第四章

拉普拉斯变换

4.1 引言

1、FT：研究信号与系统的有力工具

FT存在的条件

- 绝对可积 $\int_{-\infty}^{\infty} |f(t)|dt < \infty$

  周期信号、阶跃信号等不满足该条件
- 解决方法
  - 引入冲激函数 $F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}$

    $$F[\cos\omega_0 t] = \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$$
  - 拉氏变换

2、LT变换

- 在傅氏积分公式中引入一个负指数的时间函数作为收敛函数，以保证整个积分的收敛
- 将FT由频域ω推广至复频域S
- FT与LT的特点
  - FT的频谱结构、频宽以及系统响应具有鲜明的物理意义
  - LT变换简单且更容易计算
  - LT可处时的信号更广
  - 应用LT解微分方程时，可把系统的初始贮能的作用计入，比经典法更简单，引入系统函数
  - 复频域在研究系统特性时比频域更具有普编的意义

3、本章讨论内容

- LT的定义及基本性质
- 用LT分析线性系统
  - 微分方程的LT变换法
  - S域元件模型
- 系统函数
- 利用零极点分析系统响应
- LT与FT的关系

4.2 LT定义与存在条件

1、定义

– FT存在的充分条件 $\int_{-\infty}^{\infty} | f(t) | dt < \infty$

$f(t) = e^{\lambda t}u(t), \lambda > 0$，指数增长函数

FT存在？

$$F[e^{-(\sigma-\lambda)t}u(t)] = \frac{1}{\sigma - \lambda + j\omega} = \frac{1}{s - \lambda}$$

– 乘上一个收敛因子 $f(t)e^{-\sigma t}$

$$F[f(t)e^{-\sigma t}] = \int_{-\infty}^{\infty} f(t)e^{-\sigma t}e^{-j\omega t}dt = \int_{-\infty}^{\infty} f(t)e^{-(\sigma+j\omega)t}dt = F(\sigma + j\omega)$$

$$s = \sigma + j\omega$$

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$f(t)e^{-\sigma t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\sigma + j\omega)e^{j\omega t}d\omega$$

$$\therefore f(t) = \frac{1}{2\pi}\int_{\infty}^{\infty} F(\sigma + j\omega)e^{\sigma t}e^{j\omega t}d\omega$$

$$\because s = \sigma + j\omega, \omega : -\infty \to \infty, s : \sigma - j\infty \to \sigma + j\infty$$

$$\therefore f(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{st}ds$$

– 可得到如下变换

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{st}ds$$

← 双边LT

– 单边LT变换

$$F(s)=\int_0^{\infty} f(t)e^{-st}dt$$

$$f(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

$$F(s)=L[f(t)]$$

$$f(t)=L^{-1}[F(s)]$$

$F(s)$是$f(t)$的象函数，$f(t)$是F(s)的原函数

– 单边LT变换的积分下限

$$F_+(s)=\int_{0+}^{\infty} f(t)e^{-st}dt, F_-(s)=\int_{0-}^{\infty} f(t)e^{-st}dt$$

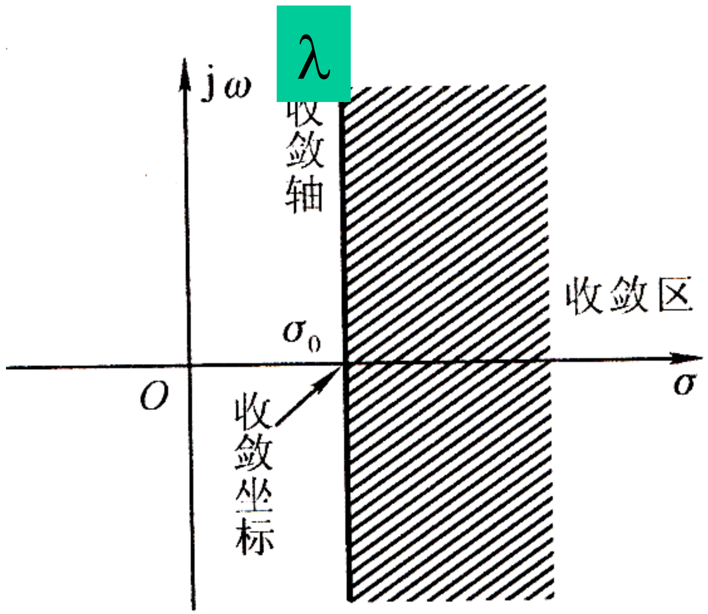
$$F_-(s)=F_+(s)+\int_{0-}^{0+} f(t)e^{-st}dt$$

$$F(s)=\int_{0-}^{\infty} f(t)e^{-st}dt$$

2、LT存在的条件

有的函数不存在FT，但存在LT
但并非所有函数的LT均存在

$$e^{\lambda t}u(t)\ (\lambda>0)\rightarrow\frac{1}{s-\lambda}$$ 存在条件：$\sigma>\lambda$

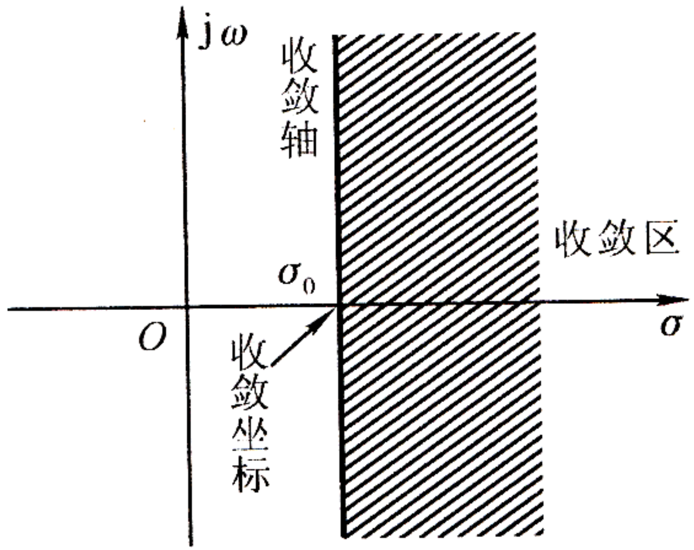


– FT存在条件

$$\int_{-\infty}^{\infty} | f(t) | dt < \infty$$

– LT存在条件

$$\int_{0}^{\infty} | f(t) | e^{-\sigma t} dt < \infty$$



$$若\lim_{t\to\infty} f(t) e^{-\sigma t} = 0 \qquad (\sigma > \sigma_0)$$

LT存在

指数阶函数

$$f(t) = e^{3t}$$

$$\int_{0}^{\infty} | e^{3t} | e^{-\sigma t} dt < \infty 不易判断$$

$$\lim_{t\to\infty} e^{3t} e^{-\sigma t} = \lim_{t\to\infty} e^{(3-\sigma)t}$$

$\sigma > 3$时 $e^{(3-\sigma)t}$是收敛函数

4.3　常用信号的LT

1、t的指数信号

$$e^{\lambda t} \Rightarrow \frac{1}{s-\lambda} \qquad (\sigma > \lambda)$$

$$e^{-\lambda t} \Rightarrow \frac{1}{s+\lambda} \qquad (\sigma > -\lambda)$$

$$e^{j\omega_0 t} \Rightarrow \frac{1}{s-j\omega_0} \qquad (\sigma > 0)$$

$$e^{(\sigma_0+j\omega_0)t} \Rightarrow \frac{1}{s-\sigma_0-j\omega_0} \qquad (\sigma > \sigma_0)$$

$$u(t) \Rightarrow \frac{1}{s} \qquad (\sigma > 0)$$

$$\sin\omega_0 t \Rightarrow \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t}) \Rightarrow \frac{\omega_0}{s^2 + \omega_0{}^2} \qquad (\sigma > 0)$$

$$\cos\omega_0 t \Rightarrow \frac{s}{s^2 + \omega_0{}^2} \qquad (\sigma > 0)$$

$$e^{-\lambda t}\sin\omega_0 t \Rightarrow \frac{\omega_0}{(s+\lambda)^2 + \omega_0{}^2} \qquad (\sigma > -\lambda)$$

$$\delta(\mathrm{t}) \Rightarrow 1$$

$$e^{-\lambda t}\sin\omega_0 t \Rightarrow \frac{1}{2j} L[e^{-(\lambda - j\varpi_0)t} - e^{-(\lambda + j\varpi_0)t}]$$

$$= \frac{1}{2j}\left(\frac{1}{s+\lambda - j\omega_0} - \frac{1}{s+\lambda + j\omega_0}\right) = \frac{\omega_0}{(s+\lambda)^2 + \omega_0^2}$$

收斂域?

2、t的正幂信号

$$t^n \Leftrightarrow \frac{n!}{s^{n+1}} \qquad (\sigma > 0)$$

$$t \Leftrightarrow \frac{1}{s^2} \qquad (\sigma > 0)$$

$$t^n e^{-\lambda t} \Leftrightarrow \frac{n!}{(s+\lambda)^{n+1}} \qquad (\sigma > -\lambda)$$

$$L[t^n] = \int_0^\infty t^n e^{-st} dt = -t^n \frac{1}{s} e^{-st} \Big|_0^\infty + \frac{n}{s} \int_0^\infty t^{n-1} e^{-st} dt$$

$$= \frac{n}{s} \int_0^\infty t^{n-1} e^{-st} dt = \frac{n}{s} L[t^{n-1}]$$

$$L[t^n] = \frac{n}{s} \frac{n-1}{s} \cdots \frac{1}{s} L[t^0] = \frac{n!}{s^{n+1}}$$

4.4　LT性质

1、线性（叠加性）

$$c_1 f_1(t) + c_2 f_2(t) \Leftrightarrow c_1 F_1(s) + c_2 F_2(s)$$

$$L[\sin \omega_0 t] = \frac{1}{2j} L[e^{j\omega_0 t} - e^{-j\omega_0 t}] = \frac{1}{2j}\left(\frac{1}{s - j\omega_0} - \frac{1}{s + j\omega_0}\right)$$

$$= \frac{\omega_0}{s^2 + \omega_0^2}$$

2、时移特性

$$f(t-t_0) \Leftrightarrow e^{-j\omega t_0}F(\omega)$$

$$f(t-t_0)u(t-t_0) \Leftrightarrow e^{-st_0}F(s)$$

$$f(t-t_0)u(t) \Leftrightarrow e^{-st_0}[F(s)+\int_{-t_0}^{0} f(t)e^{-st}dt \qquad \text{右移}$$

$$f(t+t_0)u(t) \Leftrightarrow e^{st_0}[F(s)-\int_{0}^{t_0} f(t)e^{-st}dt \qquad \text{左移}$$

$$L[f(t-t_0)u(t-t_0)] = \int_0^{\infty} f(t-t_0)u(t-t_0)e^{-st}dt$$

$$= \int_{t_0}^{\infty} f(t-t_0)e^{-st}dt$$

$$= e^{-st_0}\int_0^{\infty} f(x)e^{-sx}dx = e^{-st_0}F(s)$$

$$L[f(t-t_0)u(t)]=\int_0^{\infty} f(t-t_0)e^{-st}dt$$

$$=\int_{-t_0}^{\infty} f(t)e^{-sx}e^{-st_0}dx$$

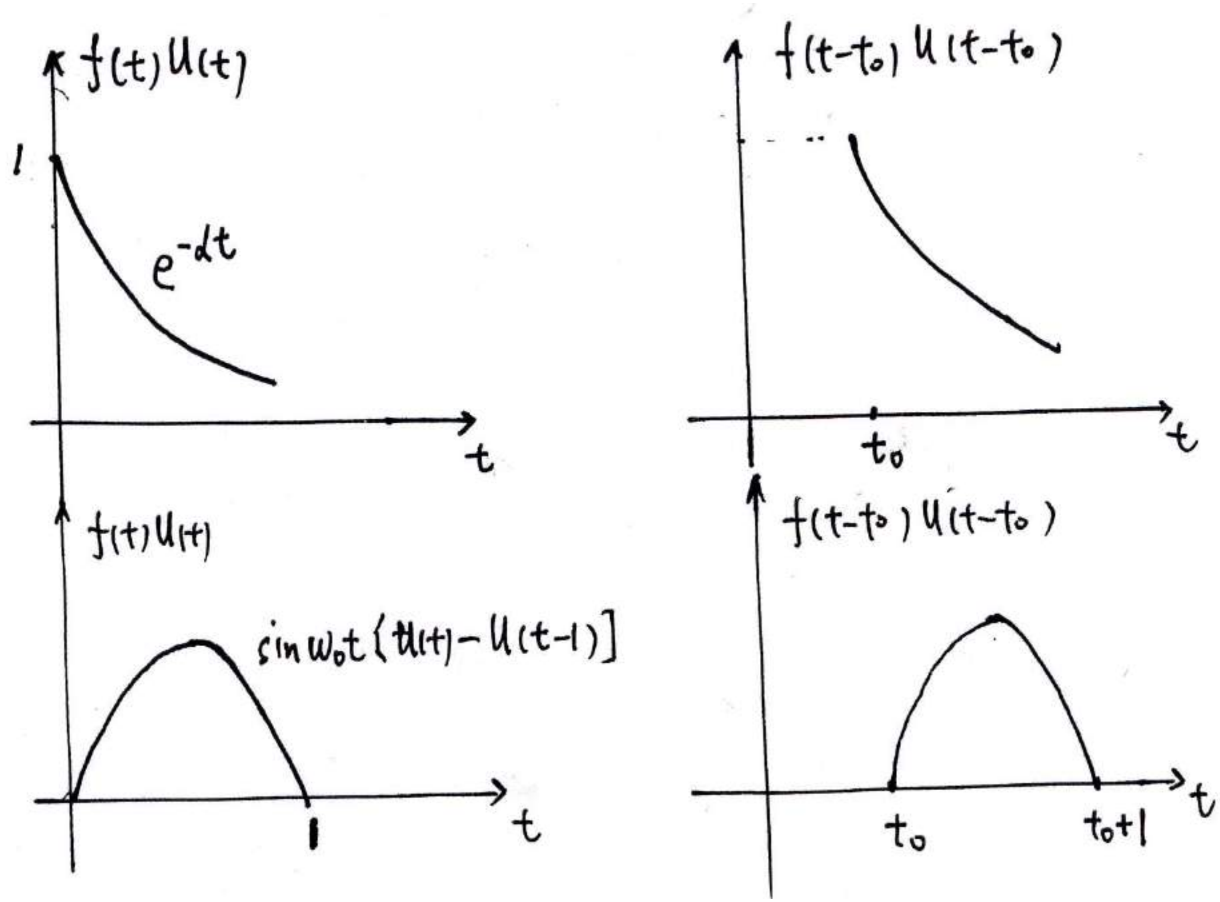
$$=e^{-st_0}[F(s)+\int_{-t_0}^{0} f(t)e^{-st}dt]$$

$$L[f(t+t_0)u(t)]=\int_0^{\infty} f(t+t_0)e^{-st}dt$$

$$=\int_{t_0}^{\infty} f(t)e^{-sx}e^{st_0}dx$$

$$=e^{st_0}[F(s)-\int_0^{t_0} f(t)e^{-st}dt]$$

例：求图中所示信号推迟t_0后的LT



$$L[f(t-t_0)u(t)] = e^{-st_0}F(s) = \frac{1}{s+\alpha}e^{-st_0}$$

$$L[\sin\omega(t-t_0)u(t-t_0) + \sin\omega(t-t_1)u(t-t_1)]$$

$$= \frac{\omega_0}{s^2+\omega_0^2}(e^{-st_0} + e^{-st_1})$$

$$= \frac{\omega_0 e^{-st_0}}{s^2+\omega_0^2}(1+e^{-s})$$

例：求周期矩形脉冲的LT

$$f_T(t) = f_1(t) + f_1(t-T) + \ldots$$

$$F_T(s) = F_1(s) + e^{-sT} F_1(s) + \ldots$$

$$= F_1(s)(1 + e^{-sT} + e^{-2sT} + \ldots) = F_1(s) \frac{1}{1-e^{-sT}}$$

周期矩形脉冲

$$\mathrm{F}_1(\mathrm{s}) = \frac{\mathrm{E}}{\mathrm{s}}(1-e^{-s\tau})$$

$$F_T(s) = \frac{E}{s} \frac{1-e^{-s\tau}}{1-e^{-sT}}$$

3、s域平移（频移特性）

$$f(t)e^{j\omega_0 t} \Leftrightarrow F(\omega-\omega_0)$$

$$f(t)e^{-s_0 t} \Leftrightarrow F(s+s_0)$$

$$f(t)e^{s_0 t} \Leftrightarrow F(s-s_0)$$

$$e^{-j\omega_0 t}u(t) \Leftrightarrow \frac{1}{s+j\omega_0}$$

$$e^{-at}\cos\omega_0 t \Leftrightarrow \frac{s+a}{(s+a)^2+\omega_0{}^2}$$

4、标度变换特性

$$f(at) \Leftrightarrow \frac{1}{a}F(\frac{s}{a}) \qquad \mathrm{a} > 0$$

例：求$L[(5t-3)^n U(5t-3)] \qquad n \geq 0$

$$(1) t^n u(t) \Leftrightarrow \frac{n!}{s^{n+1}}$$

$$5t^n u(5t) \Leftrightarrow \frac{1}{5}\frac{n!}{(\frac{s}{5})^{n+1}} = \frac{n!5^n}{s^{n+1}}$$

$$(5t-3)^n U(5t-3) \Leftrightarrow \frac{n!5^n}{s^{n+1}} e^{-\frac{3}{5}s}$$

(2)时移一标度变换

5、时间微分

$$\frac{df(t)}{dt} \Leftrightarrow j\omega F(\omega)$$

$$\frac{df(t)}{dt} \Leftrightarrow sF(s) - f(0^-)$$

$$L[\frac{df(t)}{dt}] = \int_{0-}^{\infty} \frac{df(t)}{dt} e^{-st} dt$$

分部积分

$$= f(t)e^{-st} |_{0-}^{\infty} - \int_{0-}^{\infty} f(t)(-s)e^{-st} dt$$

$$= s\int_{0-}^{\infty} f(t)e^{-st} dt - f(0-) = sF(s) - f(0-)$$

高阶微分的LT

$$L[\frac{d^2 f(t)}{dt^2}] = s[sF(s) - f(0-)] - f'(0-)$$

$$= s^2 F(s) - sf(0-) - f'(0-)$$

$$L[\frac{d^n f(t)}{dt^n}] = s^n F(s) - s^{n-1} f(0-) - s^{n-2} f'(0-) - \ldots - f^{(n-1)}(0-)$$

例：已知流经电感的电流$i_L(t)$的LT为$I_L(s)$，

求电感电压$v_L(t)$的LT

$$\because v_L(t) = \mathrm{L}\frac{di_L(t)}{dt}$$

$$\therefore V_L(s) = L[sI_L(s) - i_L(0-)] = sLI_L(s) - Li_L(0-)$$

↑ 电感元件的S域模型

例：已知：$f(t)=\begin{cases}-1 & t<0 \\ e^{-at} & t>0\end{cases}$，求$L[\frac{df(t)}{dt}]$

$t=0$为间断点，$f(0-)=-1, f(0+)=1$

$$f(t)=-u(-t)+e^{-at}u(t)$$

$$F(s)=\int_{0-}^{\infty}e^{-at}e^{-st}dt=\frac{1}{s+a}$$

$$\frac{df(t)}{dt}=\delta(t)+e^{-at}\delta(t)-ae^{-at}u(t)=2\delta(t)-ae^{-at}u(t)$$

$$L[\frac{df(t)}{dt}]=2-\frac{a}{s+a}=\frac{2s+a}{s+a}$$

微分特性：

$$L[\frac{df(t)}{dt}]=s\frac{1}{s+a}+1=\frac{2s+a}{s+a}$$

例：试求$L[\frac{d}{dt}(\cos\omega_0 t)]$及$L[\frac{d}{dt}(\cos\omega_0 t)u(t))]$

$$L[\frac{d}{dt}\cos\omega_0 t] = s\frac{s}{s^2+\omega_0^2} - 1 = \frac{-\omega_0^2}{s^2+\omega_0^2}$$

$$L[\frac{d}{dt}\cos\omega_0 t u(t)] = s\frac{s}{s^2+\omega_0^2} - 0 = \frac{s^2}{s^2+\omega_0^2}$$

6、积分特性

$$\int_{-\infty}^{t} f(\tau)d\tau \Leftrightarrow \frac{F(s)}{s} + \frac{f^{(-1)}(0-)}{s}$$

$$\int_{-\infty}^{t} f(\tau)d\tau = \int_{-\infty}^{0-} f(\tau)d\tau + \int_{0-}^{t} f(\tau)d\tau$$

$$\int_{-\infty}^{0-} f(\tau)d\tau = f^{(-1)}(0-), L[\int_{-\infty}^{0-} f(\tau)d\tau] \Leftrightarrow \frac{f^{(-1)}(0-)}{s}$$

$$L[\int_{0-}^{t} f(\tau)d\tau] = \int_{0-}^{\infty}\int_{0-}^{t} f(\tau)d\tau e^{-st}dt = \int_{0-}^{\infty}\int_{0-}^{t} f(\tau)d\tau \frac{-1}{s} d(e^{-st})$$

$$= \frac{1}{s} e^{-st}\int_{0-}^{t} f(\tau)d\tau \Big|_{0-}^{\infty} + \frac{1}{s}\int_{0-}^{\infty} e^{-st} f(t)dt = \frac{F(s)}{s}$$

$$\therefore L[\int_{-\infty}^{t} f(\tau)d\tau] = \frac{F(s)}{s} + \frac{f^{(-1)}(0-)}{s}$$

例：已知流经电容的电流$i_c(t)$的LT为$\mathrm{I}_c(s)$

求电容电压$v_c(t)$的LT

$$\because v_c(t) = \frac{1}{c}\int_{-\infty}^{t} i_c(\tau)d\tau$$

$$\therefore V_c(s) = \frac{1}{c}[\frac{I_c(s)}{s} + \frac{i_c^{\ -1}(0-)}{s}] = \frac{I_c(s)}{sc} + \frac{v_c(0-)}{s}$$

7、初值定理

如果：$f(t)$在$t=0$处有冲激$k\delta(t), t=0$处

有间断点，$A=f(0+)-f(0-)$

则：$\lim\limits_{s\to\infty}[sF(s)-ks]=f(0+)=\lim\limits_{t\to 0+}f(t)$

如果：无冲激$k=0$

则：$f(0+)=\lim\limits_{s\to\infty}sF(s)$

$$f(t)=k\delta(t)+Au(t)+f_0(t)$$

$$f'(t)=k\delta'(t)+A\delta(t)+f'_0(t)$$

$$L[f'(t)]=ks+A+\int_{0-}^{\infty}f'_0(t)e^{-st}dt$$

$$L[f'(t)]=sF(s)-f(0-)$$

$$\lim_{s\to\infty}[sF(s)-ks]=f(0+)+\lim_{s\to\infty}\int_{0-}^{\infty}f'_0(t)e^{-st}dt$$

例：已知$L[u(t)]=\dfrac{1}{s}$，求初值

$$f(0+)=\lim_{s\to\infty} sF(s)=1$$

例：已知$F(s)=\dfrac{2s}{s+1}$，求初值

$$F(s)=2-\frac{2}{s+1},\ f(t)=2\delta(t)-2e^{-t}$$

$$f(0+)=\lim_{s\to\infty}[sF(s)-2s]=\lim_{s\to\infty}[\frac{-2s}{s+1}]=-2$$

8、终值定理

如果有终值 $f(\infty)$存在，则：

$$f(\infty)=\lim_{s\to 0} sF(s)$$

$$L[f'(t)]=sF(s)-f(0-)$$

$$\lim_{s\to 0}\int_{0-}^{\infty} f'(t)e^{-st}dt=\lim_{s\to 0}[sF(s)-f(0-)]$$

$$\lim_{s\to 0}\int_{0-}^{\infty} f'(t)e^{-st}dt= f(\infty)-f(0-)$$

$$\therefore\ f(\infty)=\lim_{s\to 0}[sF(s)]$$

例：若$F(s)=\dfrac{a}{s(s+a)},(a>0)$，求$f(\infty)$

$p1=0$（虚轴上单极点），$p2=-a$（左半平面）

$$f(\infty)=\lim_{s\to 0}\frac{sa}{s(s+a)}=1$$

$$F(s)=\frac{1}{s}-\frac{1}{s+a},\ f(t)=u(t)-e^{-at}u(t)$$

$$f(\infty)=1$$

例：如图RC电路，电路中接入阶跃电压，求$V_R(0+),V_R(\infty)$?

