega_1)$$

$$\therefore \cos\omega_1 t \Leftrightarrow \pi[\delta(\omega-\omega_1)+\delta(\omega+\omega_1)]$$

同理 $\sin\omega_1 t \Leftrightarrow j\pi[\delta(\omega+\omega_1)-\delta(\omega-\omega_1)]$

$$f(t) = \cos\omega_1(t) \Leftrightarrow F_n = 1/2, n = \pm 1$$

$$f(t) = \sin\omega_1(t) \Leftrightarrow F_n = \pm 1/2j, n = \pm 1$$



2、一般的周期信号

- 周期信号的FT是由一系列在谐频处的冲激函数组成，冲激的强度是谱系数的2π倍

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{jn\omega_1 t}$$

$$F(\omega)=2\pi\sum_{n=-\infty}^{\infty}F_n\delta(\omega-n\omega_1)$$

– 周期性脉冲序列的Fn与单脉冲信号FT关系

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t}$$

$$F_n = \frac{1}{T_1} \int_{-T_1/2}^{T_1/2} f(t) e^{-jn\omega_1 t} dt$$

$$F_{SI}(\omega) = \int_{-T_1/2}^{T_1/2} f(t) e^{-j\omega t} dt$$

$$F_n = \frac{1}{T_1} F_{SI}(\omega)\Big|_{\omega = n\omega_1}$$

3、周期单位冲激序列的FS及FT

$$\delta_T(t)=\delta(t)+\delta(t-T_1)+\delta(t-2T_1)+\cdots+\delta(t-nT_1)+\cdots=\sum_{n=-\infty}^{\infty}\delta(t-nT_1)$$

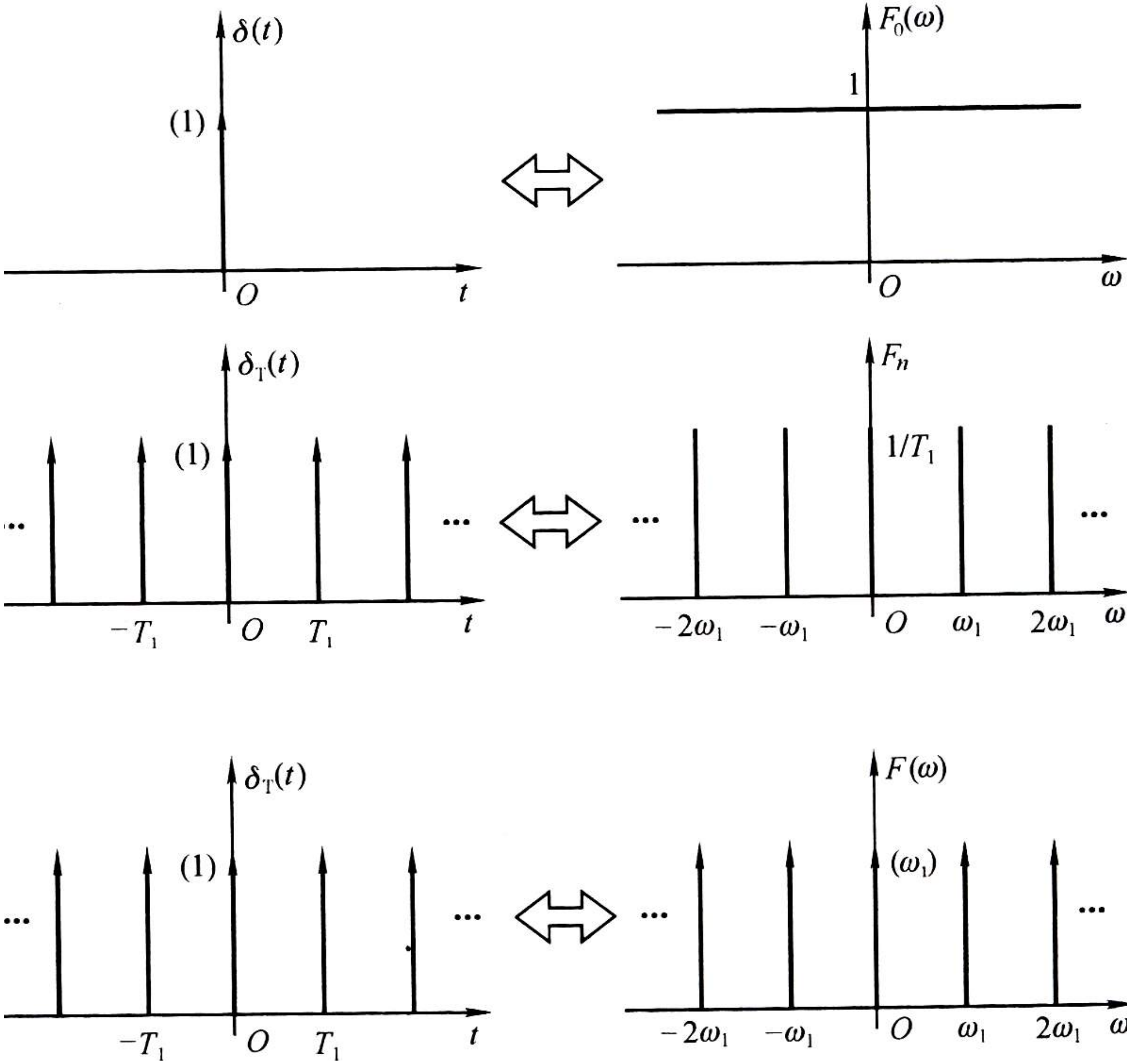
$$\delta_T(t)=\sum_{n=-\infty}^{\infty}F_n e^{jn\omega_1 t}$$