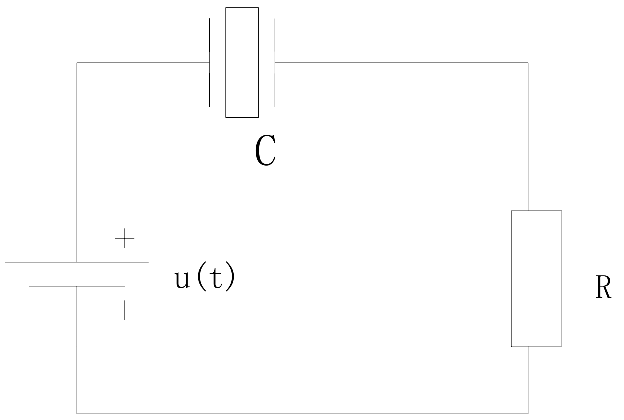
$$V_R(t)+\frac{1}{C}\int_{-\infty}^{t}\frac{V_R(\tau)}{R}d\tau=u(t)$$

$$V_R'(t)+\frac{1}{RC}V_R(t)=\delta(t)$$

$$sV_R(s)+\frac{1}{RC}V_R(s)=1$$

$$V_R(s)=\frac{RC}{RCs+1}$$

$$V_R(0+)=\lim_{s\to\infty}sV_R(s)=1,V_R(\infty)=\lim_{s\to 0}sV_R(s)=0$$



4.5 ILT

1、利用LT求解微分方程

- 建立微分方程
- LT
- ILT得到时域解
  - 基本定义 $f(t)=\dfrac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}F(s)e^{st}ds$
  - 求解方法
    - 查表法
    - 部分分式展开法
    - 留数法

$$F(s) = \frac{s^2 + 3s + 1}{s + 1}$$

$$F(s) = s + 2 - \frac{1}{s + 1}$$

$$\therefore\ f(t) = \delta'(t) + 2\delta(t) - e^{-t}(t \geq 0)$$

$$F(s) = \frac{1 - 2e^{-as}}{s + 1}$$

$$F(s) = \frac{1}{s + 1} - 2\frac{e^{-as}}{s + 1}$$

$$\therefore\ f(t) = e^{-t}u(t) - 2e^{-(t-a)}u(t - a)$$

2、部分分式展开法

– F(s)为有理函数

– F(s)为真分式 ←（长除法）

• 商＋余项

冲激函数及其导数的线性组合

部分分式展开法步骤：

•B(s)因式分解，求极点

•区分极点类型，求系数

•查表求f(t)

$$F(s)=\frac{A(s)}{B(s)}=\frac{a_0s^m+a_1s^{m-1}+\ldots+a_{m-1}s+a_m}{b_0s^n+b_1s^{n-1}+\ldots+b_{n-1}s+b_n}$$

$A(s)=0\rightarrow z_1,z_2\ldots z_m$零点

$B(s)=0\rightarrow p_1,p_2\ldots p_n$极点

– 2.1 极点为实数（无重根）

$$F(s)=\frac{A(s)}{B(s)}=\frac{A(s)}{(s-p_1)(s-p_2)\ldots(s-p_n)}=\frac{k_1}{s-p_1}+\frac{k_2}{s-p_2}+\ldots+\frac{k_n}{s-p_n}$$

$$k_j=(s-p_j)F(s)|_{s=p_j}$$

例：已知$F(s)=\dfrac{s^2+s+2}{s^3+3s^2+2s}$，求$f(t)$

$$F(s)=\frac{s^2+s+2}{s(s+1)(s+2)}=\frac{k_1}{s}+\frac{k_2}{s+1}+\frac{k_3}{s+2}$$

$$k_1=sF(s)|_{s=0}=1$$

$$k_2=(s+1)F(s)|_{s=-1}=-2$$

$$k_3=(s+2)F(s)|_{s=-2}=2$$

$$f(t)=(1-2e^{-t}+2e^{-2t})u(t)$$

– 2.2 极点为共轭复数

- 复数极点共轭成对（B(s)为实系数多项式）

设$p_{1,2} = -\alpha \pm j\beta$

$$F(s) = \frac{A(s)}{(s+\alpha - j\beta)(s+\alpha + j\beta)B_1(s)} = \frac{k_1}{s+\alpha - j\beta} + \frac{k_2}{s+\alpha + j\beta} + \frac{A_1(s)}{B_1(s)}$$

$$k_1 = (s+\alpha - j\beta)F(s)|_{s=-\alpha+j\beta} = \frac{A(p_1)}{2j\beta B_1(p_1)}$$

$$k_2 = (s+\alpha + j\beta)F(s)|_{s=-\alpha-j\beta} = \frac{A(p_1^*)}{-2j\beta B_1(p_1^*)} = k_1^*$$

- ILT的时间函数必是振荡型

$$k_1 = c + jd, k_2 = c - jd$$

$$F(s) = \frac{c+jd}{s+\alpha - j\beta} + \frac{c-jd}{s+\alpha + j\beta}$$

$$f(t) = k_1 e^{(-\alpha + j\beta)t} + k_2 e^{(-\alpha - j\beta)t} = 2e^{-\alpha t}[c\cos\beta t - d\sin\beta t]$$

– 2.3 极点为二阶及高阶

设$s = p_1$为三重极点

$$F(s) = \frac{A(s)}{B(s)} = \frac{c_3}{s-p_1} + \frac{c_2}{(s-p_1)^2} + \frac{c_1}{(s-p_1)^3} + \frac{A_1(s)}{B_1(s)}$$

令$(s-p_1)^3 F(s) = W$

$$W = c_3(s-p_1)^2 + c_2(s-p_1) + c_1 + (s-p_1)^3 \frac{A_1(s)}{B_1(s)}$$

→ u

$$\frac{dW}{ds} = 2c_3(s-p_1) + c_2 + \frac{du}{ds}$$

$$\frac{d^2W}{ds^2} = 2c_3 + \frac{d^2u}{ds^2}$$

$$u, \frac{du}{ds}, \frac{d^2u}{ds^2}\Big|_{s=p_1} = 0$$

$$\therefore c_1 = W\big|_{s=p_1} \qquad c_2 = \frac{dW}{ds}\Big|_{s=p_1} \qquad c_3 = \frac{1}{2}\frac{d^2W}{ds^2}\Big|_{s=p_1}$$

若p为k阶极点，$W=(s-p)^k F(s)$

$$c_i=\frac{1}{(i-1)!}\frac{d^{i-1}W}{ds^{i-1}}|_{s=p}\qquad \mathrm{i}=1,2,\ldots.\mathrm{k}$$

例：$F(s)=\frac{3}{(s+1)^3 s^2}$，求$f(t)$

$$F(s)=\frac{c_3}{s+1}+\frac{c_2}{(s+1)^2}+\frac{c_1}{(s+1)^3}+\frac{c_2'}{s}+\frac{c_1'}{s^2}$$

$$c_1=(s+1)^3F(s)|_{s=-1}=3\qquad c_2=\frac{d}{ds}(s+1)^3F(s)|_{s=-1}=6$$

$$c_3=\frac{d^2}{2ds^2}(s+1)^3F(s)|_{s=-1}=9$$

$$c_1'=s^2F(s)|_{s=0}=3\qquad c_2'=\frac{d}{ds}^2F(s)|_{s=0}=-9$$

$$\therefore F(s)=\frac{9}{s+1}+\frac{6}{(s+1)^2}+\frac{3}{(s+1)^3}+\frac{-9}{s}+\frac{3}{s^2}$$

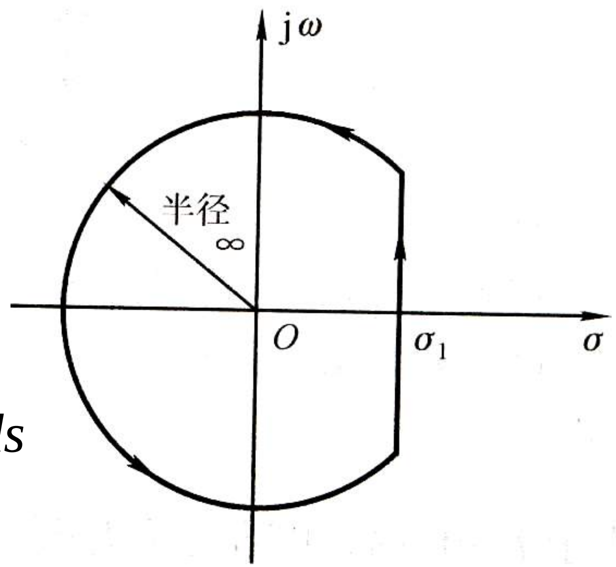
$$t^n e^{-at}\Leftrightarrow\frac{n!}{(s+a)^{n+1}}$$

$$\therefore f(t)=(9e^{-t}+6te^{-t}+\frac{3}{2}t^2e^{-t}-9+3t)u(t)$$

3、留数法

– 留数定理：若函数g(s)在闭合区域中除有限个奇点外处处解析，则有 $\oint_c g(s)ds = 2\pi j\sum[g(s)\text{的留数}]$

$\lim\limits_{s\to\infty} F(s) = 0$时利用约当引理



$$f(t) = \frac{1}{2\pi j}\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)e^{st}ds = \frac{1}{2\pi j}\oint_c F(s)e^{st}ds$$

$$= \sum \mathrm{Res}[F(s)e^{st}]$$

p_i一阶极点：$r_i = (s - p_i)F(s)e^{st}|_{s=pi}$

p_iK阶极点：$r_i = \frac{1}{(k-1)!}\frac{d^{k-1}}{ds^{k-1}}[(s-p_i)^k F(s)e^{st}]|_{s=pi}$

例：$F(s)=\dfrac{2s^2+3s+3}{(s+1)(s+2)(s+3)}$，用留数法求 $f(t)$

$$r_1=[(s+1)F(s)e^{st}]|_{s=-1}=e^{-t}$$

$$r_2=[(s+2)F(s)e^{st}]|_{s=-2}=-5e^{-2t}$$

$$r_3=[(s+3)F(s)e^{st}]|_{s=-3}=6e^{-3t}$$

$$\therefore\ f(t)=e^{-t}-5e^{-2t}+6e^{-3t}(t\geq 0)$$

例：已知$F(s)=\dfrac{s+2}{s(s+1)^2}$，用留数定理求$f(t)$

$$r_1=\frac{s+2}{(s+1)^2}e^{st}|_{s=0}=2$$

$$r_2=\frac{d}{ds}\frac{s+2}{s}e^{st}|_{s=-1}=-(2+t)e^{-t}$$

$$\therefore\ f(t)=2-(2+t)e^{-t}(t\geq 0)$$

4.6 微分方程的LT解

1、二阶和一阶常系数线性微分方程

$$a_0\frac{d^2r}{dt^2}+a_1\frac{dr}{dt}+a_2r(t)=b_0\frac{de(t)}{dt}+b_1e(t) \quad \longleftarrow \quad a_0=1$$

$$r(t)\to R(s), r'(t)\to sR(s)-r(0-), r''(t)\to s^2R(s)-sr(0-)-r'(0-)$$

$$e(t)\to E(s), e'(t)\to sE(s)$$

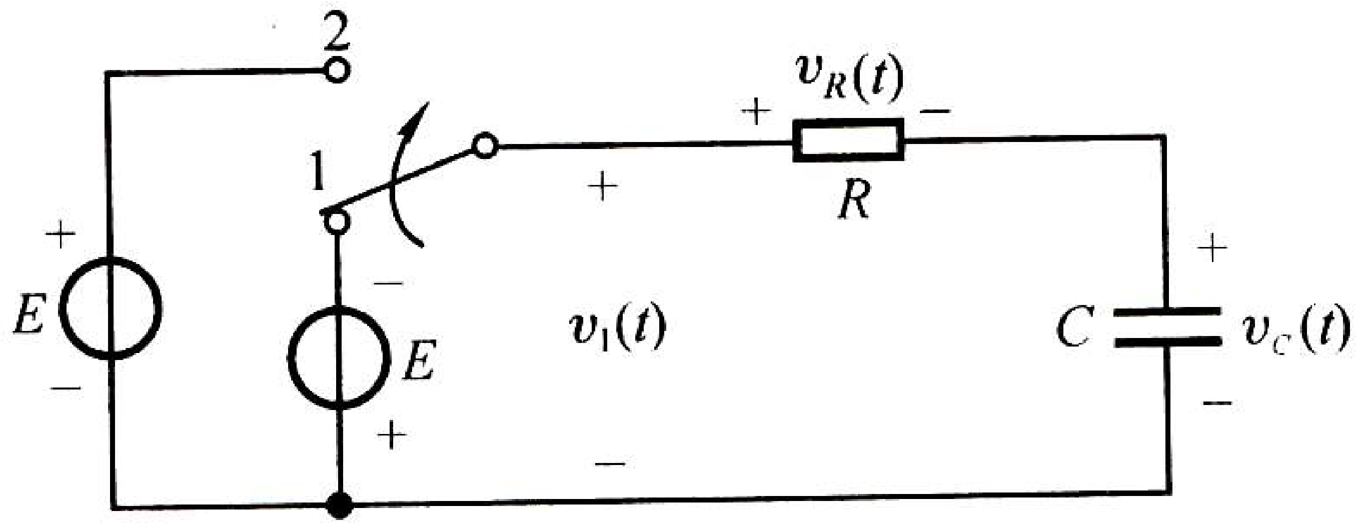
$$\therefore R(s)=\frac{b_0s+b_1}{s^2+a_1s+a_2}E(s)+\frac{(s+a_1)r(0-)+r'(0-)}{s^2+a_1s+a_2}$$

$$\downarrow \qquad\qquad \downarrow$$

$$R_{zs}(s)=H(s)E(s) \qquad\qquad R_{zI}(s)$$

$$R(s)=\frac{b_0s+b_1}{a_1s+a_2}E(s)+\frac{a_1r(0-)}{a_1s+a_2} \quad \longleftarrow \quad \text{一阶系统}$$

例：如图，当$t<0$时，开关位于“1”，电路的状态处于稳定，$t=0$时，打向“2”，求$v_c(t)$及$v_R(t)$？



$$RC\frac{dv_c(t)}{dt}+v_c(t)=E$$

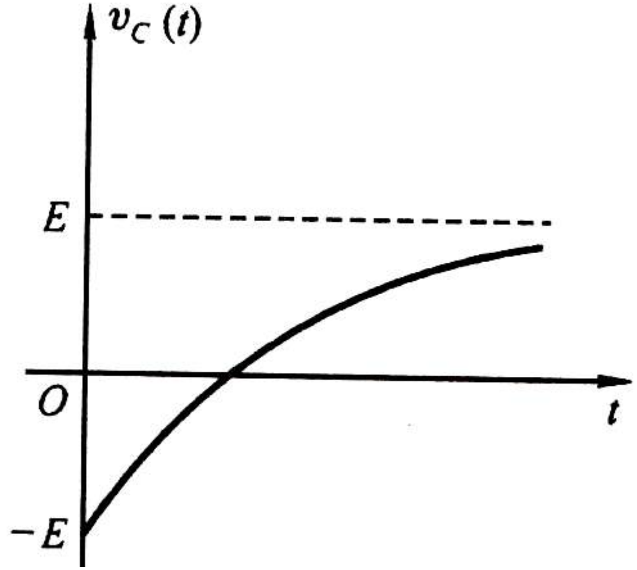
$$v_c(0-)=-E$$

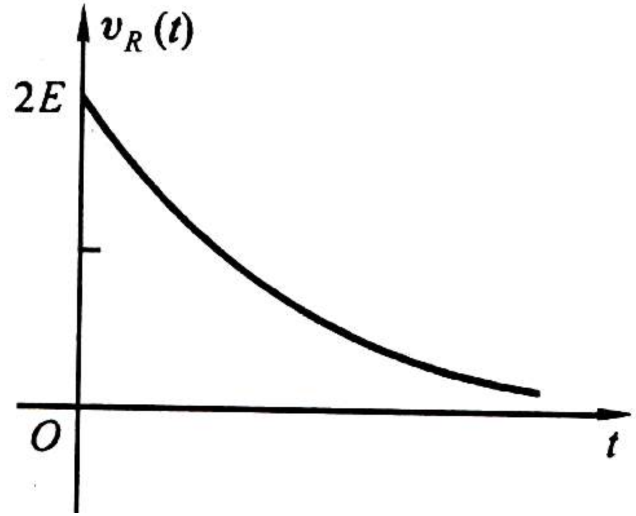
$$RC(sVc(s)+E)+Vc(s)=\frac{E}{s}$$

$$\therefore V_c(s)=\frac{E(\frac{1}{RC}-s)}{s(s+\frac{1}{RC})}=E(\frac{1}{s}-\frac{2}{s+\frac{1}{RC}})$$

$$\therefore v_c(t)=E-2Ee^{-\frac{1}{RC}t}(t\geq 0)$$

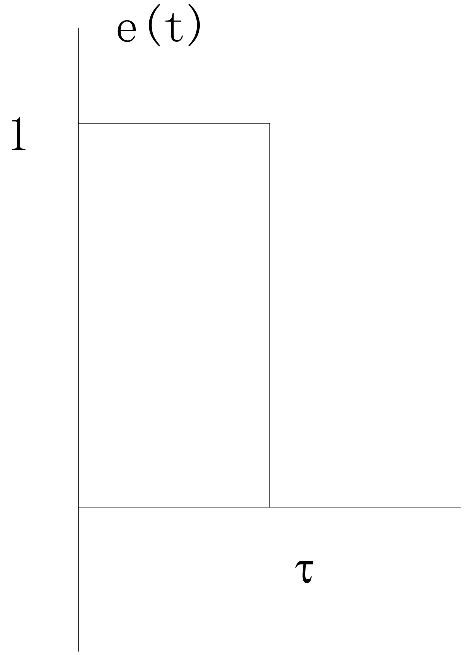
$$\therefore v_R(t)=E-v_c(t)=2Ee^{-\frac{1}{RC}t}u(t)$$