$$F_n=\frac{1}{T_1}\int_{-T_1/2}^{T_1/2}\delta_T(t)e^{-jn\omega_1 t}dt=\frac{1}{T_1}\int_{-T_1/2}^{T_1/2}\delta(t)e^{-jn\omega_1 t}dt=\frac{1}{T_1}$$

$$\therefore \delta_T(t)=\frac{1}{T_1}\sum_{n=-\infty}^{\infty}e^{jn\omega_1 t}$$

$$F(\omega)=2\pi\sum_{n=-\infty}^{\infty}\frac{1}{T_1}\delta(\omega-n\omega_1)=\omega_1\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_1)$$

T_1间隔越大，频域间隔ω_1越小，幅值也越小



δ(t)
(1)
O
t
F_0(ω)
1
O
ω
δ_T(t)
(1)
…
−T_1
O
T_1
t
F_n
1/T_1
…
−2ω_1
−ω_1
O
ω_1
2ω_1
ω
δ_T(t)
(1)
…
−T_1
O
T_1
t
F(ω)
(ω_1)
…
−2ω_1
−ω_1
O
ω_1
2ω_1
ω

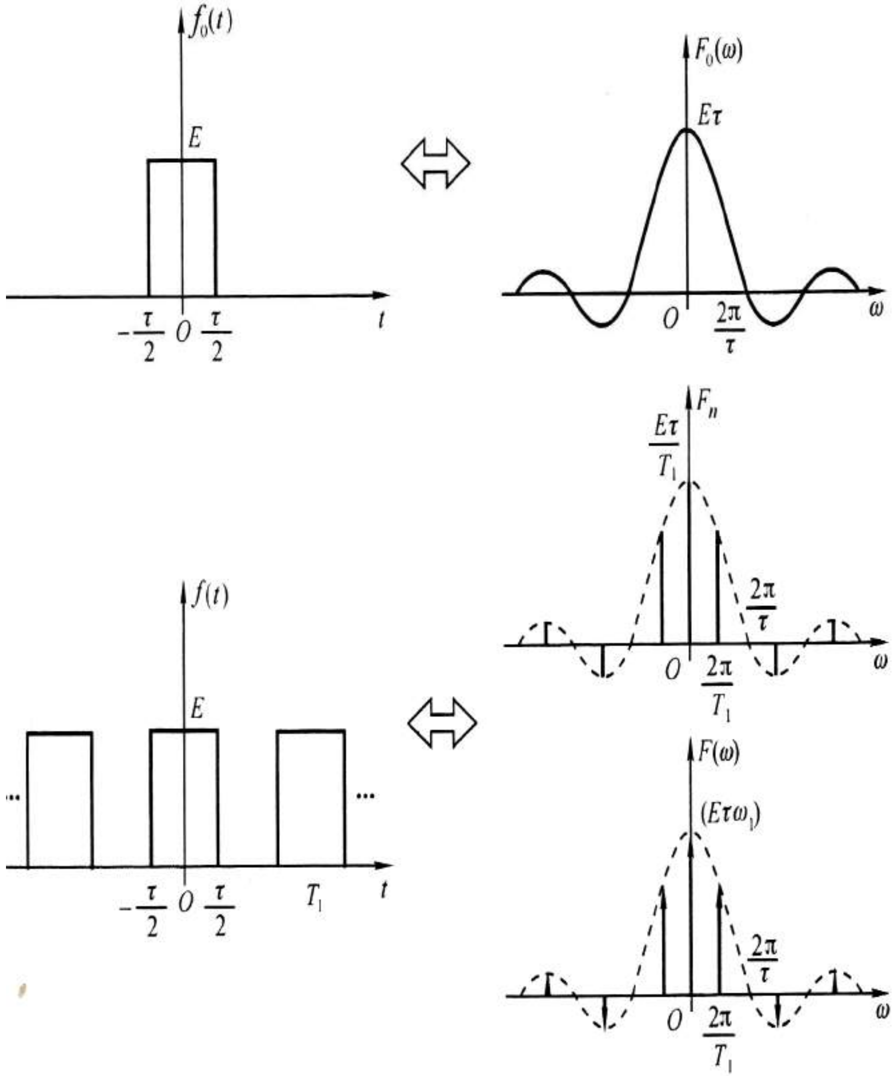
4、周期信号的FT

$$f_p(t)=\sum_{n=-\infty}^{\infty} f(t-nT_1)=\sum_{n=-\infty}^{\infty}\int f(\tau)\delta(t-nT_1-\tau)d\tau$$