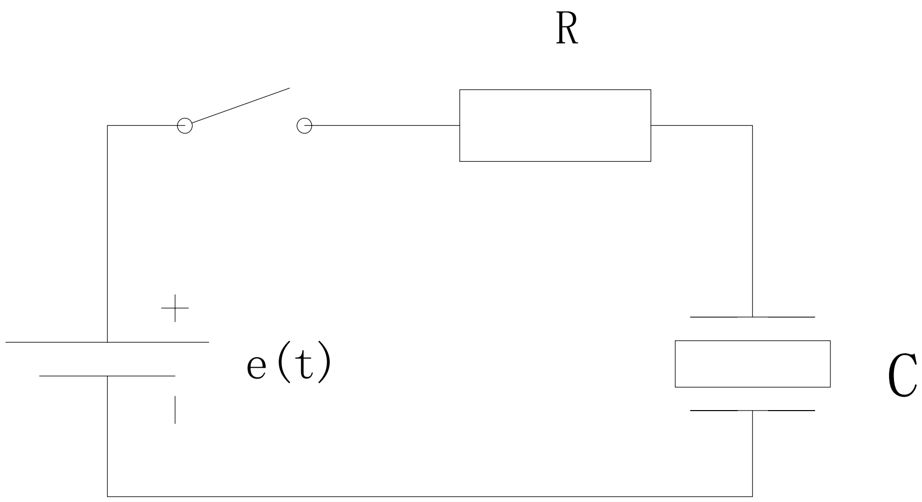




例：电路及输入如图，在$t=0$时合上开关，求输出电压$v_c(t)$

$v_c(0-)=0$





$$Rc\frac{dv_c}{dt}+v_c=e(t)$$

$$e(t)=u(t)-u(t-\tau)$$

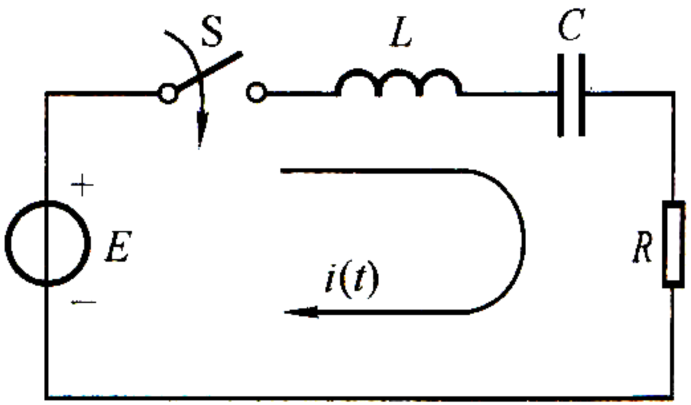
$$Rc[sV_c(s)]+V_c(s)=\frac{1}{s}-\frac{1}{s}e^{-s\tau}$$

$$\therefore V_c(s)=\frac{1-e^{-s\tau}}{s(Rcs+1)}=\frac{1}{s(Rcs+1)}-\frac{e^{-s\tau}}{s(Rcs+1)}$$

$$\frac{1}{s(Rcs+1)}=\frac{1}{s}-\frac{1}{s+\frac{1}{Rc}}$$

$$\therefore v_c(t)=(1-e^{-\frac{t}{Rc}})u(t)-(1-e^{-\frac{t-\tau}{Rc}})u(t-\tau)$$

例：如图电路起始状态为0，$t=0$开关闭合，接入直流电源E，求电流$i(t)$波形



$$v_L + v_R + v_C = Eu(t)$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} i d\tau = Eu(t)$$

$$i'' + \frac{R}{L}i' + \frac{i}{LC} = \frac{E}{L}\delta(t)$$

$$I(s) = \frac{E/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$p_1=-\frac{R}{2L}+\sqrt{(R/2L)^2-1/LC}=-\alpha+\sqrt{\alpha^2-\omega_0^2}$$

$$p_2=-\frac{R}{2L}-\sqrt{(R/2L)^2-1/LC}=-\alpha-\sqrt{\alpha^2-\omega_0^2}$$

$$I(s)=\frac{E}{L}\frac{1}{(s-p_1)(s-p_2)}=\frac{E}{L}\frac{1}{p_1-p_2}(\frac{1}{s-p_1}-\frac{1}{s-p_2})$$

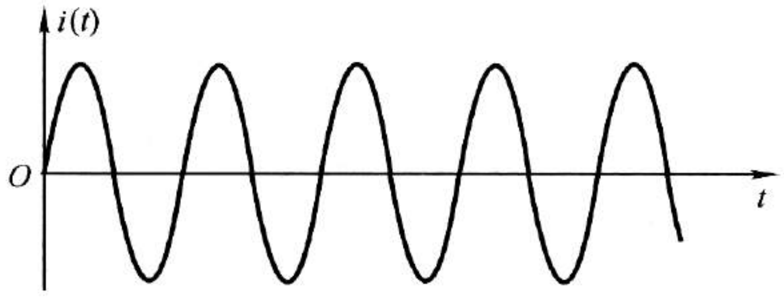
$$\therefore i(t)=\frac{E}{L(p_1-p_2)}(e^{p_1t}-e^{p_2t})$$

(1) $\alpha=0(R=0,$ 无损耗LC电路）

$$p_{1,2}=\pm j\omega_0$$

$$i(t)=\frac{E}{L}\frac{1}{2j\omega_0}(e^{j\omega_0t}-e^{-j\omega_0t})$$

$$=E\sqrt{C/L}\sin\omega_0 t$$

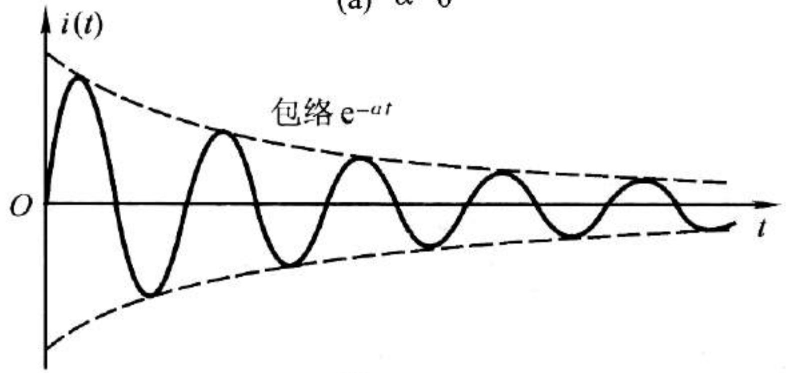


(a) $\alpha=0$

(2) $\alpha<\omega_0$(低阻尼LC回路）

$$\omega_d=\sqrt{\omega_0^2-\alpha^2},\ p_{1,2}=-\alpha\pm j\omega_d$$

$$i(t)=\frac{E}{L}\frac{1}{2j\omega_d}[e^{(-\alpha+j\omega_d)t}-e^{(-\alpha-j\omega_d)t}]$$



(b) $\alpha<\omega_0$

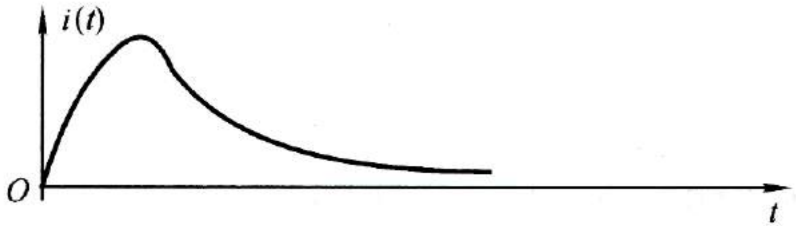
(3)$\alpha = \omega_0$(临界状态）

$$p_{1,2} = -\alpha$$

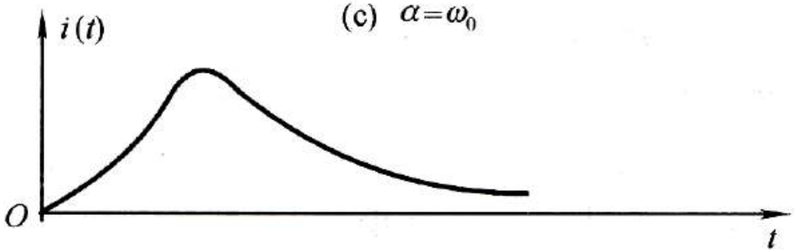
$$i(t) = \frac{E}{L} t e^{-\alpha t}$$

(4)$\alpha > \omega_0$(高阻尼LC回路）

$$i(t) = \frac{E}{L} \frac{1}{\sqrt{\alpha^2 - \omega_0^2}} e^{-\alpha t} \sinh(\sqrt{\alpha^2 - \omega_0^2} t)$$



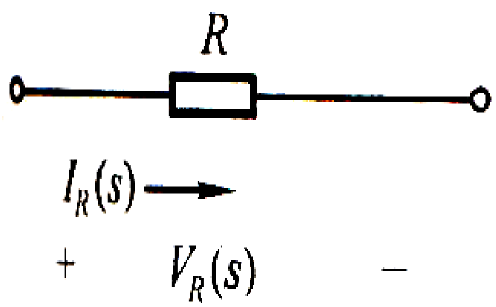
(c) $\alpha=\omega_0$



(d) $\alpha>\omega_0$

2、S域元件模型

– 电阻R



$$v_R(t) = Ri_R(t)$$

$$V_R(s) = RI_R(s)$$

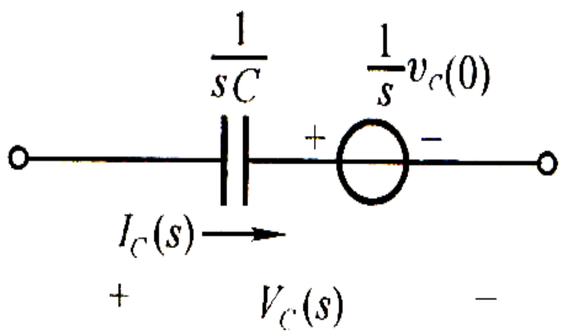
– 电容C

$$v_c(t) = v_c(0-)u(t) + \frac{1}{C}\int_{0-}^{t} i_c(\tau)d\tau$$

$$\frac{dv_c(t)}{dt} = v_c(0-)\delta(t) + \frac{i_c(t)}{C}$$

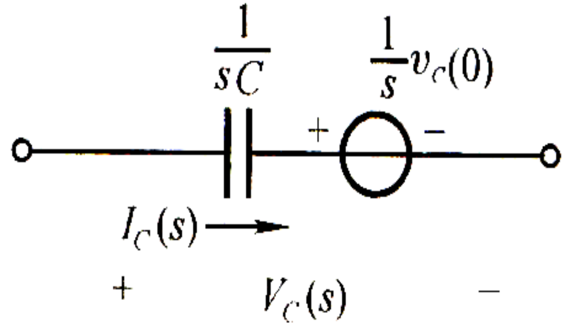
$$\therefore I_c(s) = csV_c(s) - cv_c(0-)$$

$$\therefore V_c(s) = \frac{I_c(s)}{sc} + \frac{v_c(0-)}{s}$$



– 电压源转化为等效电流源

- 和电压源串联的元件改成与电流源并联
- 等效电流源电流＝有串联元件的电压源两端短接的电流

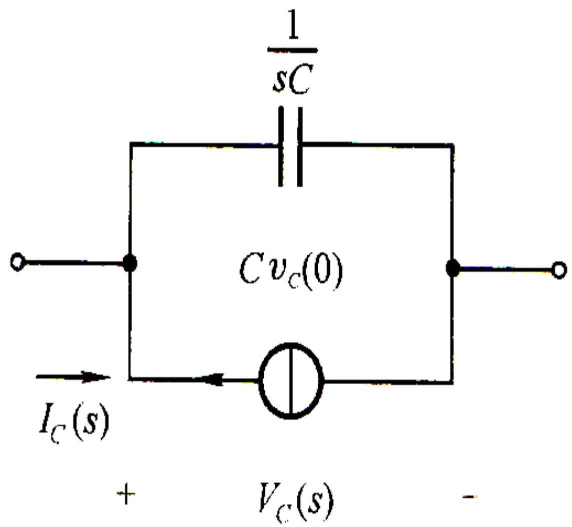


$$i_s(t) = c\frac{d}{dt}[v_c(0-)u(t)] = cv_c(0-)\delta(t)$$

$$i_c(t) = c\frac{dv_c(t)}{dt} - cv_c(0-)\delta(t)$$

$$I_c(s) = csV_c(s) - cV_c(0-)$$

$$I_c(s) = \frac{V_c(s)}{\frac{1}{sc}} - cV_c(0-)$$



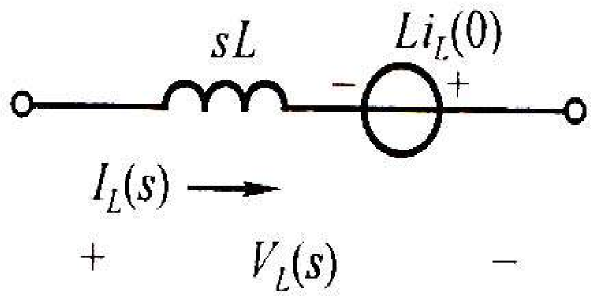
– 电感L

$$i_L(t)=i_L(0-)u(t)+\frac{1}{L}\int_{0-}^{t}v_L(\tau)d\tau$$

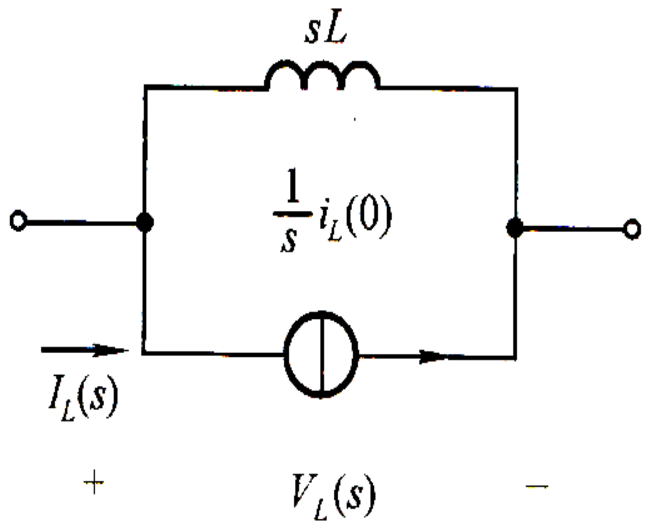
$$v_L(t)=L\frac{di_L(t)}{dt}-Li_L(0-)\delta(t)$$

$$\therefore V_L(s)=sLI_L(s)-Li_L(0-)$$

$$\therefore I_L(s)=\frac{V_L(s)}{sL}+\frac{i_L(0-)}{s}$$

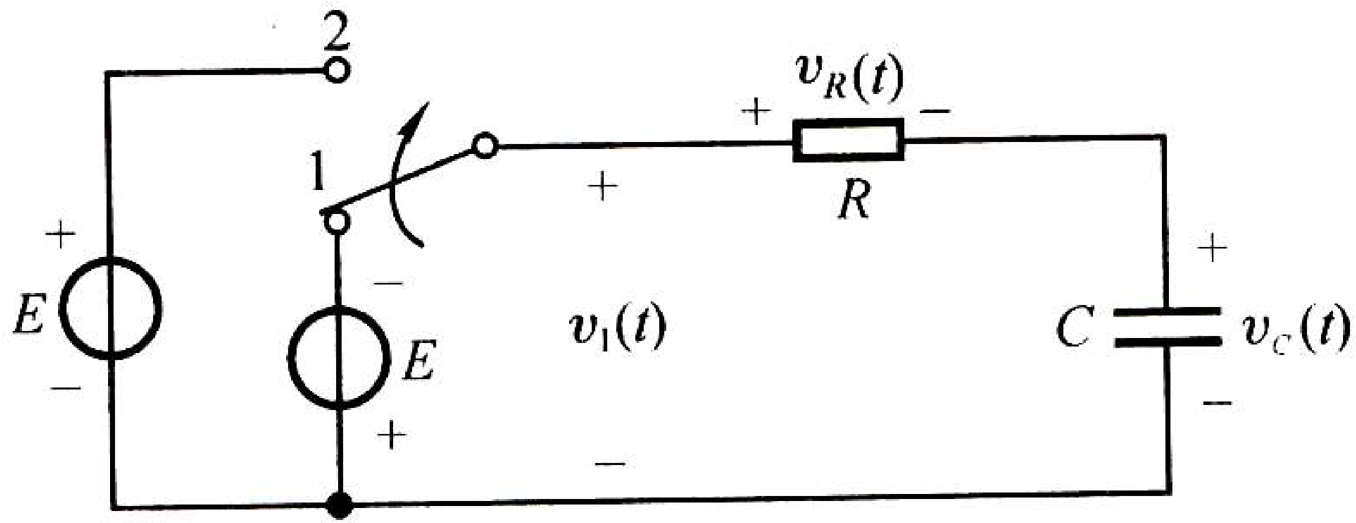


电感



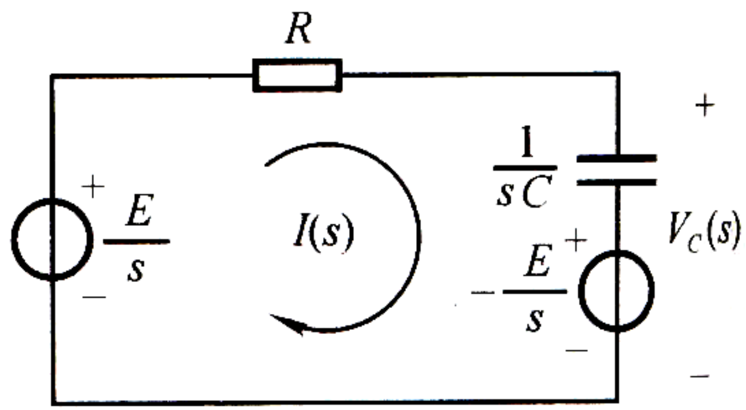
电感

例：如图，当$t<0$时，开关位于“1”，电路的状态处于稳定，$t=0$时，打向“2”，求$v_c(t)$及$v_R(t)$？



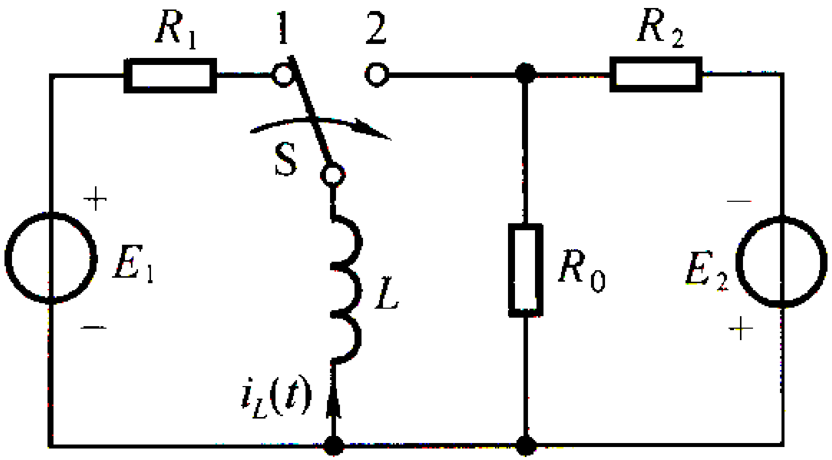
$$(R+\frac{1}{sc})I(s)=\frac{E}{s}+\frac{E}{s}$$

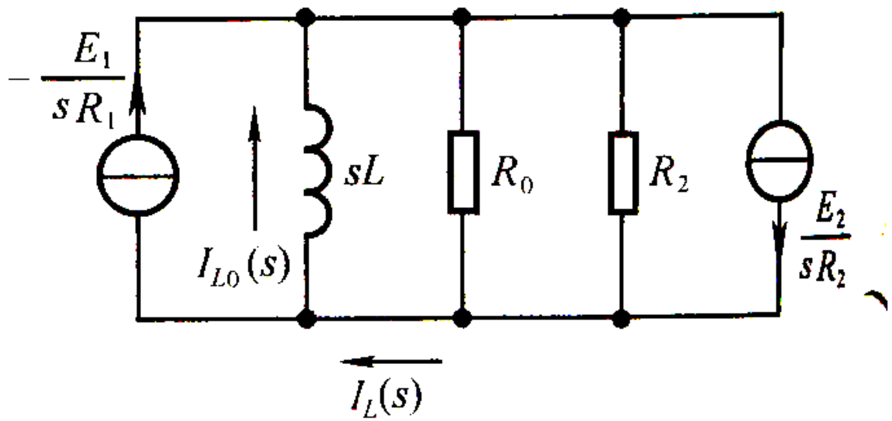
$$\therefore I(s)=\frac{2E}{s(R+\frac{1}{sc})}$$



$$\therefore V_c(s)=\frac{I(s)}{sc}-\frac{E}{s}=\frac{E(\frac{1}{Rc}-s)}{s(s+\frac{1}{Rc})}$$

例：如图所示电路，当t<0时，开关位于“1”，电路的状态已稳定，t=0时开关打到“2”，求$i_L(t)$





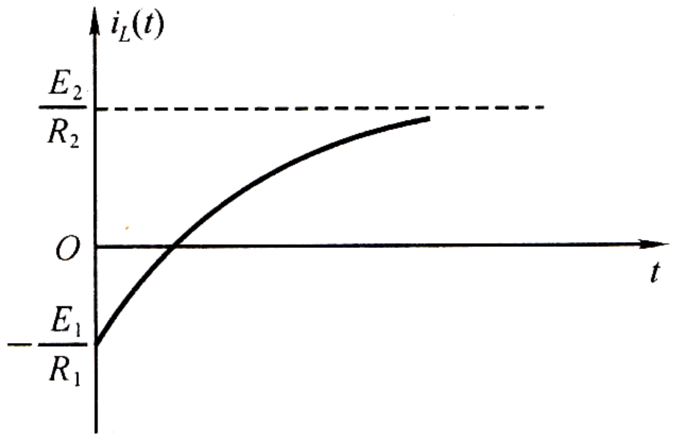
$$i_L(0-) = -E_1 / R_1$$

$$I_{L0}(s)sL = \left(\frac{E_1}{sR_1} + \frac{E_2}{sR_2}\right)\frac{1}{\frac{1}{R_0} + \frac{1}{R_2} + \frac{1}{sL}}$$

$$I_{L0}(s) = \left(\frac{E_1}{R_1} + \frac{E_2}{R_2}\right)\left(\frac{1}{s} - \frac{1}{s+1/L}\right)$$

$$I_L(s) = I_{L0}(s) - \frac{E_1}{sR_1} = \frac{E_2}{sR_2} - \left(\frac{E_1}{R_1} + \frac{E_2}{R_2}\right)\frac{1}{s+1/L}$$

$$\therefore i_L(t) = \left[\frac{E_2}{R_2} - \left(\frac{E_1}{R_1} + \frac{E_2}{R_2}\right)e^{-\frac{t}{\tau}}\right]u(t)$$



例：如图，已知$e(t)=10u(t)$,电路参数为$c=1F, R_{12}=\frac{1}{5}\Omega, R_2=1\Omega,$
$L=1/2H$,起始条件$v_c(0-)=-5V, i_L(0-)=4A,$方向如图，求$i_1(t)$

4.7 卷积定理

1、时域卷积

$$f_1(t) * f_2(t) \Leftrightarrow F_1(s) \cdot F_2(s)$$

2、复频域卷积

$$f_1(t) f_2(t) \Leftrightarrow \frac{1}{2\pi j} F_1(s) * F_2(s)$$

例：利用卷积定理，解积分方程

$$y(t)+\int_0^t g(t-\tau)y(\tau)d\tau = f(t) \qquad t\geq 0$$

已知: $f(t)=\sin t\,, g(t)=2\cos t, \quad t\geq 0$

$$Y(s)+G(s)Y(s)=F(s)$$

$$Y(s)=\frac{F(s)}{1+G(s)}$$

$$F(s)=\frac{1}{s^2+1}, G(s)=\frac{2s}{s^2+1}$$

$$Y(s)=\frac{1}{(s+1)^2}$$

$$\therefore y(t)=te^{-t}u(t)$$

4.8　系统函数

1、系统函数的定义

$$r_{zs}(t) = e(t) * h(t)$$

$$R_{zs}(s) = E(s)H(s)$$

$$\therefore h(t) \Leftrightarrow H(s), \quad H(s) = R_{zs}(s) / E(s)$$

频域：$H(j\omega) = R_{zs}(j\omega) / E(j\omega)$

Z域：$H(z) = R_{zs}(z) / E(z)$

- 系统函数在网络理论中应用广泛—网络函数
- H(jω)反映系统稳态下的频响特性
- H(s)具有较丰富的内容

$$H(s)|_{s=j\omega} = H(j\omega)$$

$$r_{zs}(t) = e(t) * h(t)$$

$$e(t) = \delta(t), r_{zs}(t) = h(t)$$

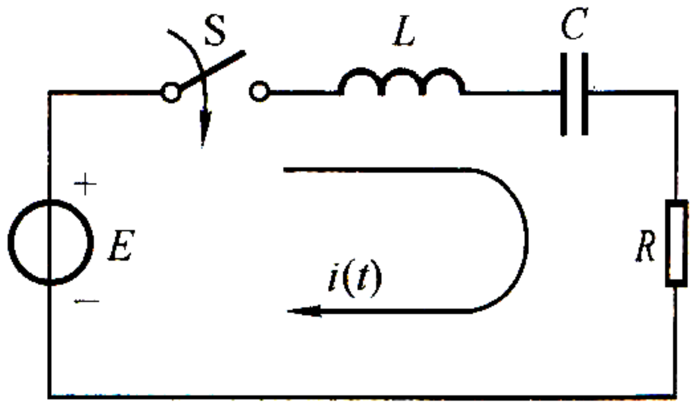
$$L[\delta(t)] = 1, R_{zs}(s) = H(s)$$

$$h(t) \Leftrightarrow H(s)$$

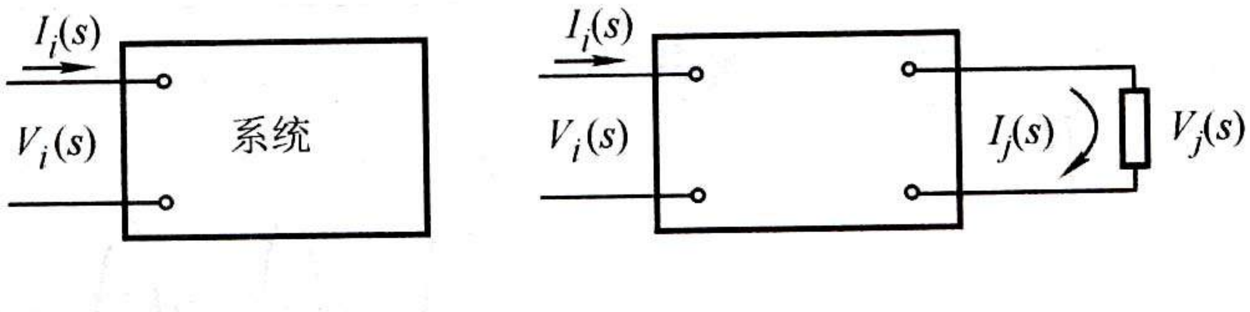
例：

$$I(s)=\frac{E(s)}{R+sL+\frac{1}{sc}}$$

$$H(s)=\frac{I(s)}{E(s)}=\frac{1}{R+sL+\frac{1}{sc}}$$

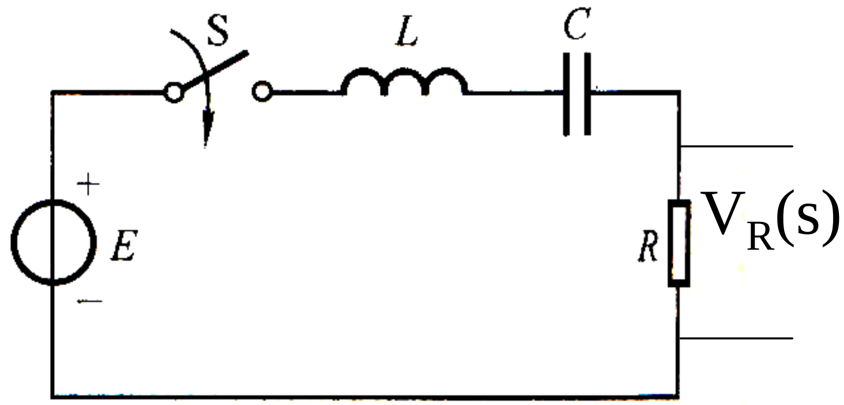


2、网络系统函数



(a) (b)

- 单口网络（策动点阻抗或导纳）
- 双端口网络（转移或传输函数）
- 激励与响应在同一端口，响应为电压，激励为电流，策动点阻抗(反之，策动点导纳)
- 激励与响应不在同一端口，转移函数(传输函数)，阻抗、导纳、电压比或电流比

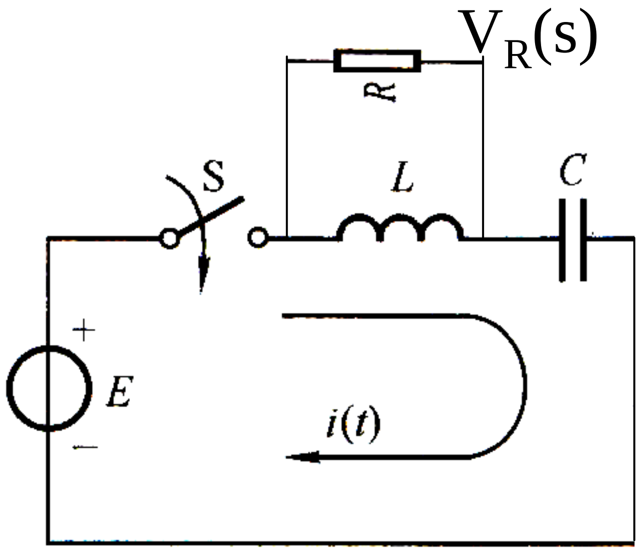


例：

$$H(s)=\frac{V_R(s)}{E(s)}=\frac{R}{R+sL+\frac{1}{sc}}$$

传输函数H(s)为传输电压比

要计算h(t),可以利用H(s)



例：R=1Ω, L=1H, C=1F, 求h(t)

$$H(s)=\frac{V_R(s)}{E(s)}=\frac{\dfrac{RsL}{R+sL}}{\dfrac{RsL}{R+sL}+\dfrac{1}{sc}}=\frac{\dfrac{s}{1+s}}{\dfrac{1}{s}+\dfrac{s}{1+s}}$$

$$=\frac{s^2}{s^2+s+1}=1-\frac{s+1}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$e^{-at}\sin\omega t-->\frac{\omega}{(s+a)^2+\omega^2},e^{-at}\cos\omega t-->\frac{s+a}{(s+a)^2+\omega^2}$$

$$h(t)=\delta(t)-e^{-\frac{t}{2}}[\cos\frac{\sqrt{3}}{2}t+\frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t)u(t)$$

3、计算H(s)的一般方法

- 作出S域模型图
- KVL或KCL列出方程
- 求H(s)=R(s)/E(s)

若网络有L个回路，L个电压和L个电流，可列出L个线性方程

$$\begin{pmatrix} Z_{11}(s) & Z_{12}(s) & \cdots & Z_{1L}(s) \\ Z_{21}(s) & Z_{22}(s) & \cdots & Z_{2L}(s) \\ \vdots & \vdots & \vdots & \vdots \\ Z_{L1}(s) & Z_{L2}(s) & \cdots & Z_{LL}(s) \end{pmatrix} \begin{pmatrix} I_1(s) \\ I_2(s) \\ \vdots \\ I_L(s) \end{pmatrix} = \begin{pmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_L(s) \end{pmatrix}$$

$$ZI = V$$

$$I = Z^{-1}V$$

$$I_k(s) = \frac{\Delta_{1k}}{\Delta}V_1(s) + \frac{\Delta_{2k}}{\Delta}V_2(s) + \cdots + \frac{\Delta_{Lk}}{\Delta}V_L(s)$$

$$\therefore \frac{I_k(s)}{V_j(s)} = \frac{\Delta_{jk}}{\Delta} = Y_{kj}(s)$$

Δ_{jk}为Δ中去掉第j行第k列剩下的子行列式$*(-1)^{j+k}$

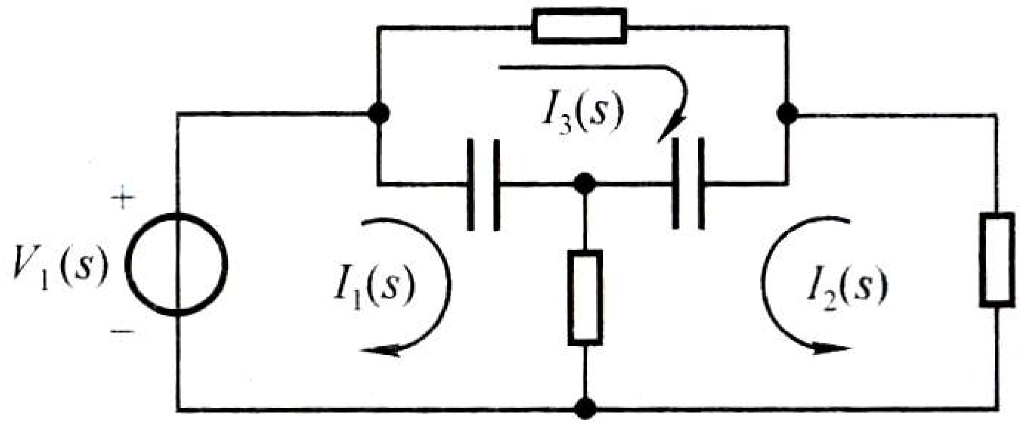
例：如图电容为1F，试求电路的转移导纳函数$Y_{21}(s)=I_2(s)/V_1(s)$

$$\begin{pmatrix} 1+1/s & 1 & -1/s \\ 1 & 2+1/s & 1/s \\ -1/s & 1/s & 1+2/s \end{pmatrix}\begin{pmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{pmatrix}=\begin{pmatrix} V_1(s) \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta=\begin{vmatrix} 1+1/s & 1 & -1/s \\ 1 & 2+1/s & 1/s \\ -1/s & 1/s & 1+2/s \end{vmatrix}=\frac{s^2+5s+2}{s^2}$$

$$\Delta_{12}=(-1)^{1+2}\begin{vmatrix} 1 & 1/s \\ -1/s & 1+2/s \end{vmatrix}=-\frac{s^2+2s+1}{s^2}$$

$$\therefore Y_{21}(s)=\frac{\Delta_{12}}{\Delta}=-\frac{s^2+2s+1}{s^2+5s+2}$$



4.9 周期信号与抽样信号的LT

1、周期信号的LT

$$f_T = f_1(t) + f_1(t-T) + f_1(t-2T) + \ldots = \sum_{n=0}^{\infty} f_1(t-nT)$$

$$F_T(s) = F_1(s) + F_1(s)e^{-sT} + F_1(s)e^{-2sT} + \ldots$$

$$= F_1(s)\sum_{n=0}^{\infty} e^{-nsT} = F_1(s)\frac{1}{1-e^{-sT}}$$

↓

周期化定理

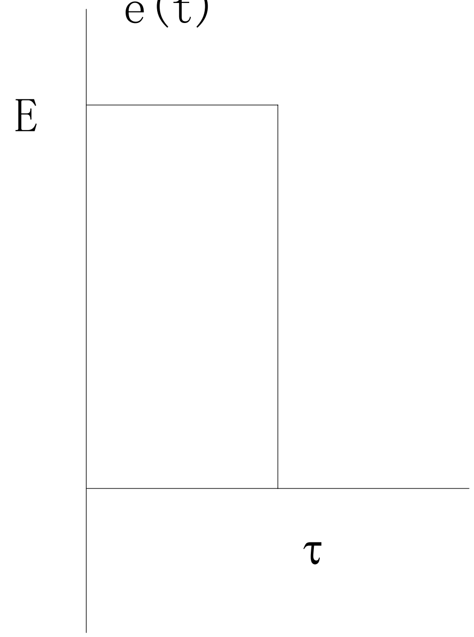
周期信号的单边LT=第一个周期内的函数的LT除以(1-e-^{sT})

– 周期性矩形脉冲

$$f_1(t)=\begin{cases}E & 0<\mathrm{t}<\tau\\ 0 & \tau<\mathrm{t}<\mathrm{T}\end{cases}$$

$$F_1(s)=\int_0^T f_1(t)e^{-st}dt=\int_0^\tau Ee^{-st}dt=\frac{E}{s}(1-e^{-s\tau})$$

$$F_T(s)=\frac{E}{s}\frac{1-e^{-s\tau}}{1-e^{-sT}}$$



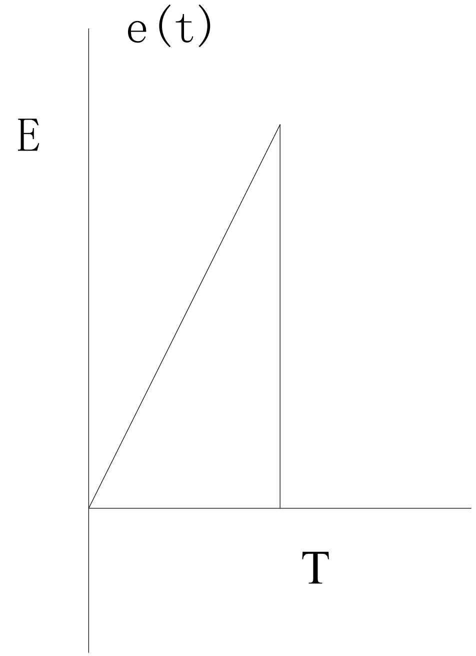
– 周期性锯齿波脉冲

$$f_1(t)=\frac{t}{T}[u(t)-u(t-T)]$$

$$=\frac{t}{T}u(t)-\frac{t-T}{T}u(t-T)-u(t-T)$$

$$F_1(s)=\frac{1}{Ts^2}-\frac{1}{Ts^2}e^{-sT}-\frac{1}{s}e^{-sT}$$

$$F_T(s)=F_1(s)\frac{1}{1-e^{-sT}}$$

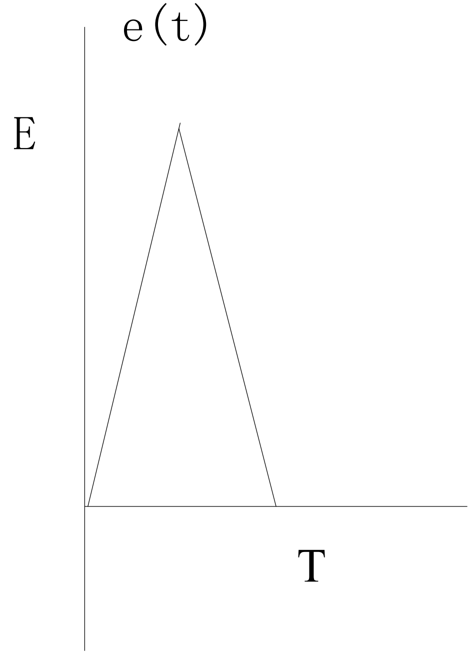


– 周期性三角脉冲

$$f_1(t) = \frac{2}{T}tu(t) - \frac{4}{T}(t - \frac{T}{2})u(t - \frac{T}{2}) + \frac{2}{T}(t - T)u(t - T)$$

$$F_1(s) = \frac{2}{Ts^2} - \frac{4}{Ts^2}e^{-\frac{T}{2}s} + \frac{2}{Ts^2}e^{-Ts}$$

$$F_T(s) = \frac{F_1(s)}{1 - e^{-sT}}$$



2、抽样信号的LT

$$\delta_T(t)=\sum_{n=0}^{\infty}\delta(t-nT)$$

$$L[\delta_T(t)]=\frac{1}{1-e^{-sT}}$$

对$f(t)$进行理想抽样，抽样周期为T_s

$$f_s(t)=f(t)\delta_{T_s}(t)=f(t)\sum_{n=0}^{\infty}\delta(t-nT_s)$$

$$F_s(s)=\int_0^{\infty}f(t)\sum_{n=0}^{\infty}\delta(t-nT_s)e^{-st}dt$$

$$=\sum_{n=0}^{\infty}\int_0^{\infty}f(t)e^{-st}\delta(t-nT_s)dt=\sum_{n=0}^{\infty}f(nT_s)e^{-snT_s}$$

抽样信号的LT为S域的级数

例：求指数抽样序列的LT

$$f_s(t) = e^{-at}\delta_T(t) \qquad \text{a} > 0$$

$$F_s(s) = \sum_{n=0}^{\infty} e^{-anT_s} e^{-nsT_s} = \frac{1}{1 - e^{-(a+s)T_s}}$$

4.10 零极点与时域特性

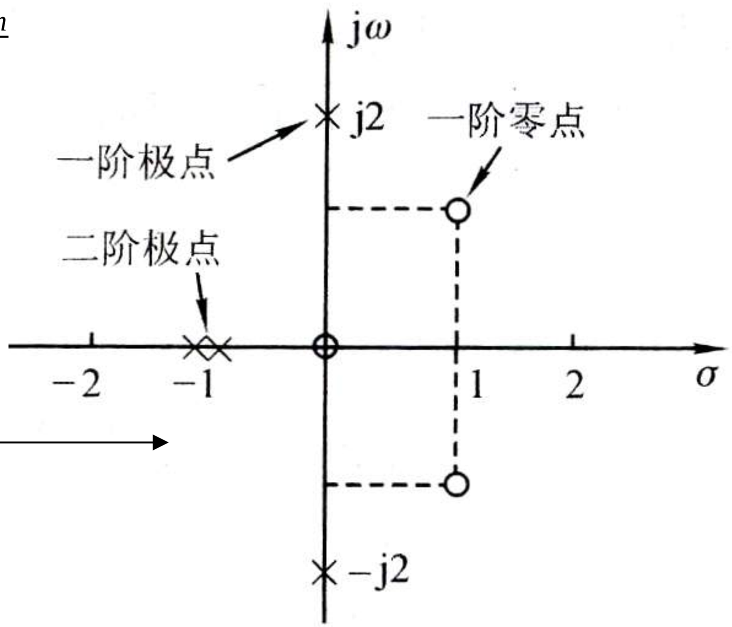
1、零极点

$$F(s)=\frac{A(s)}{B(s)}=\frac{a_0s^m+a_1s^{m-1}+\ldots+a_{m-1}s+a_m}{b_0s^n+b_1s^{n-1}+\ldots+b_{n-1}s+b_n}$$

$A(s)=0\rightarrow z_1,z_2\ldots z_m$ 零点

$B(s)=0\rightarrow p_1,p_2\ldots p_n$ 极点

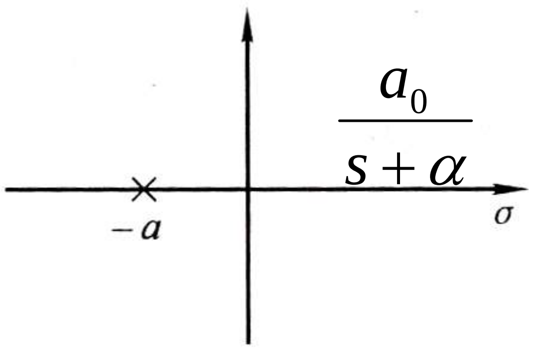
$$F(s)=\frac{s[(s-1)^2+1]}{(s+1)^2(s^2+4)}$$