$$=\int f(\tau)\sum\delta(t-nT_1-\tau)d\tau=f(t)*\delta_T(t)$$

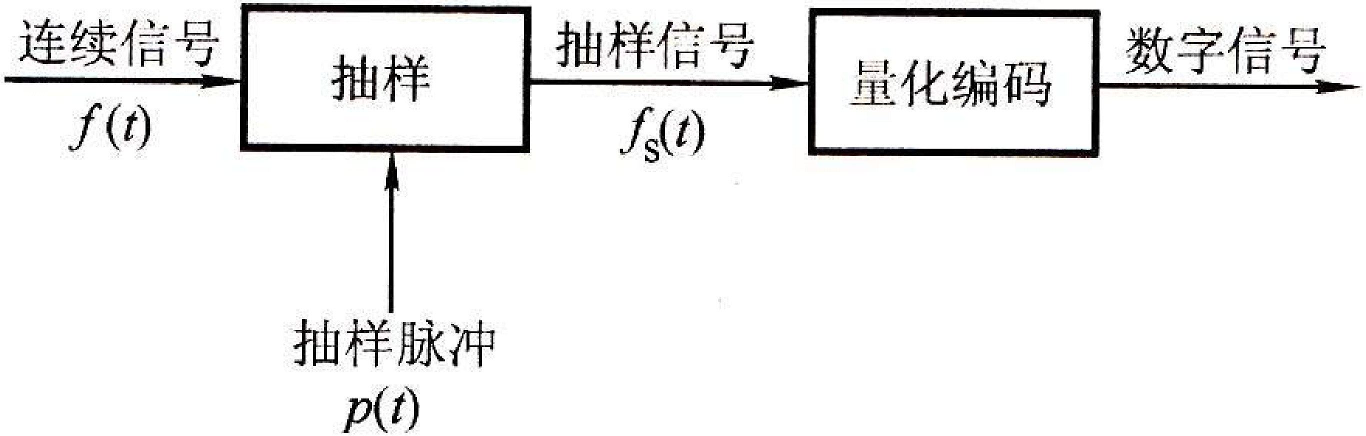
$$F[f_p(t)]=F(\omega)\cdot\omega_1\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_1)=\omega_1\sum_{n=-\infty}^{\infty}F(\omega)\delta(\omega-n\omega_1)$$

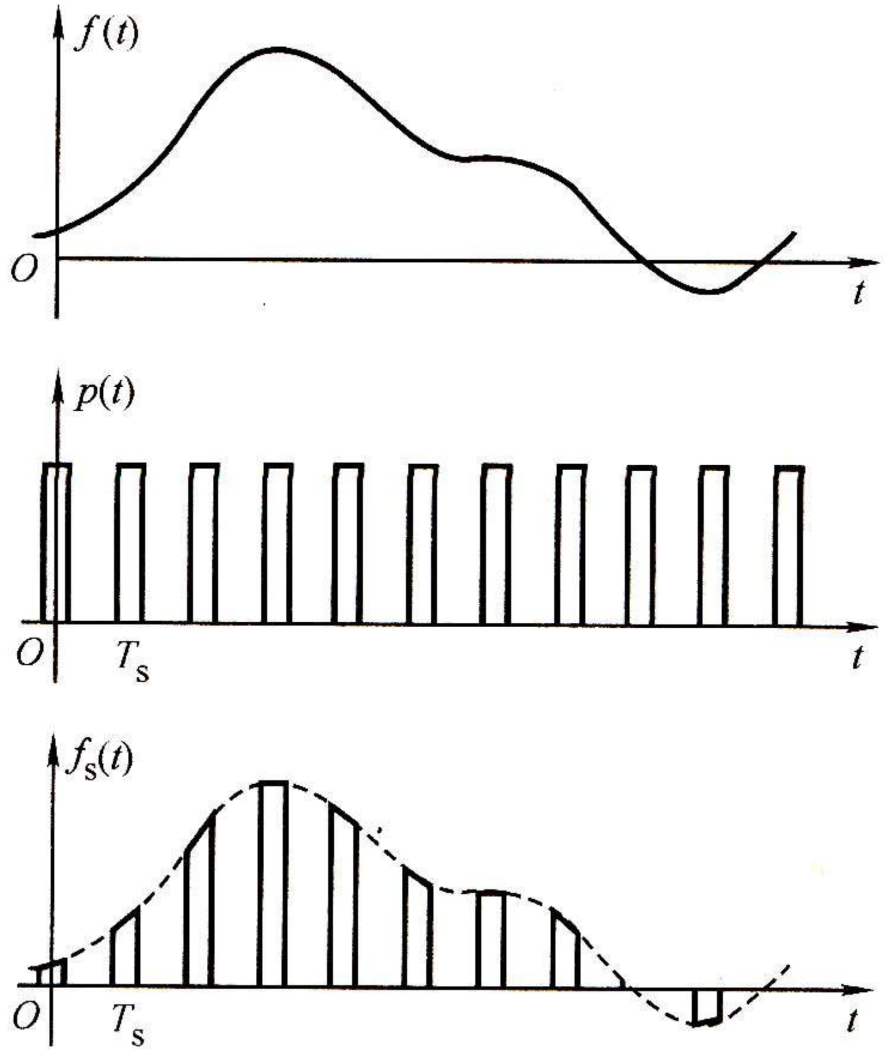
例：周期矩形脉冲信号的FS及FT



3.10　抽样信号的FT

- 信号的抽样
  - 抽样：连续信号f(t)，用该信号的等间隔的离散序列f(0),f(Ts),f(2T_S),…来表示，这一过程称为抽样





f(t)
O
t
p(t)
O
Ts
t
fs(t)
O
Ts
t

1、时域抽样

$$f_s(t) = f(t)\tilde{p}(t)$$

$$F_s(\omega) = \frac{1}{2\pi} F(\omega) * P(\omega)$$

周期信号

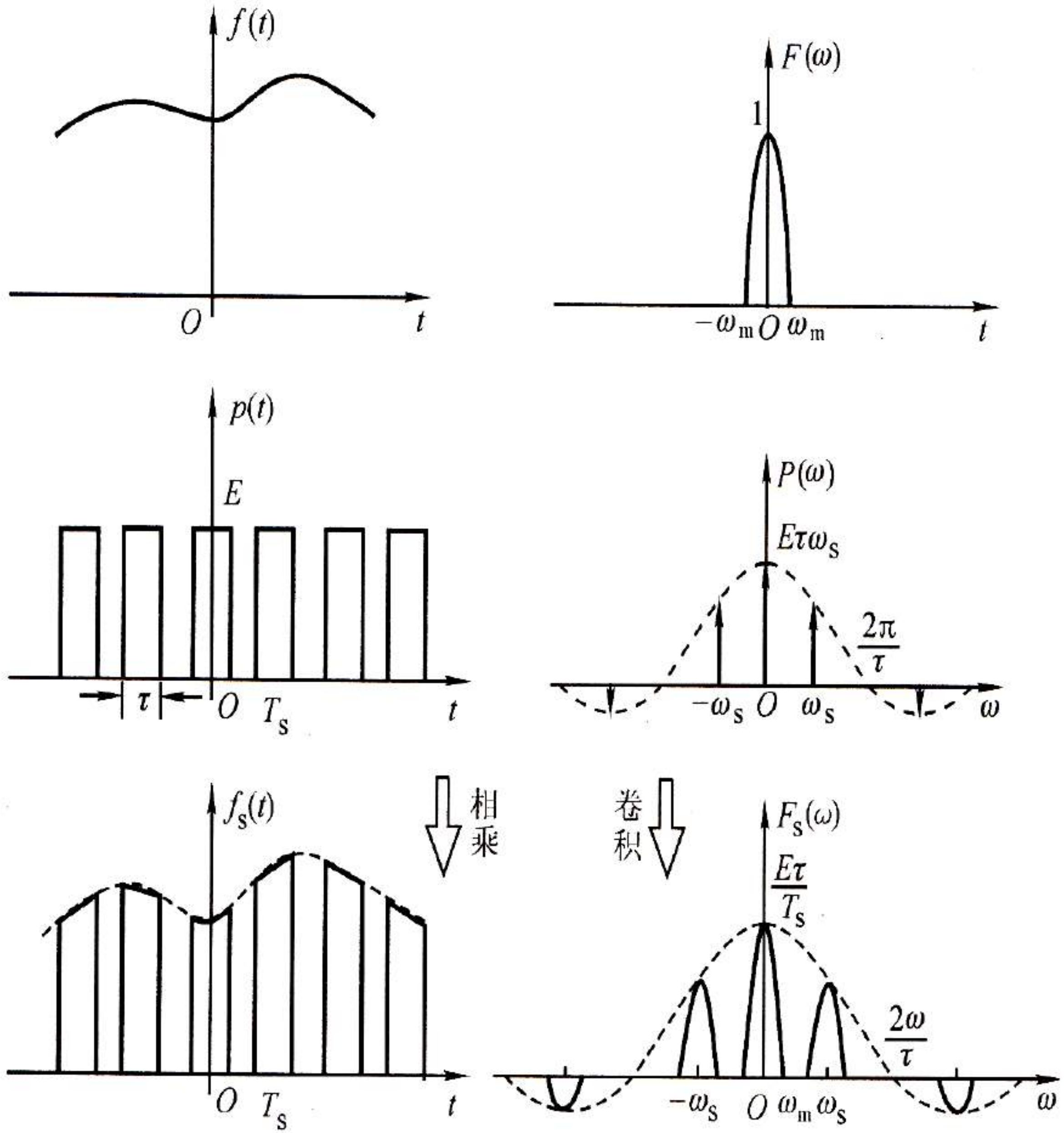
$$F[\tilde{p}(t)] = P(\omega) = 2\pi \sum_{n=-\infty}^{\infty} P_n \delta(\omega - n\omega_s)$$

$$F_s(\omega) = \frac{1}{2\pi} F(\omega) * 2\pi \sum_{n=-\infty}^{\infty} P_n \delta(\omega - n\omega_s)$$

$$= \sum_{n=-\infty}^{\infty} P_n F(\omega) * \delta(\omega - n\omega_s) = \sum_{n=-\infty}^{\infty} P_n F(\omega - n\omega_s)$$