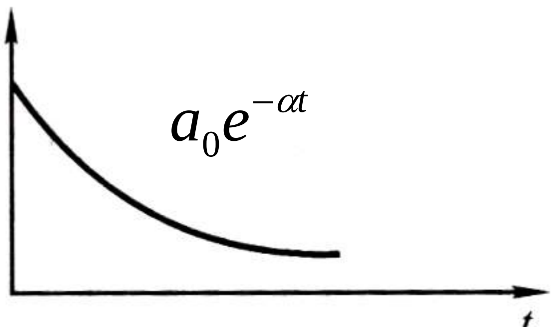


2、零极点与时域响应

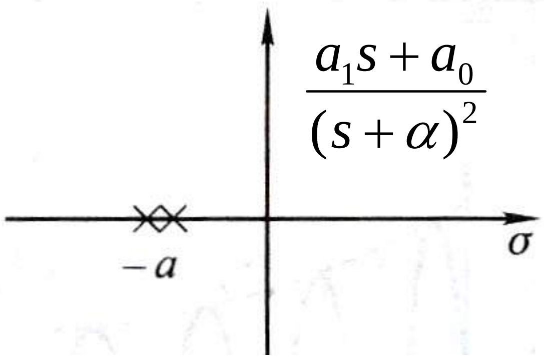
– 左半开平面内的极点

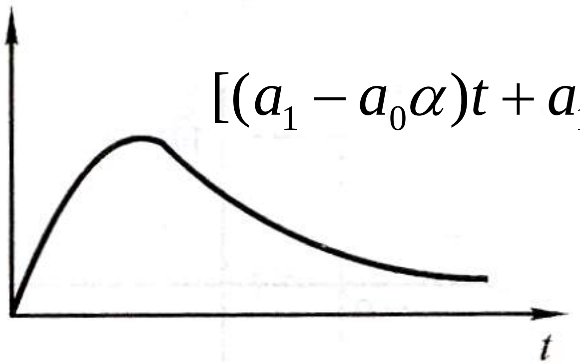
- 负实轴上单极点



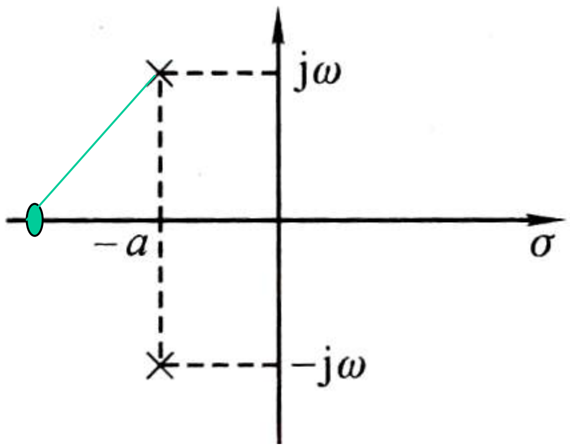


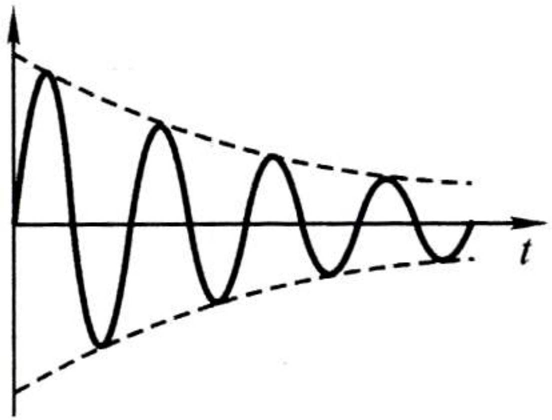
- 负实轴上二阶极点





- 左半开平面内的共轭极点





$$\frac{s+b}{(s+\alpha)^2+\beta^2}$$

$$\frac{\sqrt{(b-\alpha)^2+\beta^2}}{\beta}e^{-\alpha t}\sin(\beta t+\varphi)$$

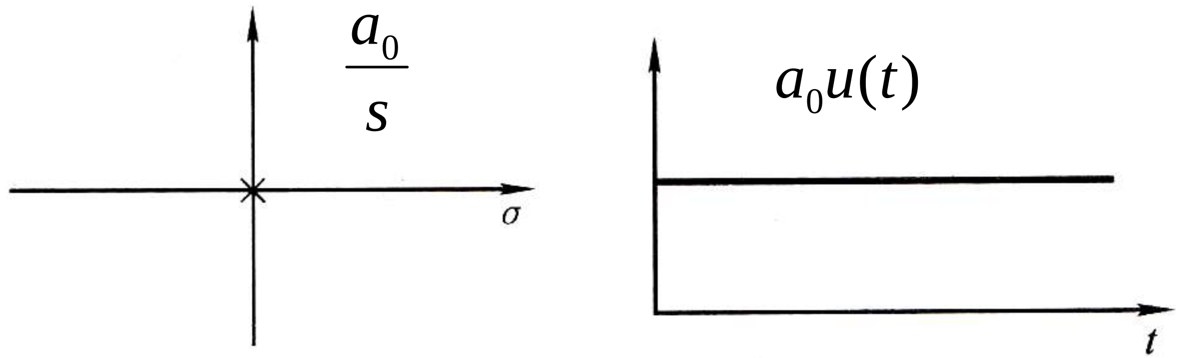
$$\varphi=\tan^{-1}\frac{\beta}{b-\alpha}$$

- 左半开平面内的m阶共轭极点

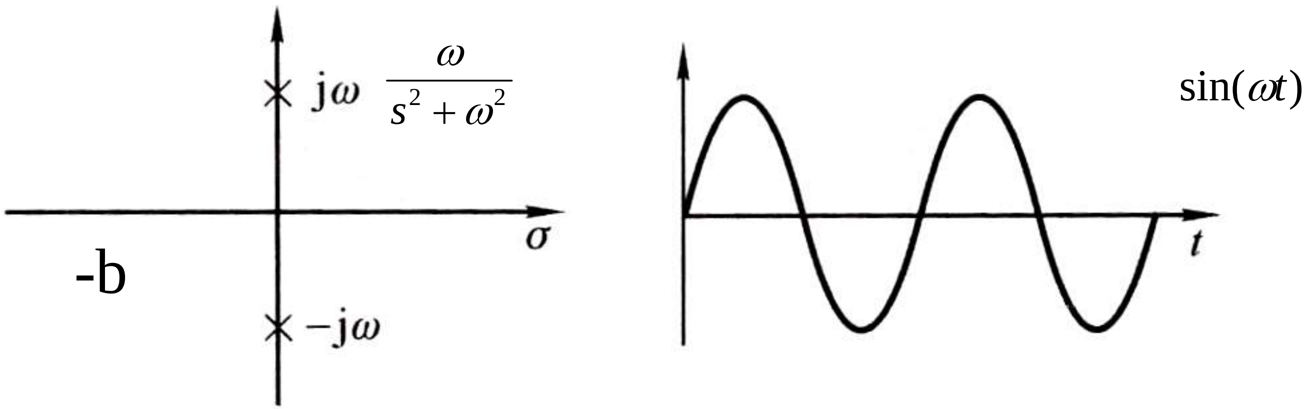
$$[(s+\alpha)^2+\beta^2]^m \rightarrow \frac{t^{k-1}}{(k-1)!}e^{-\alpha t}\sin(\beta t+\varphi)$$

– 虚轴上的极点

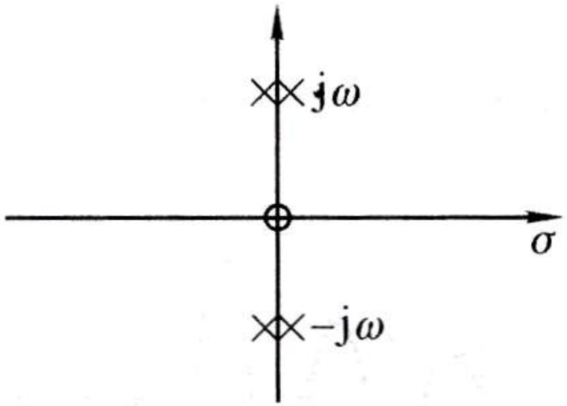
- 原点处的的单极点



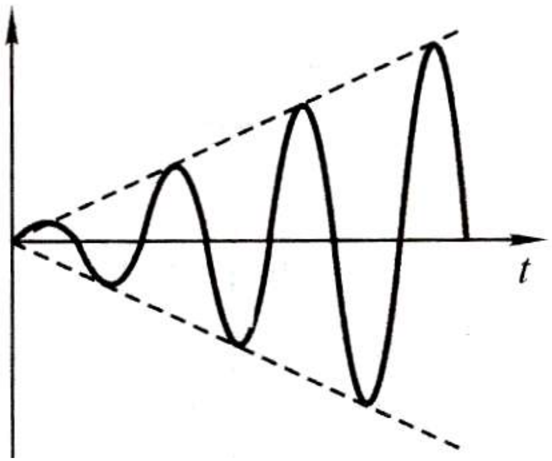
- 虚轴上共轭单极点



- 虚轴上共轭二阶极点



$$\frac{2\omega s}{(s^2+\omega^2)^2}$$



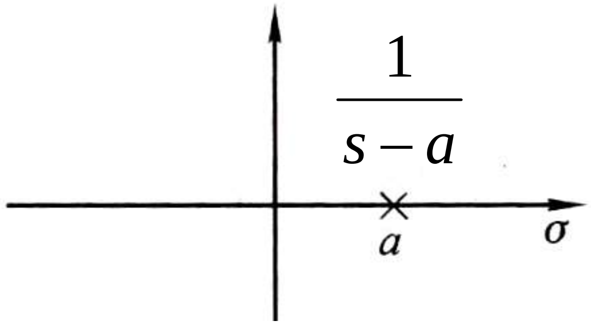
$$t\sin\omega t$$

- 虚轴上共轭m阶极点

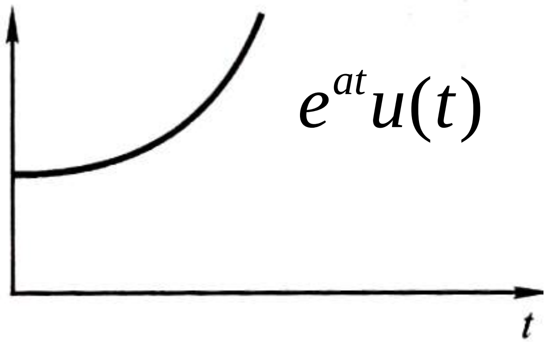
$$t^{m-1}u(t)\text{或}t^{m-1}\sin\omega t$$

– 右半开平面的极点

- 单实极点

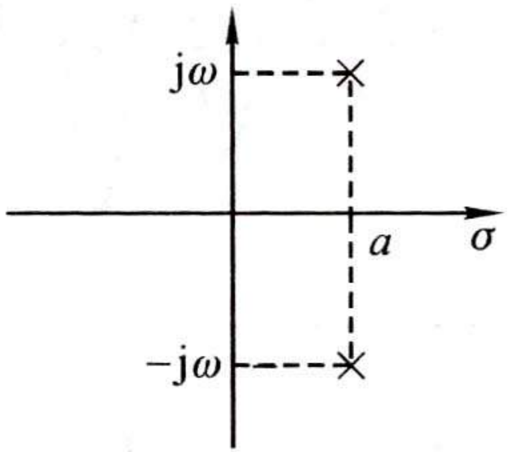


$$\frac{1}{s-a}$$

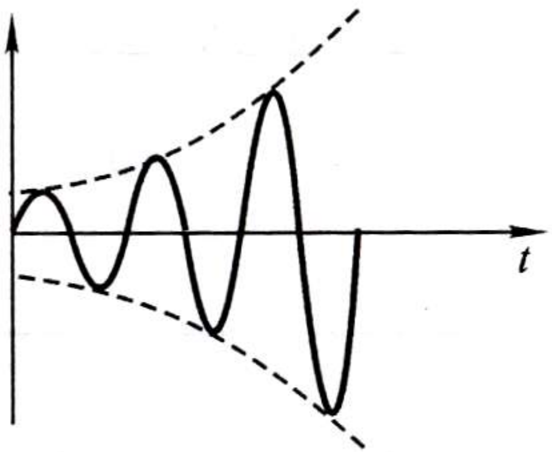


$$e^{at}u(t)$$

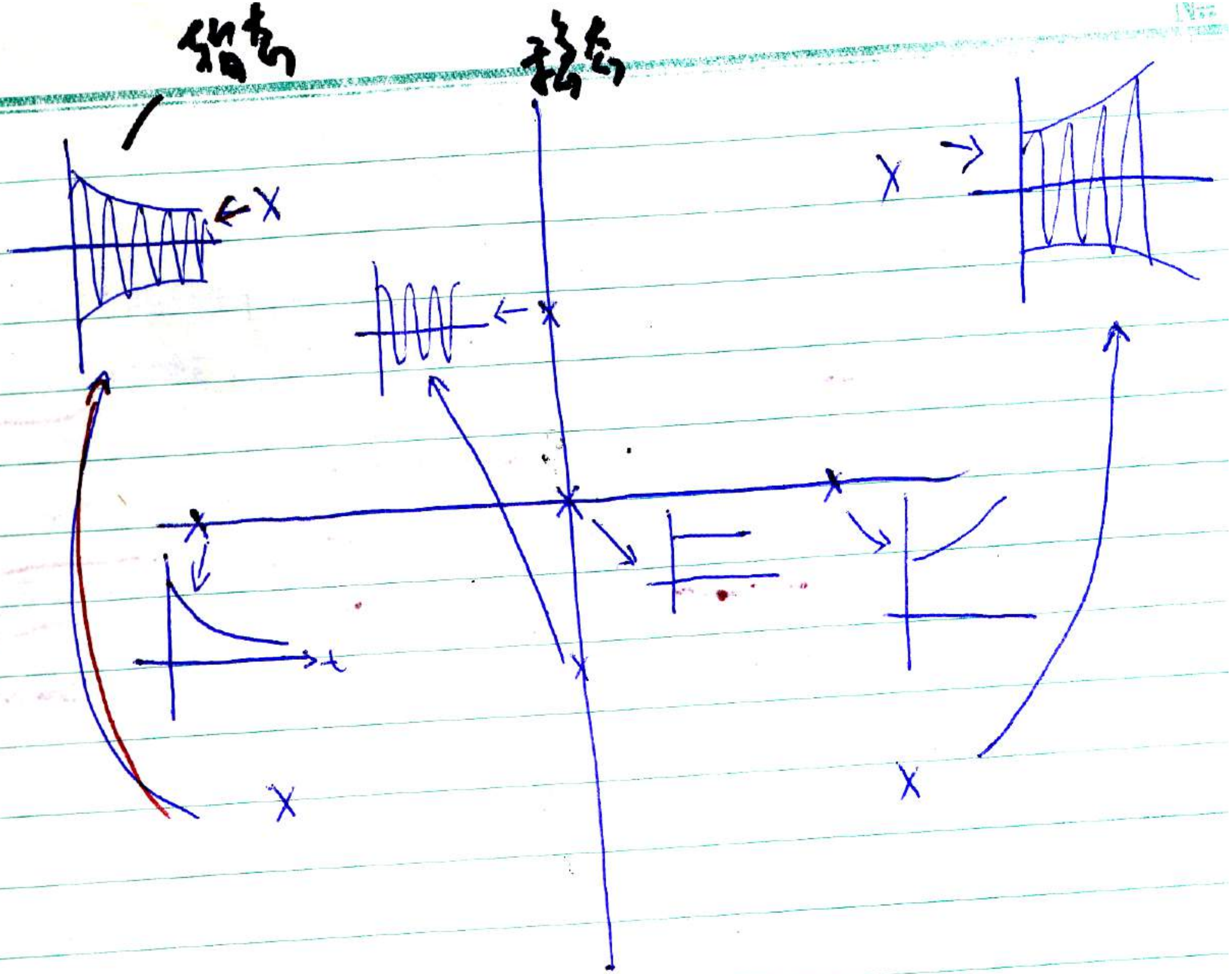
- 共轭单极点



$$\frac{\omega}{(s+a)^2+\omega^2}$$



$$e^{at}\sin\omega t$$



3、自由响应与强迫响应

- 非齐次的常微分方程描述电网络
  - 通解—自由响应(固有响应)，特征根决定于系统参数，幅度决定于初始条件和输入
  - 特解---强迫响应(受迫响应)，与系统的输入有关，也与系统参数有关
  - 稳定系统的自由响应—暂态响应
  - 稳定系统的强迫响应—稳态响应
- 稳定系统：极点在S域左半平面

例：$\frac{dr(t)}{dt}+3r(t)=3u(t), r(0-)=3/2$,求自由及强迫响应

$\alpha+3=0$, $\alpha=-3$, 3B=3, B=1

$r(t)=Ae^{-3t}+1$

$$r(0+)=r(0-)=3/2$$

$$A=1/2$$

$$r(t)=\frac{1}{2}e^{-3t}+1$$

自由　强迫

若零状态，r(0-)=0

$$r(t)=-e^{-3t}+u(t)$$

自由　强迫

$$R(s)=H(s)E(s)$$

$$H(s)=\frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)},E(s)=\frac{\prod_{l=1}^{u}(s-z_l)}{\prod_{k=1}^{v}(s-p_k)}$$

$$R(s)=\sum_{i=1}^{n}\frac{k_i}{s-p_i}+\sum_{k=1}^{v}\frac{k_k}{s-p_k}$$

$$\therefore r(t)=\sum_{i=1}^{n}k_i e^{p_i t}+\sum_{k=1}^{v}k_k e^{p_k t}$$

↓ 自由响应 ↓ 强迫响应

例：$\frac{dr(t)}{dt}+3r(t)=3u(t), r(0-)=3/2$,求自由及强迫响应

$$sR(s)-r(0-)+3R(s)=\frac{3}{s}$$

$$R(s)=\frac{1}{s+3}\frac{s+2}{s}\frac{3}{2}=\frac{1}{2(s+3)}+\frac{1}{s}$$

$$r(t)=0.5e^{-3t}+u(t)$$

自由　强迫

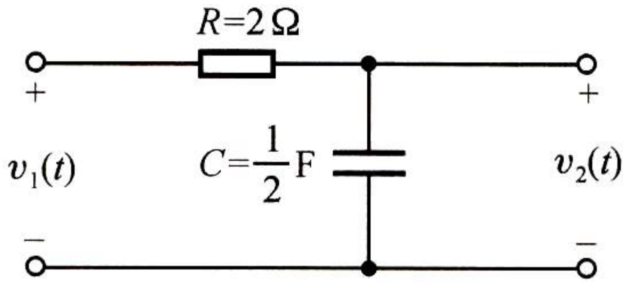
若零状态，r(0−)=0

$$R(s)=H(s)E(s)=\frac{-1}{s+3}+\frac{1}{s}$$

$$r(t)=-e^{-3t}+u(t)$$

自由　强迫

例：电路如图，输入信号$v_1(t)=10\cos(4t)u(t)$,求输出电压$v_2(t)$,并指出自由及强迫响应？



$$H(s)=\frac{V_2(s)}{V_1(s)}=\frac{\frac{1}{sc}}{R+\frac{1}{sc}}=\frac{1}{s+1}$$

$$V_1(s)=\frac{10s}{s^2+16}$$

$$V_2(s)=V_1(s)H(s)=\frac{10s}{(s^2+16)(s+1)}=\frac{\frac{10}{17}s+\frac{160}{17}}{s^2+16}-\frac{\frac{10}{17}}{s+1}$$

$$v_2(t)=-\frac{10}{17}e^{-t}+\frac{10}{17}\cos(4t)+\frac{160}{17}\sin(4t)$$

自由响应　　强迫响应

– 系统的固有频率

$$C_0 \frac{d^n r(t)}{dt^n} + C_1 \frac{d^{n-1} r(t)}{dt^{n-1}} + \ldots + C_{n-1} \frac{dr(t)}{dt} + C_n r(t)$$

$$= E_0 \frac{d^m e(t)}{dt^m} + E_1 \frac{d^{m-1} e(t)}{dt^{m-1}} + \ldots + E_{m-1} \frac{de(t)}{dt} + E_m e(t)$$

$$C_0 \alpha^n + C_1 \alpha^{n-1} + \ldots + C_{n-1} \alpha + C_n = 0$$

$\alpha_1, \alpha_2, \cdots \alpha_n$系统的固有频率

$H(s)$的n个极点

如果系统激励为电压源，响应为网孔电流，可列出如下矩阵

$$V(s)=Z(s)I(s)$$

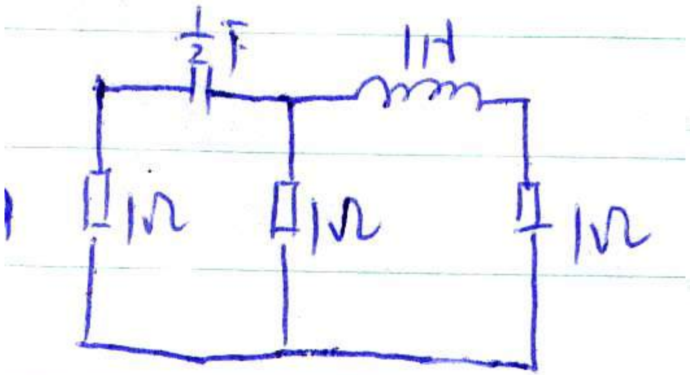
$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ \vdots & & & \\ \vdots & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

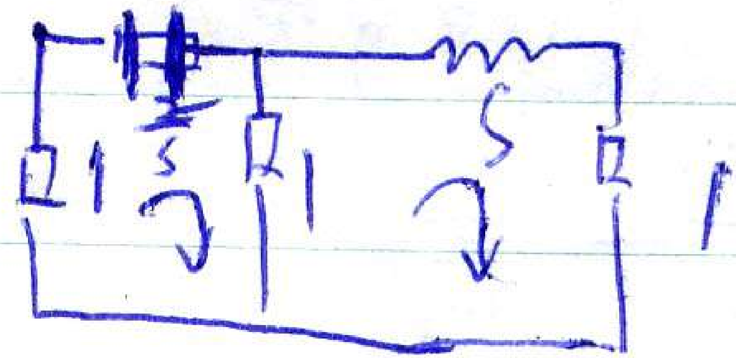
$$\Delta Z = 0$$

例：求图示系统的固有频率

$$Z(s)=\begin{bmatrix} 2+\dfrac{2}{s} & -1 \\ -1 & 2+s \end{bmatrix}, \Delta=\begin{vmatrix} 2+\dfrac{2}{s} & -1 \\ -1 & 2+s \end{vmatrix}=\frac{2s^2+5s+4}{s}=0$$

$$s_{1,2}=-\frac{5}{4}\pm\frac{\sqrt{7}}{4}j$$





– 暂态响应与稳态响应

- 暂态响应：信号接入后较短时间内出现的解 t增大，解趋于0
- 稳态响应： t增大，解不为0

系统极点
- 左半平面 → 暂态响应
- 虚轴 → 稳态响应
- 右半平面 → 不稳定系统

激励源极点
- $\mathrm{Re}[p_k] \geq 0$ → 稳态响应
- $\mathrm{Re}[p_k] < 0$ → 暂态响应

例：已知输入$e(t)=e^{-t}u(t)$，起始条件$r(0-)=2, r'(0-)=1$,

系统函数$H(s)=\dfrac{s+5}{s^2+5s+6}$,求$r(t)$,并标出自由响应，强迫响应，暂态响应及稳态响应

$$\frac{R(s)}{E(s)}=\frac{s+5}{s^2+5s+6}$$

$$s^2R(s)+5sR(s)+6R(s)=sE(s)+5E(s)$$

$$\frac{d^2r(t)}{dt^2}+5\frac{dr(t)}{dt}+6r(t)=\frac{de(t)}{dt}+5e(t)$$

$$[s^2R(s)-sr(0-)-r'(0-)]+5sR(s)-5r(0-)+6R(s)$$

$$=sE(s)+5E(s)$$

$a_1=5, a_2=6$

$$E(s)=\frac{1}{s+1}$$

$$R(s)=R_{zs}(s)+R_{zI}(s)$$

$$R_{zs}(s)=H(s)E(s)=\frac{2}{s+1}+\frac{-3}{s+2}+\frac{1}{s+3}$$

$$R_{zI}(s)=\frac{(s+a)r(0-)+r'(0-)}{s^2+5s+6}=\frac{7}{s+2}+\frac{-5}{s+3}$$

$$\therefore r_{zs}(t)=(\underbrace{2e^{-t}}_{\text{强迫}}\underbrace{-3e^{-2t}+e^{-3t}}_{\text{自由}})u(t), r_{zi}(t)=(\underbrace{7e^{-2t}-5e^{-3t}}_{\text{自由}})u(t)$$

$$\therefore r(t)=\underbrace{2e^{-t}}_{\text{强迫}}+\underbrace{4e^{-2t}-4e^{-3t}}_{\text{自由}}$$