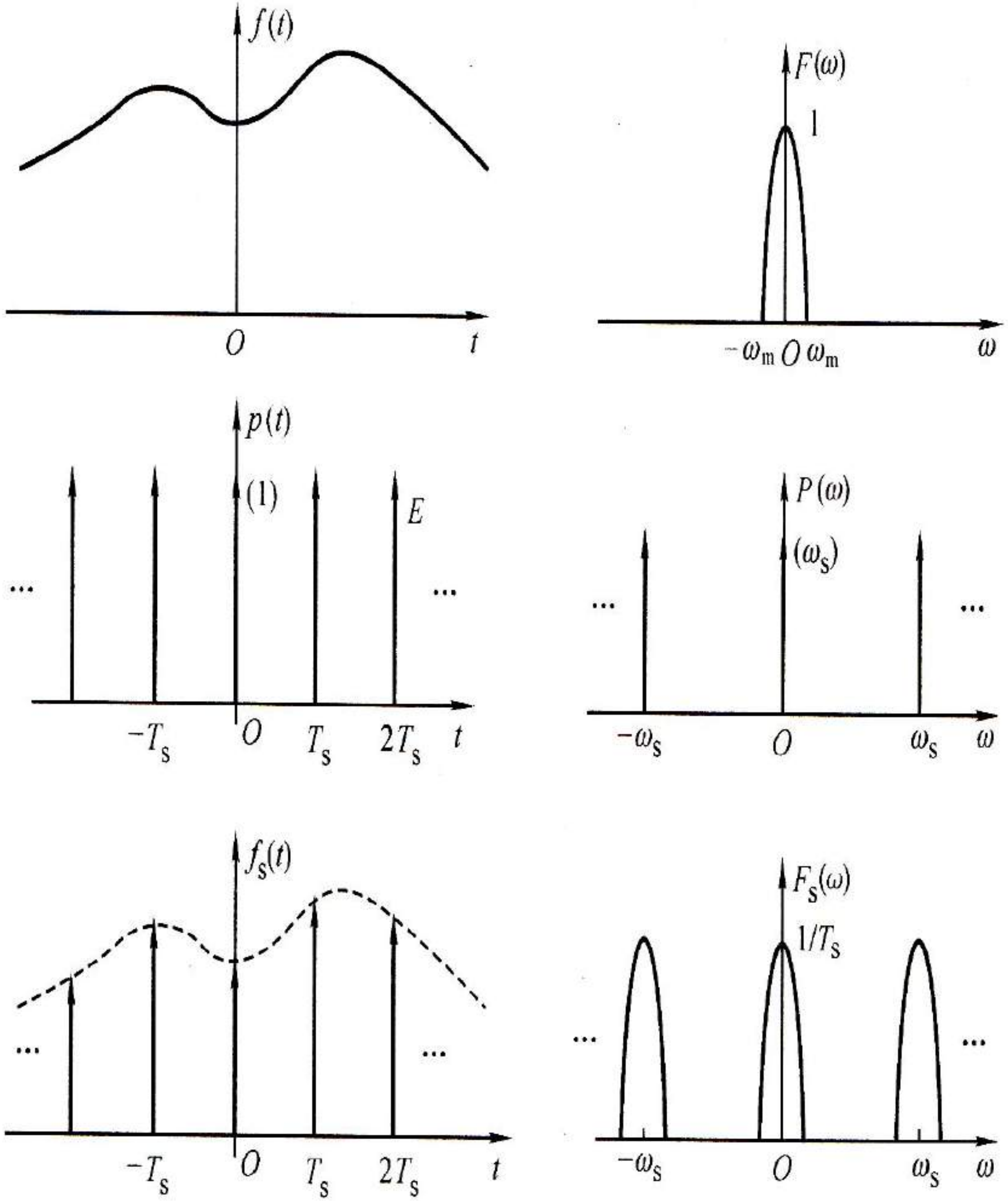
原来的频谱发生周期延拓，其重复周期为ω_s，幅度乘P_n

– 周期矩形脉冲抽样



f(t)
O
t
F(ω)
1
−ωm O ωm
t
p(t)
E
τ
O Ts
t
P(ω)
Eτωs
2π/τ
−ωs O ωs
ω
fs(t)
相乘
卷积
Fs(ω)
Eτ/Ts
2ω/τ
O Ts
t
−ωs O ωm ωs
ω

– 冲激抽样



f(t)
O
t
F(ω)
1
-ωm O ωm
ω
p(t)
(1)
E
...
...
-Ts
O
Ts
2Ts
t
P(ω)
(ωs)
...
...
-ωs
O
ωs
ω
fs(t)
...
...
-Ts
O
Ts
2Ts
t
Fs(ω)
1/Ts
...
...
-ωs
O
ωs
ω