强迫　　自由

暂态

例：已知输入$e(t)=e^{t}u(t)$，起始条件$r(0-)=2, r'(0-)=1$，

系统函数$H(s)=\dfrac{s+5}{s^{2}+5s+6}$，求$r(t)$，并标出自由响应，强迫响应，暂态响应及稳态响应

$$a_1=5, a_2=6$$

$$E(s)=\frac{1}{s-1}$$

$$R(s)=R_{zs}(s)+R_{zI}(s)$$

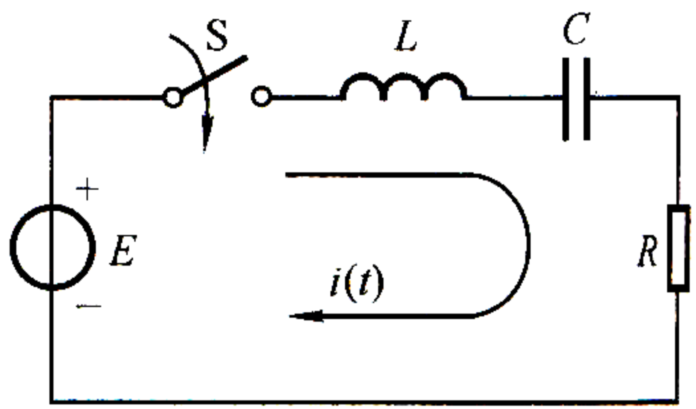
$$R_{zs}(s)=H(s)E(s)=\frac{1/2}{s-1}+\frac{-1}{s+2}+\frac{-1/2}{s+3}$$

$$R_{zI}(s)=\frac{(s+a)r(0-)+r'(0-)}{s^{2}+5s+6}=\frac{7}{s+2}+\frac{-5}{s+3}$$

$$\therefore r_{zs}(t)=(\underbrace{1/2e^{t}}_{\text{强迫}}\underbrace{-e^{-2t}-1/2e^{-3t}}_{\text{自由}})u(t), r_{zi}(t)=(\underbrace{7e^{-2t}-5e^{-3t}}_{\text{自由}})u(t)$$

$$\therefore r(t)=\underbrace{1/2e^{t}}_{\text{稳态}}+\underbrace{6e^{-2t}-5.5e^{-3t}}_{\text{暂态}}$$

例：已知输入$e(t)=E_m\sin\omega_{01}tu(t)$作用于RLC串联电路，起始条件为0，求i(t),并标出自由响应，强迫响应，暂态响应及稳态响应



$$(1)e(t)=E_m\sin\omega_{01}tu(t)$$

$$E(s)=\frac{E_m\omega_{01}}{s^2+\omega_{01}^2},\ p_{1,2}=\pm j\omega_{01}$$

(2)系统转移函数为输入导纳

$$H(s)=Y(s)=\frac{1}{SL+R+\frac{1}{sc}}=\frac{s}{s^2+\frac{R}{L}s+\frac{1}{Lc}}=\frac{s}{L(s-p_3)(s-p_4)}$$

$$p_{3,4}=-\frac{R}{2L}\pm\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}},\quad \alpha=\frac{R}{2L},\quad \omega_0=\sqrt{\frac{1}{LC}}$$

当回路具有正常Q值，$R<2\sqrt{\frac{L}{c}}$负阻尼情况下，$p_{3,4}$共轭极点

$$I(s)=H(s)E(s)=\frac{E_m\omega_{01}}{L}\frac{s}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)}$$

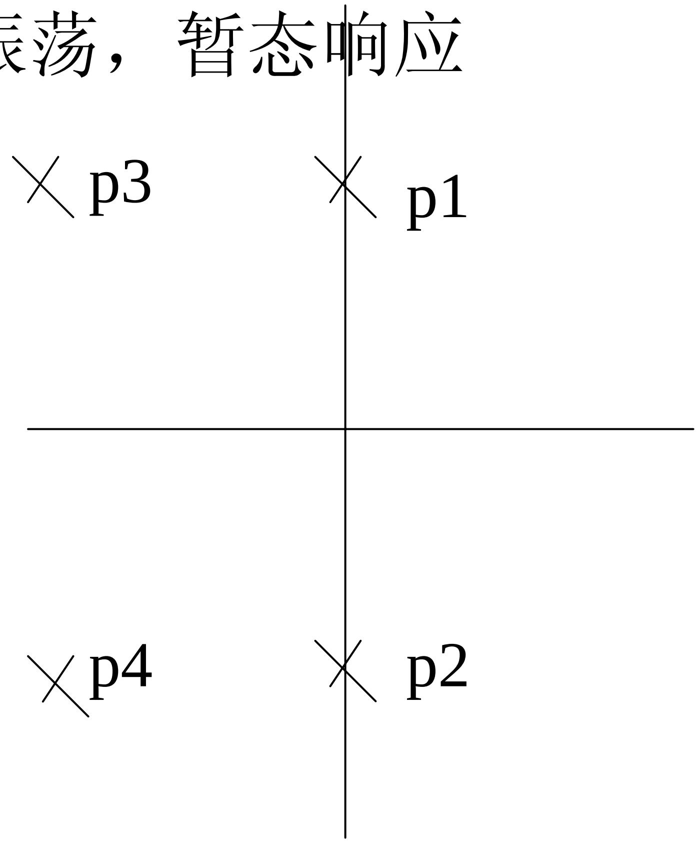
$p_{1,2} = \pm j\omega_{01}$

$p_{3,4} = -\alpha \pm j\omega_d$

源极点：强迫响应，等幅正弦振荡，稳态响应

系统极点：自由响应，衰减正弦振荡，暂态响应

$Ae^{-\alpha t}\sin(\omega_d t + \phi)$



特殊情况，电路对激励信号载频是调谐的，$\omega_0 = \omega_{01}$

$$Z(j\omega_{01}) = R$$

$$R_{es1} = [(s - p_1)I(s)e^{st}]_{s=p_1} = E_m \frac{\omega_{01}e^{st}}{(s - p_2)z(s)}|_{s=p_1}$$

$$= E_m \frac{\omega_{01}e^{j\omega_0 t}}{2j\omega_{01}R}$$

$$R_{es2} = E_m \frac{\omega_{01}e^{-j\omega_0 t}}{-2j\omega_{01}R}$$

$$i_{sr}(t) = R_{es1} + R_{es2} = \frac{E_m}{R}\sin\omega_{01}tu(t)$$

稳态响应为周期函数，其周期与激励相同

$$\mathrm{R}_{es3}=[(s-p_3)I(s)e^{st}]_{s=p_3}=E_m\frac{\omega_{01}e^{st}}{(s^2+\omega_{01}^2)(s-p_4)L}|_{s=p_3}$$

$$=\frac{E_m\omega_{01}}{L}\frac{(-\alpha+j\omega_d)e^{(-\alpha+j\omega_d)t}}{2j\omega_d(\omega_{01}^2-\omega_d^2+\alpha^2-2j\alpha\omega_d)}$$

$$=\frac{E_m\omega_{01}}{L}\frac{(-\alpha+j\omega_d)e^{(-\alpha+j\omega_d)t}}{2j\omega_d(2\alpha^2-2j\alpha\omega_d)}=\frac{E_m\omega_{01}}{-j2R\omega_d}e^{(-\alpha+j\omega_d)t}$$

高Q值电路时， $\alpha\ll\omega_0$， $\omega_d\approx\omega_0=\omega_{01}$

$$R_{es3}=\frac{E_m}{-j2R}e^{(-\alpha+j\omega_d)t},R_{es4}=R_{es3}{}^*$$

$$i_{tr}(t)=-\frac{E_m}{R}e^{-\alpha t}\sin\omega_{01}tu(t)$$

$$i(t)=i_{sr}(t)+i_{tr}(t)=\frac{E_m}{R}(1-e^{-\alpha t})\sin\omega_0 tu(t)$$

4.11　零极点与频域特性

1、频响特性

– 在正弦信号激励下稳态响应随频率的变化

$$H(j\omega)=\left|H(j\omega)\right|e^{j\varphi(\omega)}$$

幅频特性　　相频特性

其它信号也可得到频响特性，例如冲激信号

$$e(t)=E_m\sin\omega_0 t$$

$$E(s)=\frac{E_m\omega_0}{s^2+\omega_0^2},\text{源极点}p_{1,2}=\pm j\omega_0$$

$H(s)$为稳定系统，极点在左半开平面，对应暂态响应

$$R_s=H(s)E(s)=\frac{E_m\omega_0 H(s)}{s^2+\omega_0^2}$$

$$\mathrm{R}_{ss}(s)=\frac{k_{-j\omega_0}}{s+j\omega_0}+\frac{k_{j\omega_0}}{s-j\omega_0}$$

$$k_{-j\omega_0}=(s+j\omega_0)R(s)\big|_{s=-j\omega_0}=\frac{E_m\omega_0 H(-j\omega_0)}{-2j\omega_0}=\frac{E_m H(-j\omega_0)}{-2j}$$

$$k_{j\omega_0}=(s-j\omega_0)R(s)\big|_{s=j\omega_0}=\frac{E_m\omega_0 H(j\omega_0)}{2j\omega_0}=\frac{E_m H(j\omega_0)}{2j}$$

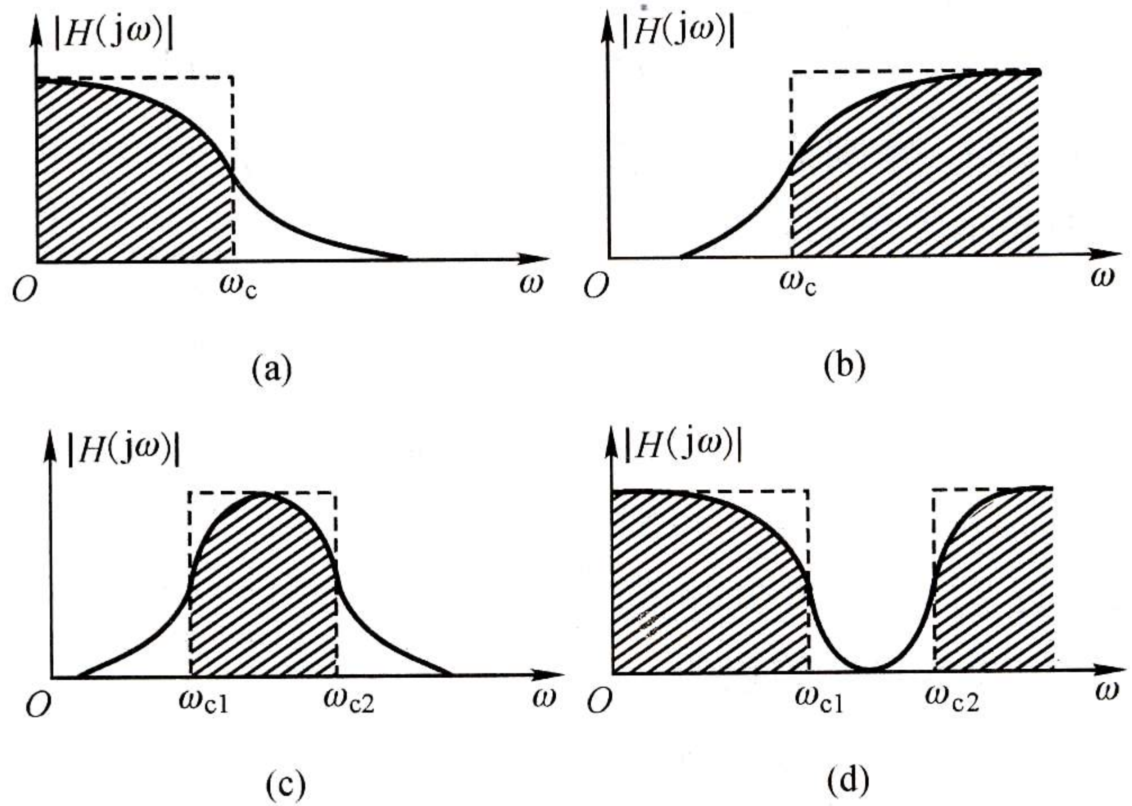
$$H(j\omega_0)=H_0e^{j\varphi_0},H(-j\omega_0)=H_0e^{-j\varphi_0}$$

$$\therefore \mathrm{R}_{ss}(s)=\frac{E_m H_0}{2j}\left(-\frac{e^{-j\varphi_0}}{s+j\omega_0}+\frac{e^{j\varphi_0}}{s-j\omega_0}\right)$$

$$\therefore r_{ss}(t)=E_m H_0\sin(\omega_0 t+\varphi_0)\longrightarrow H(j\omega)=H(s)\big|_{s=j\omega}$$

2、滤波器的滤波特性

– 根据幅频特性的不同，可划分成如下几种



截止频率——下降3dB的频率点

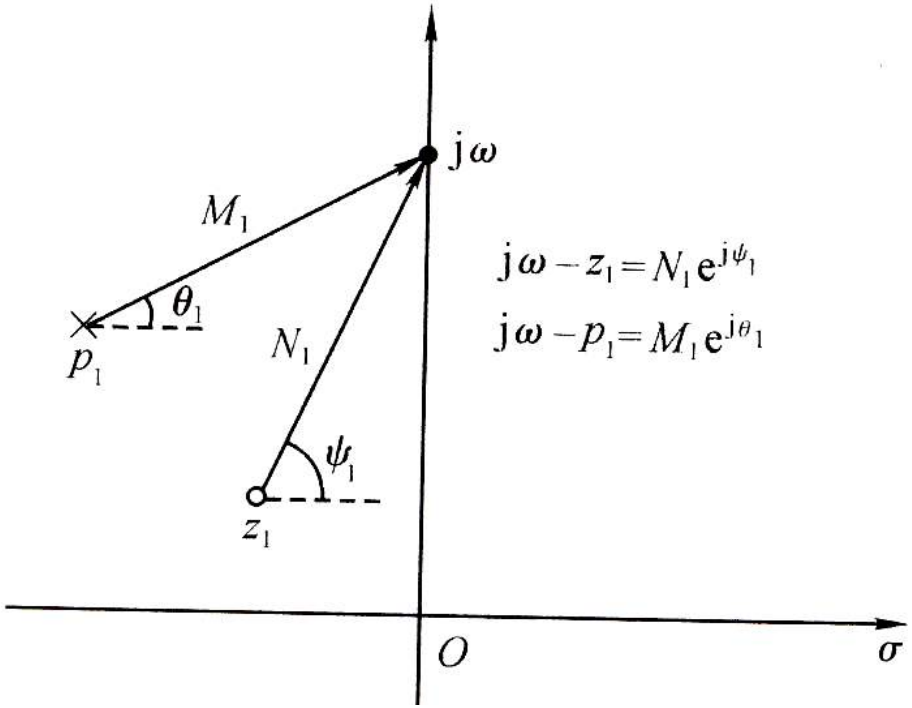
- 常用滤波器
    - Butterworth filter
    - Chebyshev filter

3、零极点决定频响曲线

$$H(s)=K\frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)}$$

s沿虚轴移动$s=j\omega$

$$H(j\omega)=K\frac{\prod_{j=1}^{m}(j\omega-z_j)}{\prod_{i=1}^{n}(j\omega-p_i)}$$



$$j\omega-p_i=M_ie^{j\theta_i}$$

$$j\omega-z_j=N_je^{j\varphi_j}$$

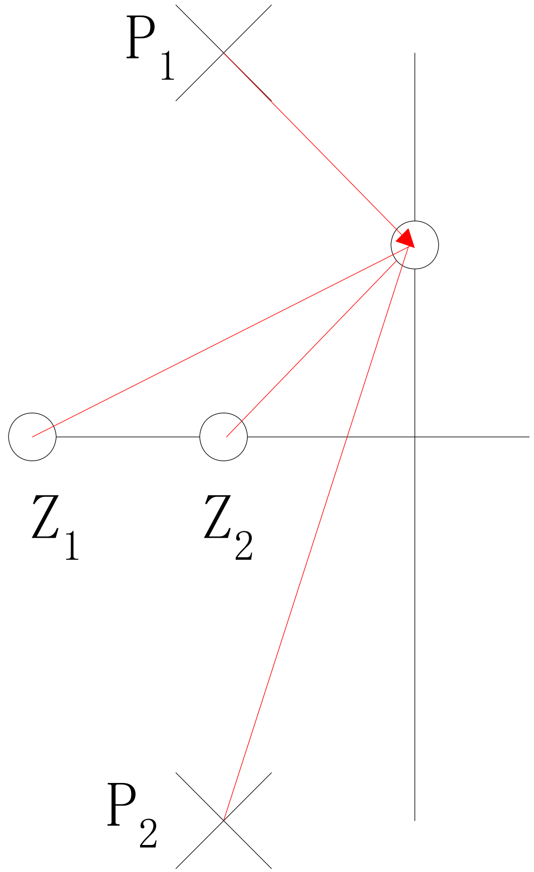
$$\therefore H(j\omega)=K\frac{N_1N_2\ldots N_m}{M_1M_2\ldots M_n}e^{j[(\varphi_1+\varphi_2+\ldots+\varphi_m)-(\theta_1+\theta_2+\ldots+\theta_n)]}$$

$$\therefore |H(j\omega)|=K\frac{N_1N_2\ldots N_m}{M_1M_2\ldots M_n},\varphi(\omega)=(\varphi_1+\varphi_2+\ldots+\varphi_m)-(\theta_1+\theta_2+\ldots+\theta_n)$$

例：已知某系统零极点分布如图，$z_1=-2, z_2=-1$

$p_1=-1+j2, p_2=-1-j2,$

$$H(j\omega)=K\frac{\sum_{j=1}^{2}(s-z_j)}{\sum_{i=1}^{2}(s-p_i)}, K=1, \text{当}\omega=1\text{时，求}H(j\omega)$$



$$M_1=\sqrt{2}, M_2=\sqrt{10}, N_1=\sqrt{5}, N_2=\sqrt{2}$$

$$\varphi_1=26.6, \varphi_2=45, \theta_1=-45, \theta_2=71.6$$

$$|H(j\omega)|=\frac{\sqrt{5}\sqrt{2}}{\sqrt{10}\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\varphi(\omega)=26.6+45-45-71.6=45$$

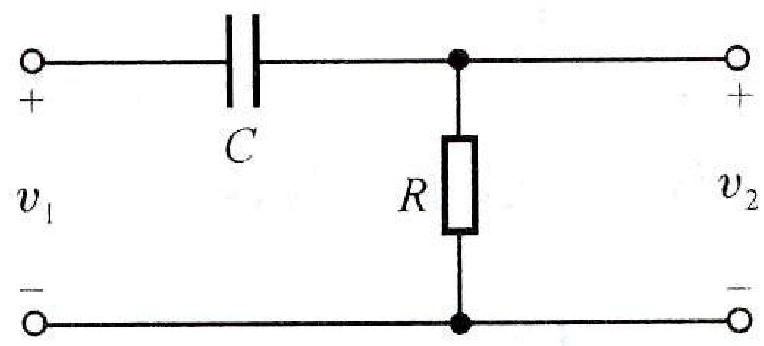
4.12 一阶及二阶系统的S域分析

1、一阶系统

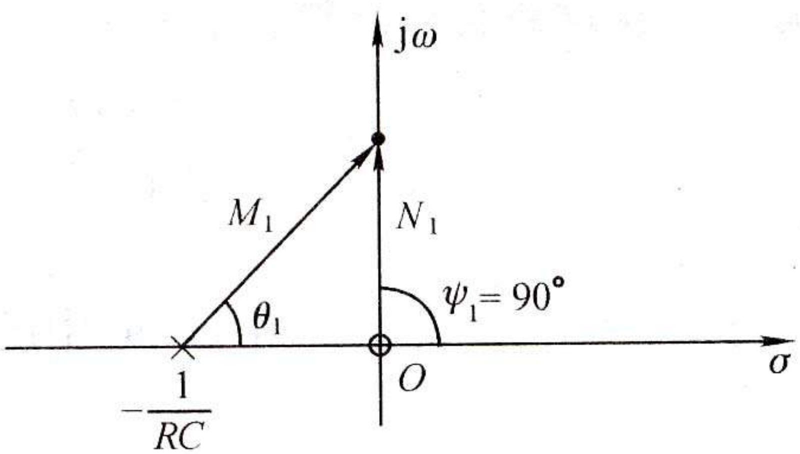
- 只含有一个储能器件，H(s)只有一个极点，且位于实轴上

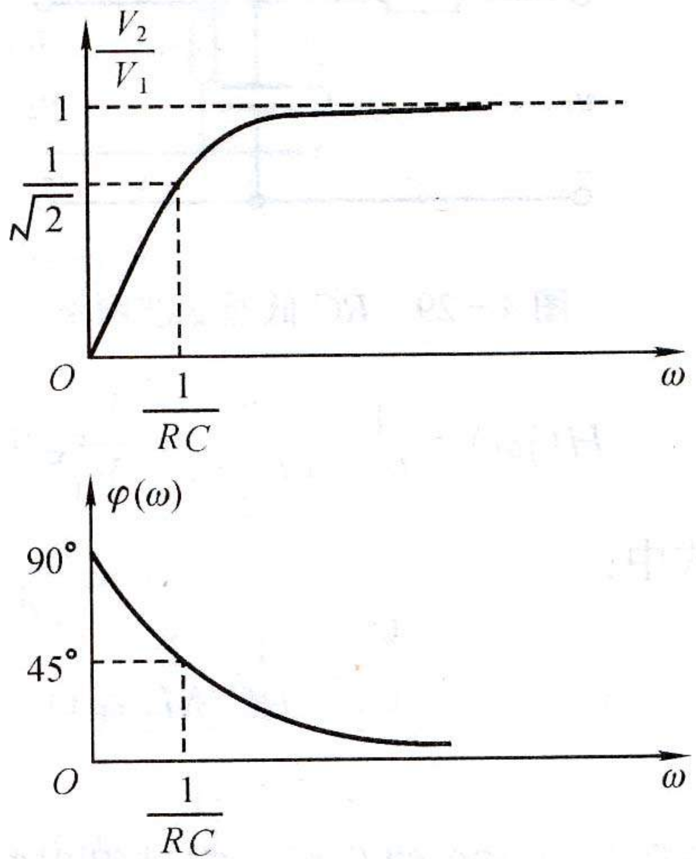
例：RC高通滤波网络的频响特性

$$H(s)=\frac{V_2(s)}{V_1(s)}=\frac{s}{s+1/RC}$$



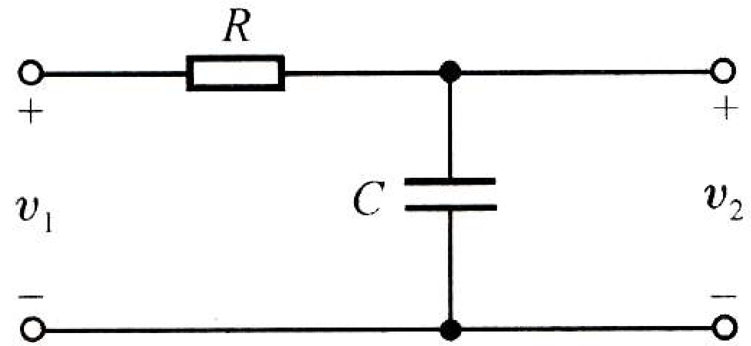
ω	N_1	M_1	N_1/M_1	φ_1	θ_1	$\varphi(\omega)$
0	0	1/RC	0	90	0	90
1/RC	1/RC	$\sqrt{2}/RC$	$\sqrt{2}/2$	90	45	45
∞	∞	∞	1	90	90	0