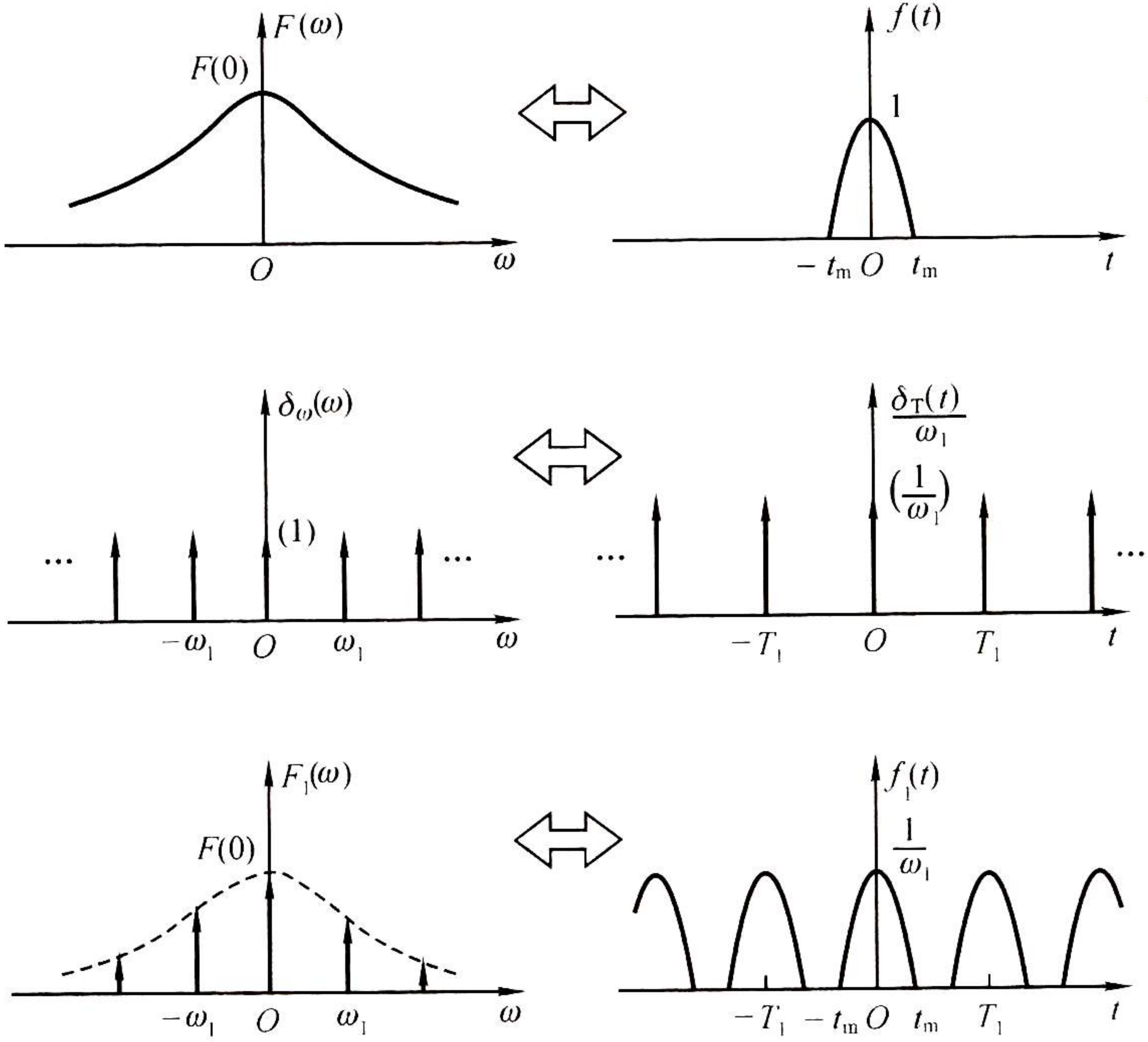
2、频域抽样

- 时域上以T_s抽样，频域上以ω_s为周期延拓

$$F_s(\omega)=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}F(\omega-n\omega_s)$$

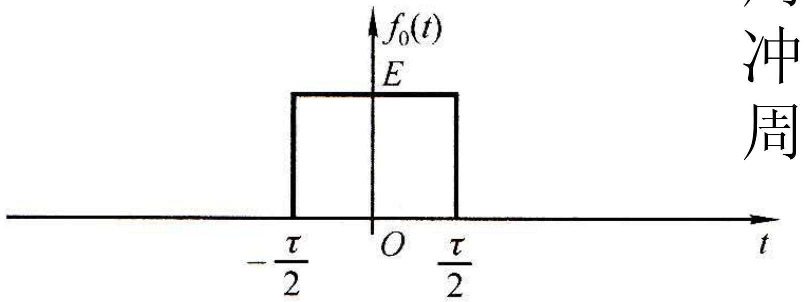
- 频域上以ω_s 抽样，时域上以T_s为周期延拓

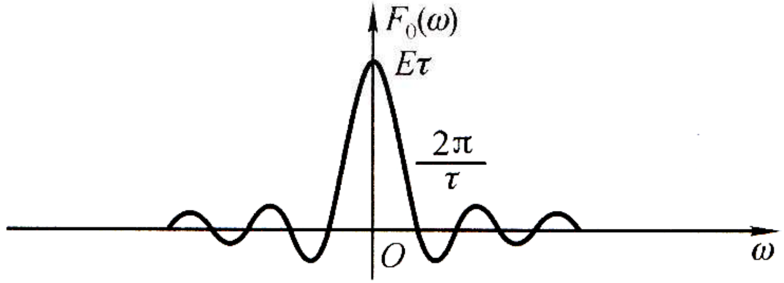
$$f_1(t)=\frac{1}{\omega_s}\sum_{n=-\infty}^{\infty}f(t-nT_s)$$

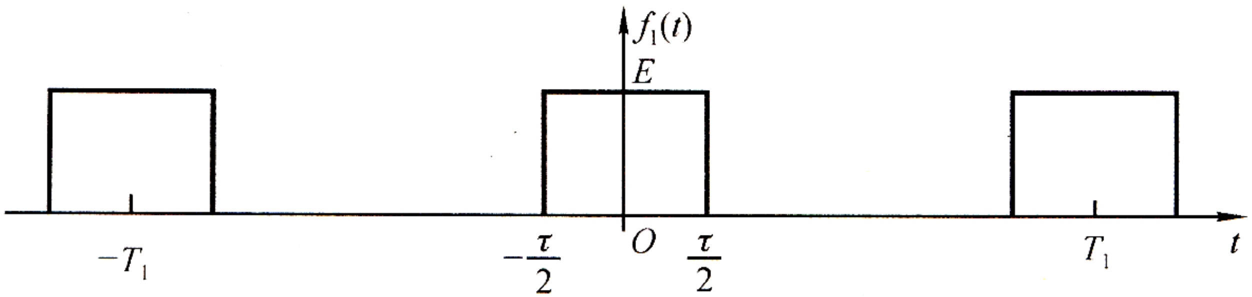

F(ω)
F(0)
O
ω
f(t)
1
− tm O tm
t
δω(ω)
(1)
… …
−ω1 O ω1
ω
δT(t)/ω1
(1/ω1)
… …
−T1 O T1
t
F1(ω)
F(0)
−ω1 O ω1
ω
f1(t)
1/ω1
−T1 − tm O tm T1
t

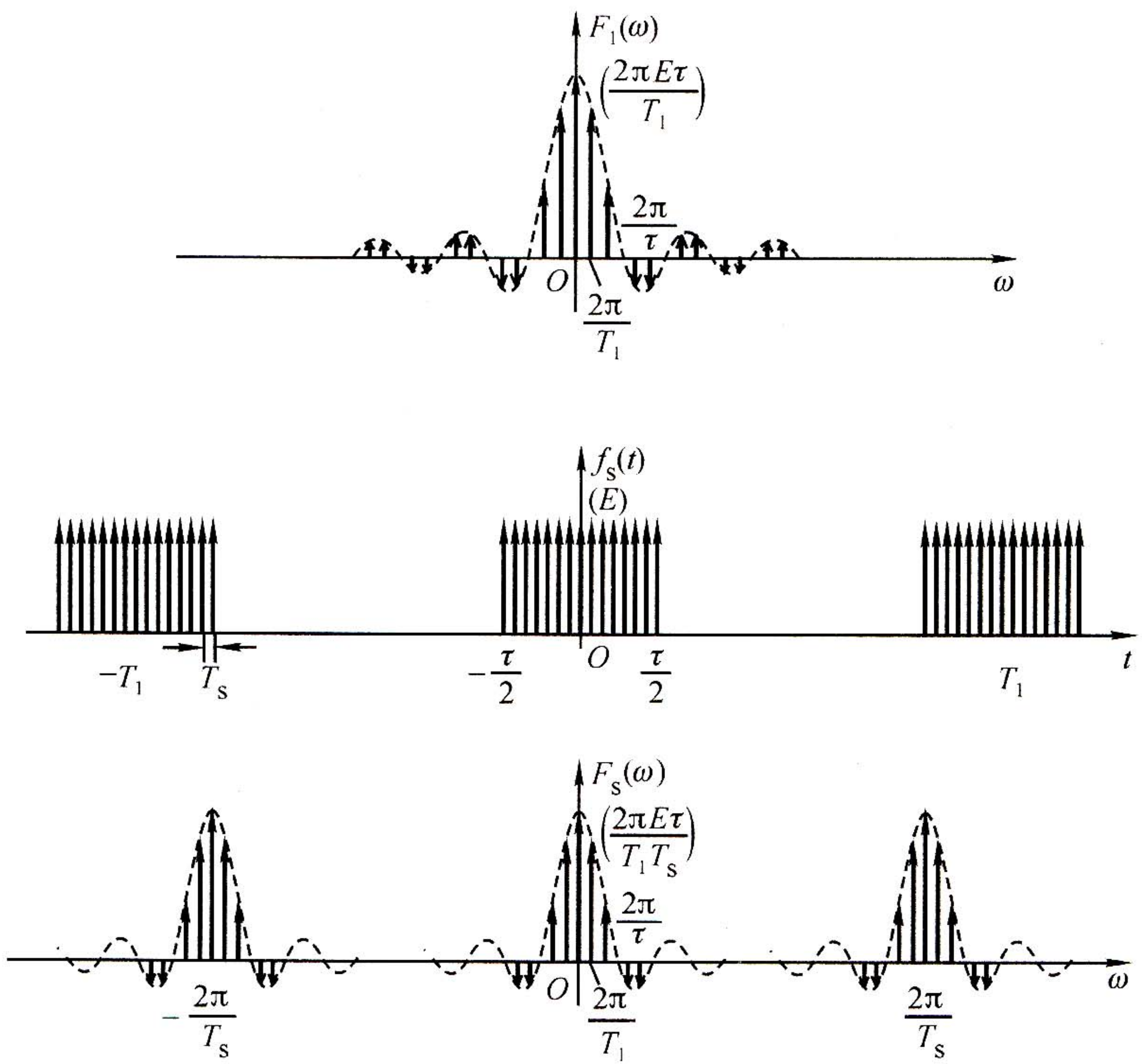
- 周期矩形抽样信号

周期信号－＞离散谱
冲激抽样－＞周期延拓
周期性的离散谱









$F_1(\omega)$
$\left(\frac{2\pi E\tau}{T_1}\right)$
$\frac{2\pi}{\tau}$
O
$\frac{2\pi}{T_1}$
ω
$f_S(t)$
(E)
$-T_1$
T_S
$-\frac{\tau}{2}$
O
$\frac{\tau}{2}$
T_1
t
$F_S(\omega)$
$\left(\frac{2\pi E\tau}{T_1 T_S}\right)$
$\frac{2\pi}{\tau}$
$-\frac{2\pi}{T_S}$
O
$\frac{2\pi}{T_1}$
$\frac{2\pi}{T_S}$
ω

$$F(\omega) = A\tau Sa(\frac{\omega\tau}{2})$$

$$F_p(\omega) = \omega_1 \sum_{n=-\infty}^{\infty} F(\omega)\delta(\omega - n\omega_1)$$

$$F_p(\omega) = \omega_1 A\tau \sum_{n=-\infty}^{\infty} Sa(\frac{n\omega_1\tau}{2})\delta(\omega - n\omega_1)$$

$$F_s(\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} F_p(\omega - m\omega_s)$$