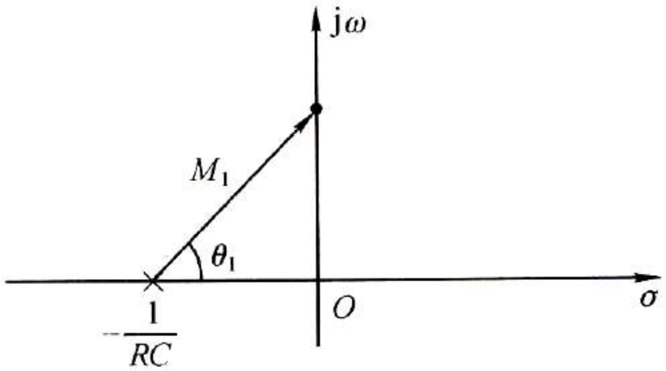




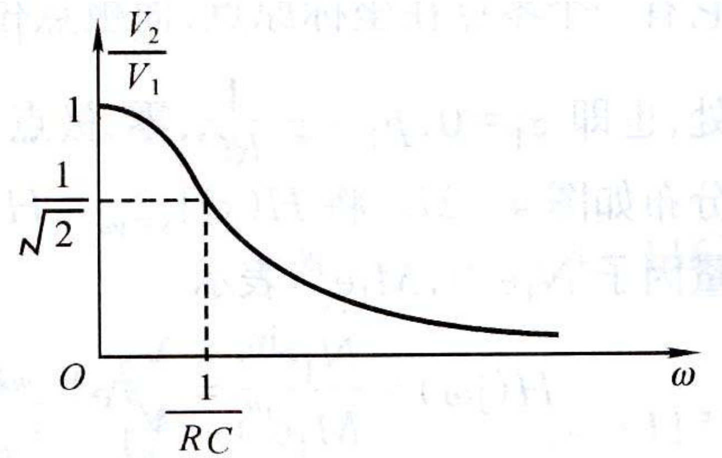
例：RC低通滤波网络的频响特性

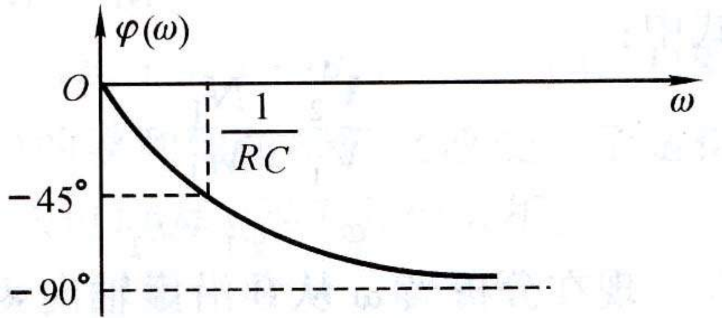
$$H(s)=\frac{V_2(s)}{V_1(s)}=\frac{1/sc}{1/sc+R}=\frac{1}{Rc}\frac{1}{s+1/Rc}$$





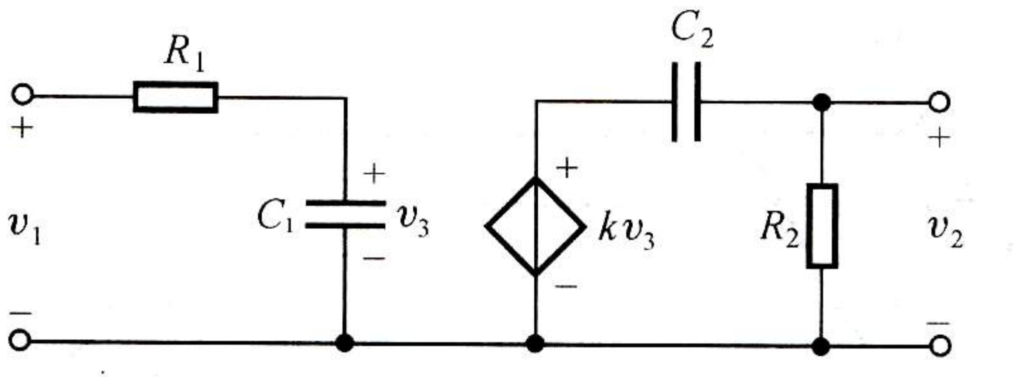
ω	M_1	$\|H(j\omega)\|$	θ_1	φ(ω)
0	1/RC	1	0	0
1/RC	$\sqrt{2}/RC$	$\sqrt{2}/2$	45	-45
∞	∞	0	90	-90





例：二阶RC滤波网络的频响特性，图中kv_3为受控电压源，且$R_1c_1 << R_2c_2$

$$H(s)=\frac{V_2(s)}{\mathrm{V}_1(s)}=\frac{k\frac{1}{sc_1}}{R_1+\frac{1}{sc_1}}\frac{R_2}{R_2+\frac{1}{sc_2}}=\frac{k}{R_1c_1}\frac{s}{(s+\frac{1}{R_1c_1})(s+\frac{1}{R_2c_2})}$$

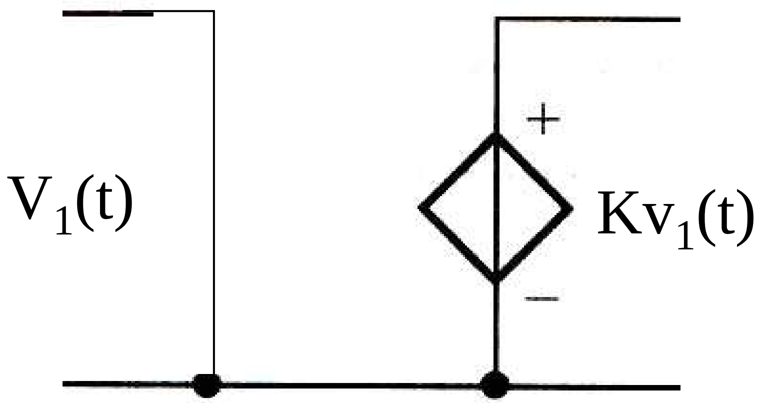


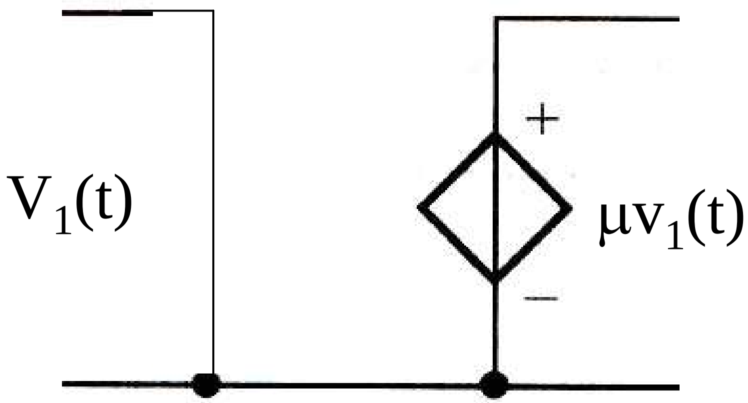
LP　　　　HP

受控源：

电压控制的电压源$v_2(t)=kv_1(t)$

电压控制的电流源$i_2(t)=\mu v_1(t)$

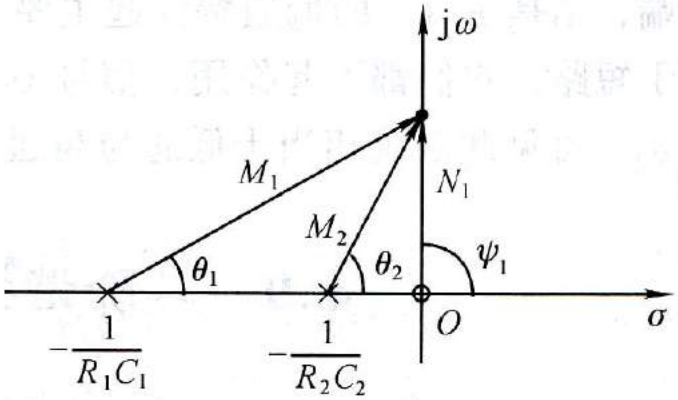




$$H(s)=\frac{V_2(s)}{V_1(s)}=\frac{k\dfrac{1}{sc_1}}{R_1+\dfrac{1}{sc_1}}\frac{R_2}{R_2+\dfrac{1}{sc_2}}$$

$$=\frac{k}{R_1c_1}\frac{s}{(s+\dfrac{1}{R_1c_1})(s+\dfrac{1}{R_2c_2})}$$

$$H(j\omega)=\frac{k}{R_1c_1}\frac{N_1}{M_1M_2}e^{j(\varphi_1-\theta_1-\theta_2)}$$



$$H(j\omega)=\frac{k}{R_1c_1}\frac{N_1}{M_1M_2}e^{j(\varphi_1-\theta_1-\theta_2)}$$

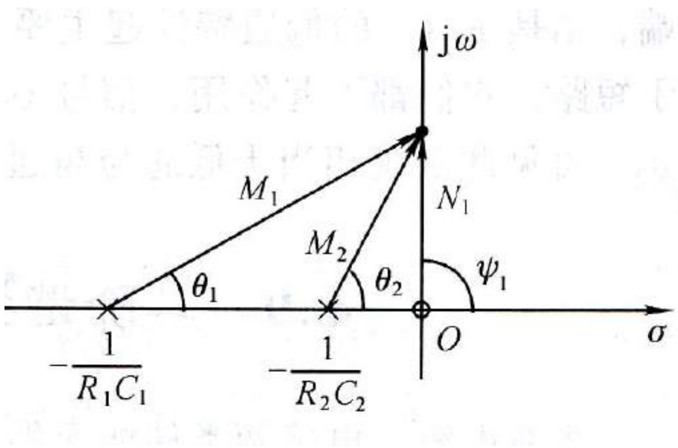
(1)ω较小时，$R_1c_1 << R_2c_2 \therefore M_1 \approx 1/R_1c_1, \theta_1 \approx 0$

特性主要由$N_1, M_2, \varphi_1, \theta_2$决定，$HP$特性

(2)ω很大时，$N_1, M_2, \varphi_1, \theta_2$的作用抵消，只有一个极点,这时为$LP$特性

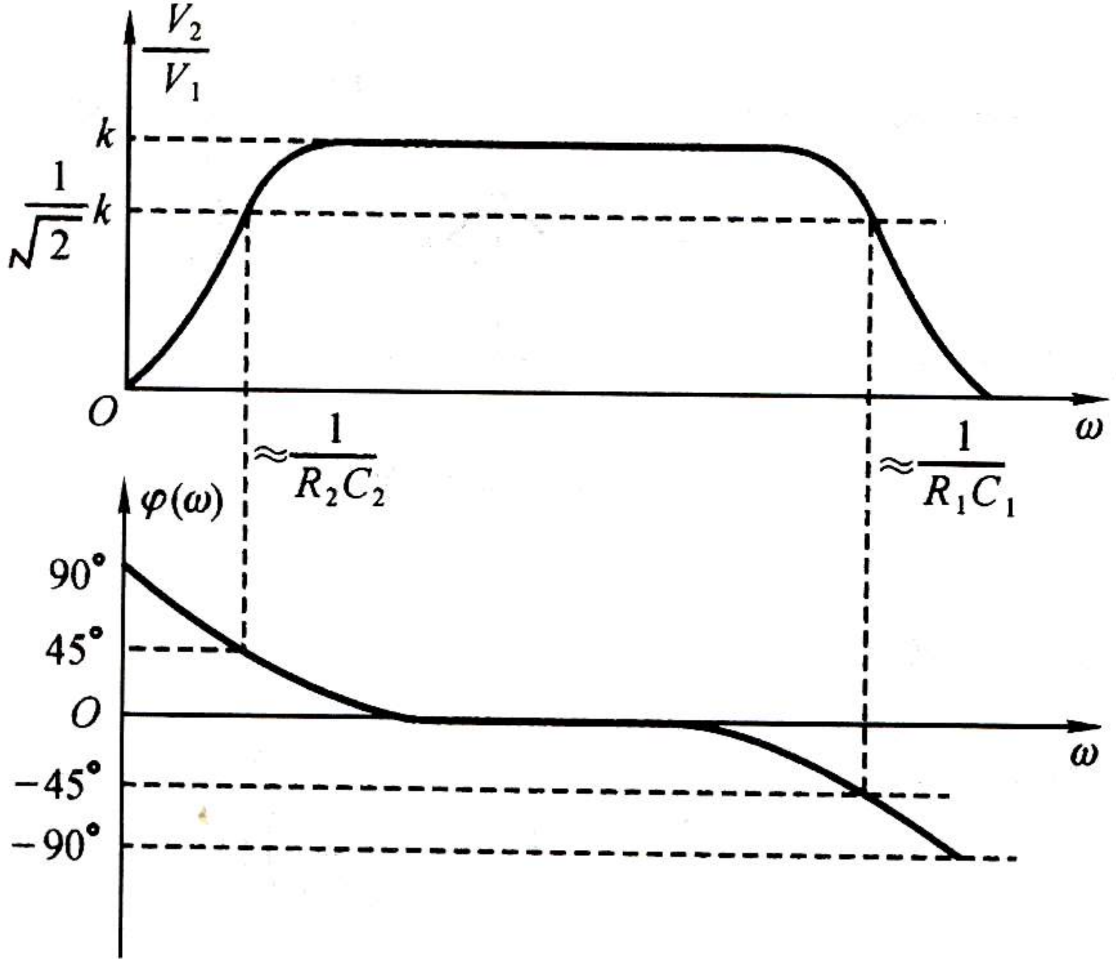
(3)ω在中频段时，设$\omega_1=\frac{1}{2R_1c_1}, \left| j\omega+\frac{1}{R_2c_2}\right| \approx \frac{1}{2R_1c_1}$

$$|H(j\omega_1)| \approx \frac{k}{R_1c_1}\frac{\frac{1}{2R_1c_1}}{\frac{\sqrt{5}}{2}\frac{1}{R_1c_1}\frac{1}{2R_1c_1}} \approx k$$



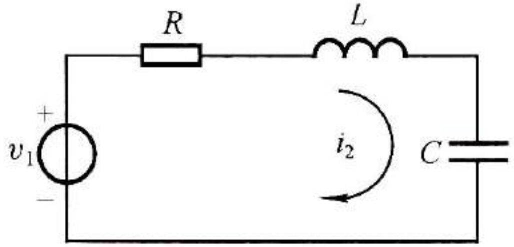
$$\omega_2=\frac{1}{3R_1c_1}, |H(j\omega_2)| \approx \frac{k}{R_1c_1}\frac{\frac{1}{3R_1c_1}}{\frac{\sqrt{10}}{3}\frac{1}{R_1c_1}\frac{1}{3R_1c_1}} \approx k$$

$\therefore$中频段是平坦特性

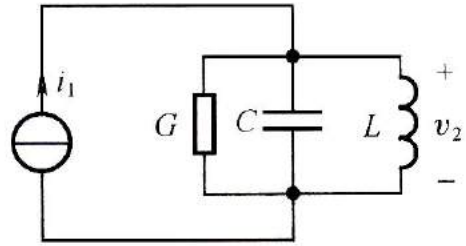


V2/V1
k
1/√2 k
O
ω
≈1/(R2C2)
≈1/(R1C1)
φ(ω)
90°
45°
O
ω
−45°
−90°

2、二阶谐振系统



(a)

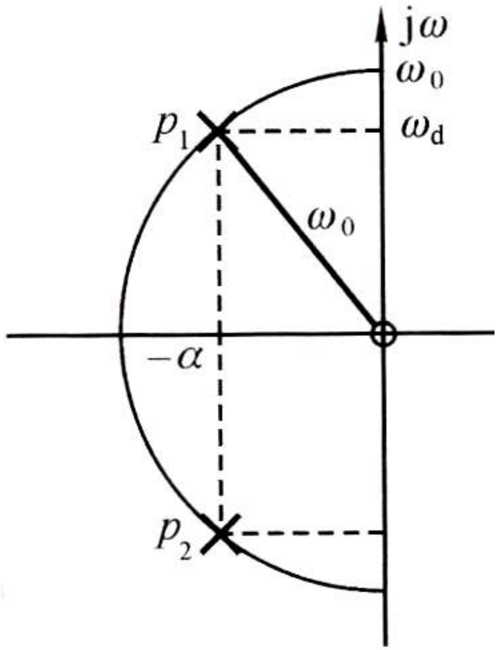


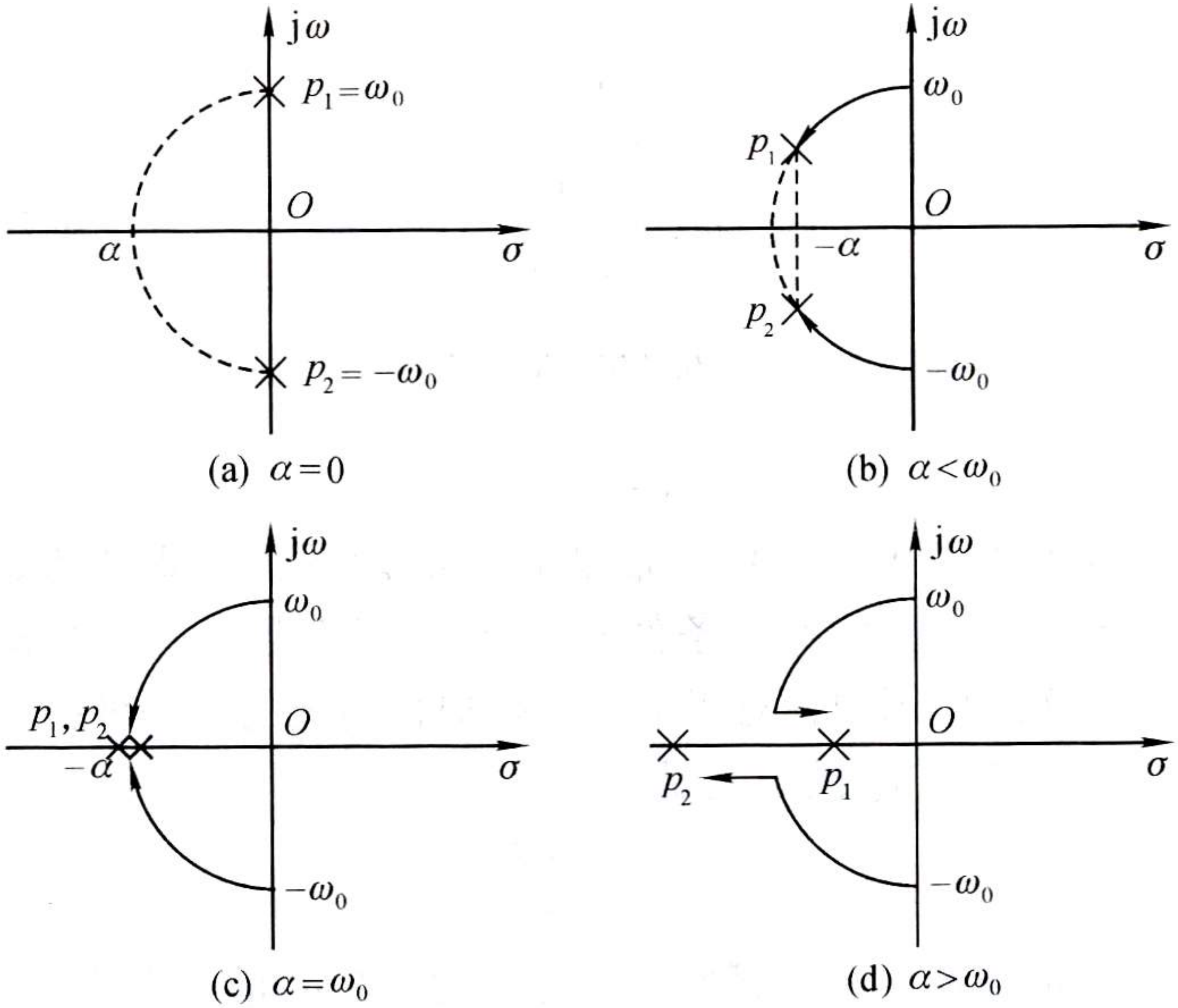
(b)

$$H(s)=\frac{V(s)}{I(s)}=\frac{1}{sc+1/R+1/sL}=\frac{1}{c}\frac{s}{s^2+\frac{1}{Rc}s+\frac{1}{Lc}}$$

$$p_{1,2}=-\frac{1}{2Rc}\pm j\sqrt{\frac{1}{Lc}-(\frac{1}{2Rc})^2}=-\alpha\pm j\sqrt{\omega_0^2-\alpha^2}=-\alpha\pm j\omega_d$$

$$H(s)=\frac{1}{c}\frac{s}{(s-p_1)(s-p_2)}$$




jω
$p_1=\omega_0$
O
α
σ
$p_2=-\omega_0$
(a) $\alpha=0$
jω
ω_0
p_1
O
$-\alpha$
σ
p_2
$-\omega_0$
(b) $\alpha<\omega_0$
jω
ω_0
p_1, p_2
O
$-\alpha$
σ
$-\omega_0$
(c) $\alpha=\omega_0$
jω
ω_0
O
σ
p_2
p_1
$-\omega_0$
(d) $\alpha>\omega_0$