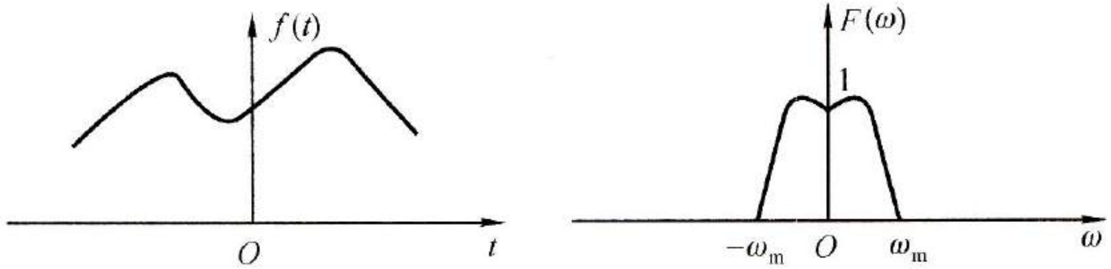
$$= \frac{\omega_1 A\tau}{T_s} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Sa(\frac{n\omega_1\tau}{2})\delta(\omega - m\omega_s - n\omega_1)$$

3.11　抽样定理

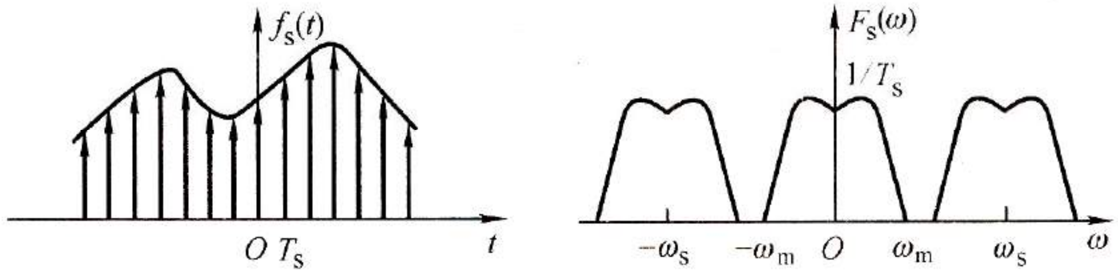
- 抽样定理：数字信号传输，数字信号处理的基本理论依据

模拟信号 —A/D→ 抽样信号、数字信号

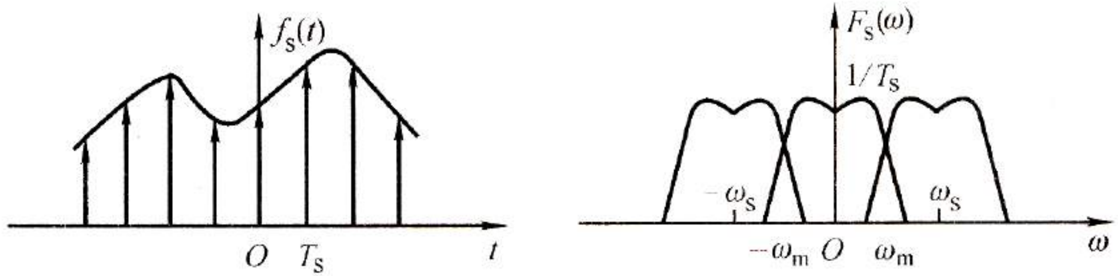
- 时域抽样定理
  - 若$f_s \geq 2f_h$,则f(nTs)可唯一表示f(t)
  - f_s Nyquist frequency



(a) 连续信号的频谱



(b) 高抽样率时的抽样信号及频谱(不混叠)



- 由抽样信号恢复原来信号

理想低通：

$$H(\omega)=\begin{cases}1 & |\omega|<\omega_c \\ 0 & |\omega|>\omega_c\end{cases}$$

$$F(\omega)=F_s(\omega)H(\omega)$$

$$f(t)=f_s(t)*h(t)$$

$$h(t)=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}e^{j\omega t}d\omega=\frac{\omega_c}{\pi}Sa(\omega_c t)$$

$$f(t)=f_s(t)*h(t)$$

$$=\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}f(\tau)\delta(\tau-nT_s).h(t-\tau)d\tau=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}f(\tau)h(t-\tau)\delta(\tau-nT_s)d\tau$$

$$=\sum_{n=-\infty}^{\infty}f(nT_s)h(t-nT_s)=\sum_{n=-\infty}^{\infty}\frac{\omega_c}{\pi}f(nT_s)Sa[\omega_c(t-nT_s)]$$

作　业

P. 160

3-4	3-12
3-20	3-29
3-33	3-37（b)(c)
3-38	3-41