损耗变化
极点变化

$$H(j\omega) = \frac{1}{c}\frac{N_1}{M_1 M_2} e^{j(\varphi_1 - \theta_1 - \theta_2)} = |H(j\omega)| e^{j\varphi(\omega)}$$

$\alpha < \omega_0$(耗损较小）

(1)$\omega = 0,$

$$N_1 = 0, M_1 = M_2 = \omega_0, \theta_1 = -\theta_2, \varphi_1 = 90^0$$

$$\therefore |H(j\omega)| = 0, \varphi(\omega) = 90^0$$

(2)$\omega = \omega_0,$

$$M_1 M_2' = 2\alpha \cdot N_1,$$

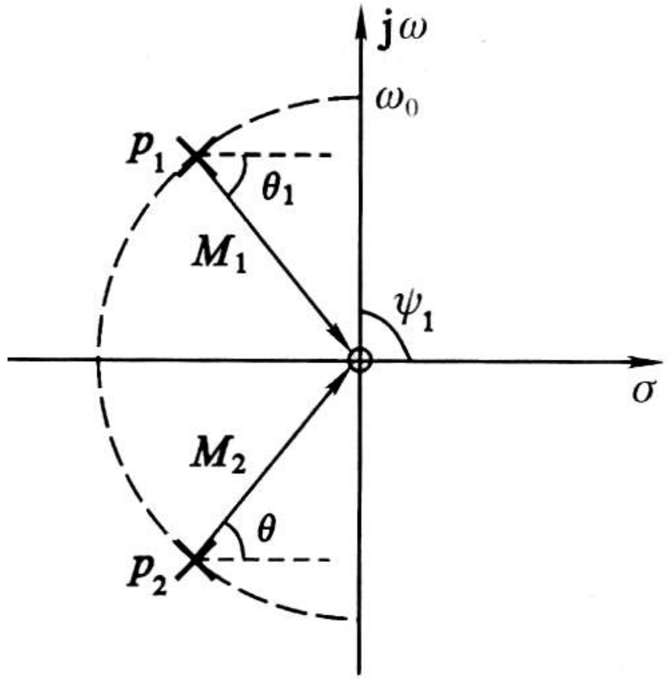
$$\therefore \frac{N_1}{cM_1M_2} = \frac{1}{c \cdot 2\alpha} = R$$

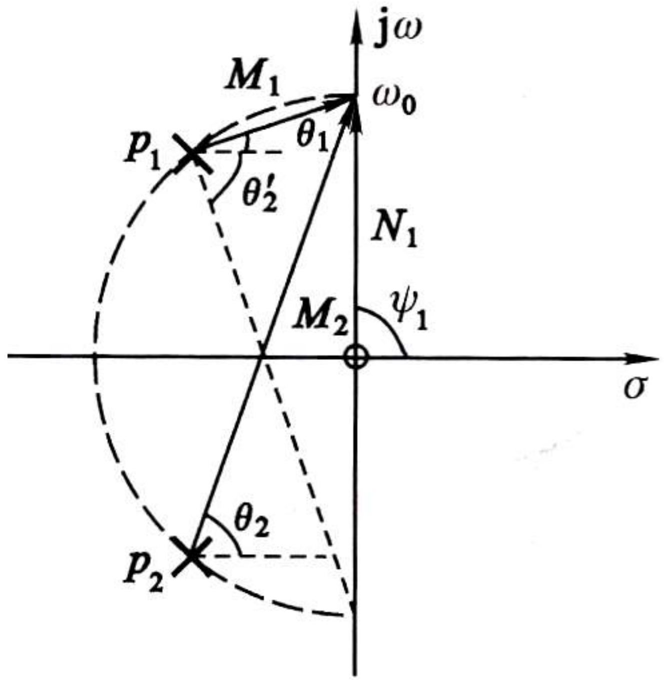
$$\varphi = \varphi_1 - \theta_1 - \theta_2 = 90 - 90 = 0$$

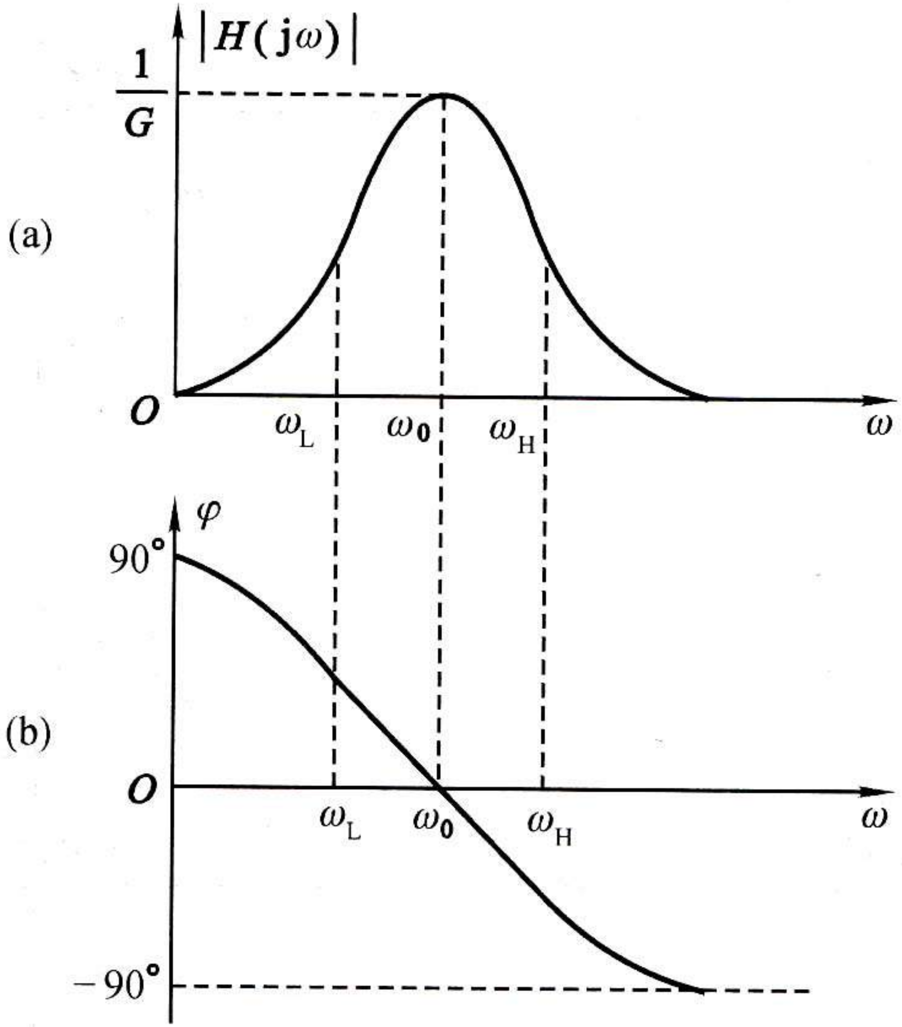
(3) ω很大时，

N_1, M_1, M_2近似相等∞

$$\therefore |H(j\omega)| = 0, \varphi(\omega) = 90^0 - 90^0 - 90^0 = -90^0$$







|H(jω)|
1/G
(a)
O
ω_L
ω_0
ω_H
ω
φ
90°
(b)
O
ω_L
ω_0
ω_H
ω
−90°

$$Q=\frac{\omega_0}{2\alpha},\text{高Q值}$$

$$\omega \approx \omega_0,$$

$$\because N_1=\omega_0,\varphi_1=90,M_2\approx 2\omega_0,\theta_2\approx 90$$

$$M_1 e^{j\theta_1}=\alpha+j(\omega-\omega_d)\approx \alpha+j(\omega-\omega_0)$$

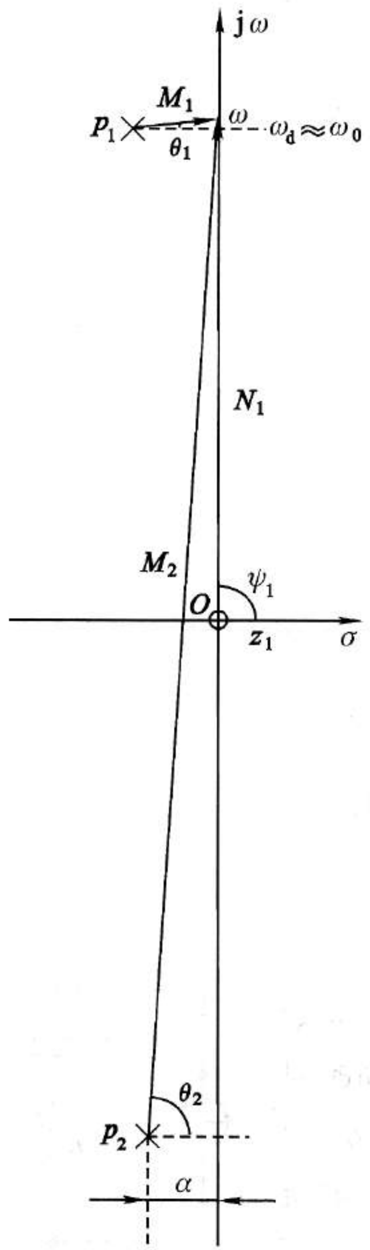
$$\therefore H(j\omega)=\frac{1}{c}\frac{\omega_0}{2\omega_0[\alpha+j(\omega-\omega_0)]}$$

$$=\frac{1}{2\alpha c}\frac{1}{[1+j\dfrac{\omega-\omega_0}{\alpha}]}=R\frac{1}{[1+j\dfrac{\omega-\omega_0}{\alpha}]}$$

$$\therefore \left|H(j\omega)\right|=\frac{R}{\sqrt{1+\dfrac{(\omega-\omega_0)^2}{\alpha^2}}},\varphi(\omega)=-tg^{-1}\frac{\omega-\omega_0}{\alpha}$$

$$(1)\omega=\omega_0,\left|H(j\omega)\right|=R,\varphi(\omega)=0$$

$$(2)\omega=\omega_0\pm\alpha,\left|H(j\omega)\right|=\frac{R}{\sqrt{2}},\varphi(\omega)=\pm 45^0$$



$$H(j\omega)=K\frac{\prod_{j=1}^{m}(j\omega-z_j)}{\prod_{i=1}^{n}(j\omega-p_i)}$$

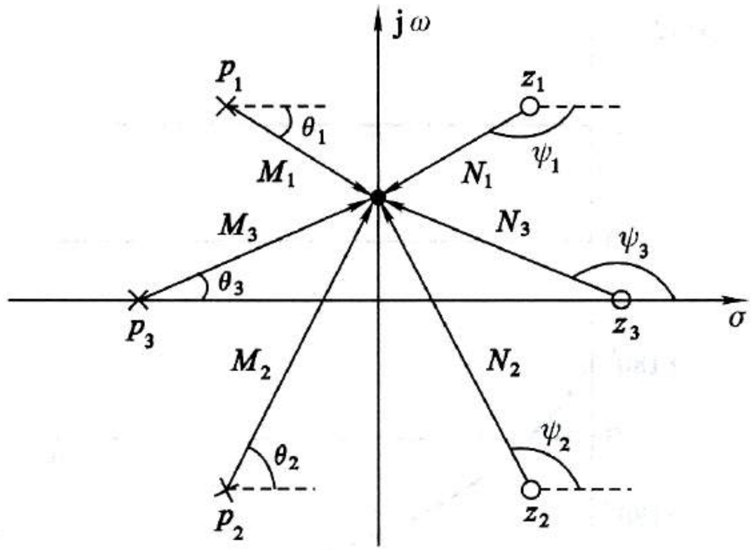
当极点在S平面的虚轴上时，出现极大值，频响曲线出现峰值

当零点在S平面的虚轴上时，出现极小值，频响曲线出现谷值

4.13 全通函数及最小相移函数

1、全通函数

- 系统的幅频特性不是频率的函数，是常数
- 任意幅频特性的信号经过该系数，幅频特性不会变化
- 全通函数的零极点分布
  - 极点在S左半平面，零点在右半平面
  - 极点数＝零点数，且与虚轴成镜像对称



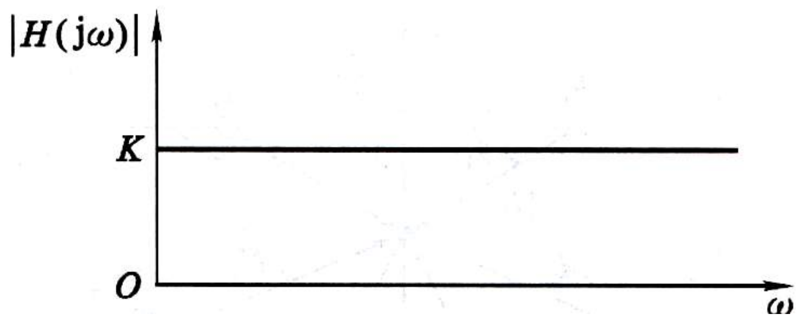
$$\frac{N_1N_2N_3}{M_1M_2M_3}=1$$

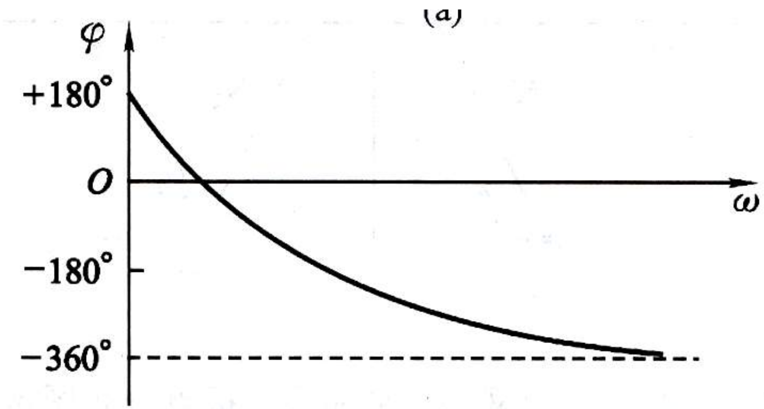
$$\therefore \left|H(j\omega)\right|=k\frac{N_1N_2N_3}{M_1M_2M_3}=k$$

(1)$\omega=0,\theta_1=-\theta_2,\varphi_1=-\varphi_2,\theta_3=0,\varphi_3=180\therefore\varphi=180$

(2)ω向上移动,$\theta_2,\theta_3\uparrow$， $\varphi_2,\varphi_3\downarrow,\theta_1$由负变正,$\varphi_1$更负,$\therefore\varphi\downarrow$

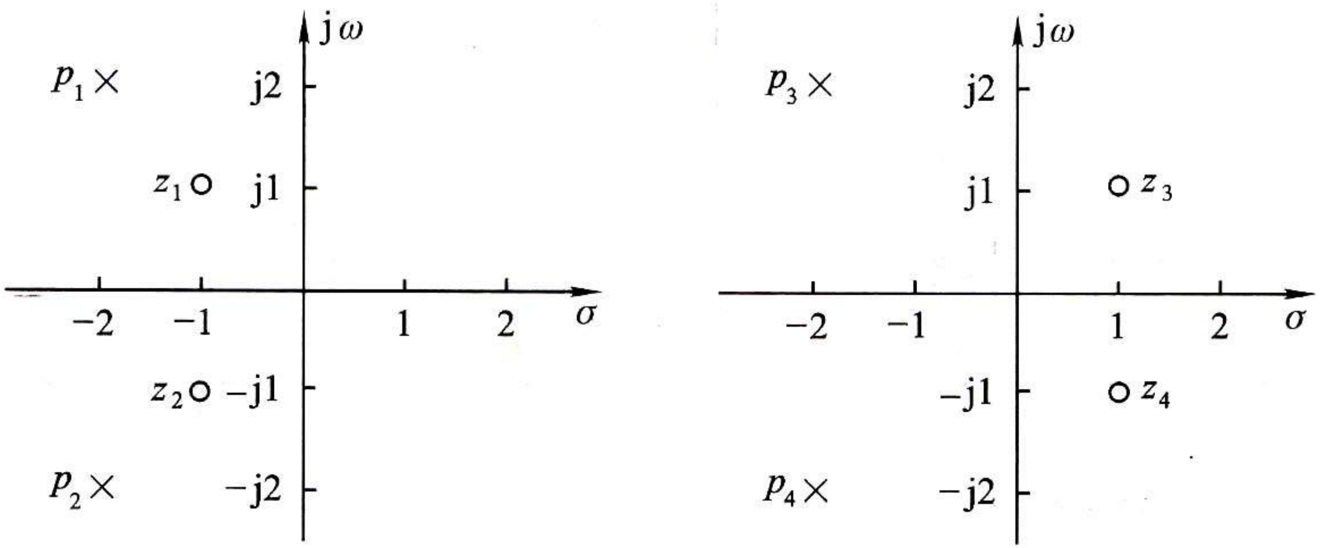
(3)$\omega=\infty,\theta_1=\theta_2=\theta_3=90,\varphi_1=-270,\varphi_2=\varphi_3=90\therefore\varphi=-360$



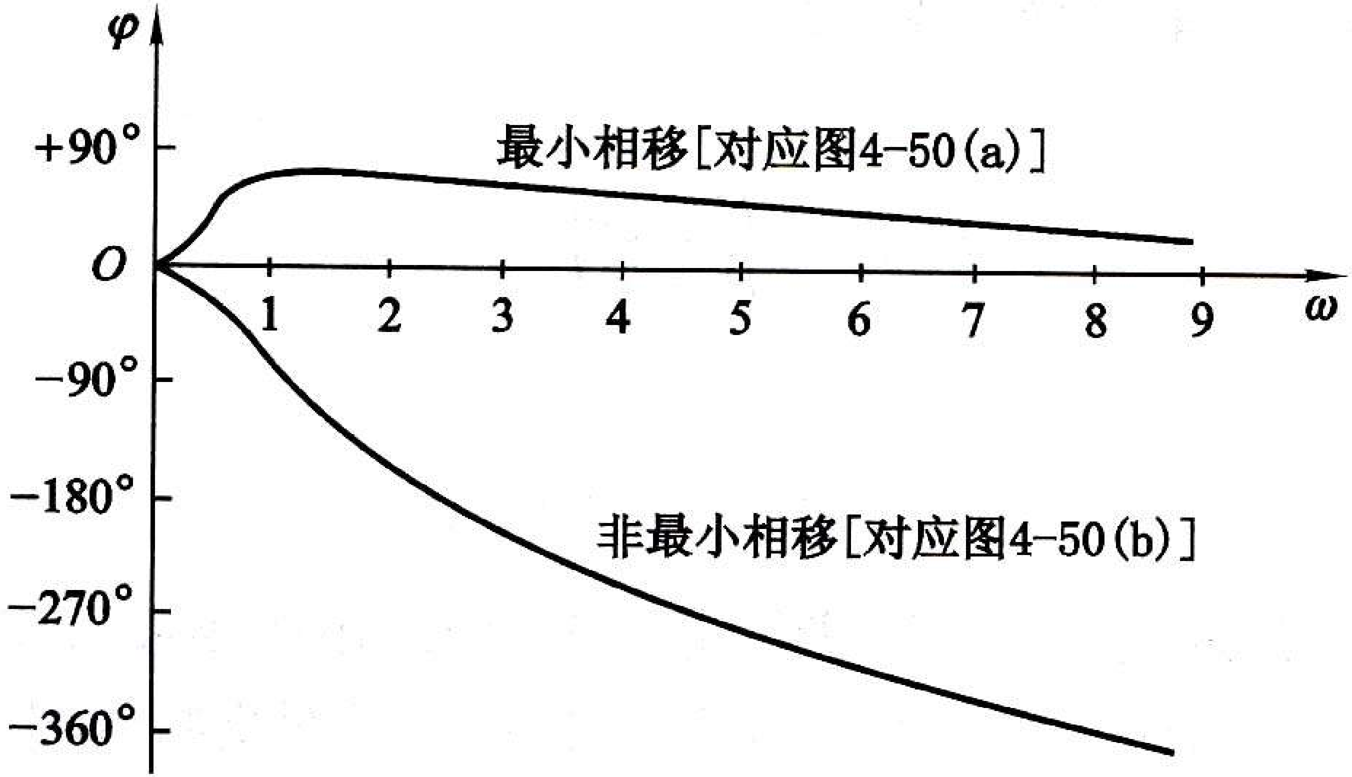


2、最小相移函数

– 系统函数的零极点都在S左半平面，零点可在虚轴上，具有最小相移



(a) (b)


φ
+90°
O
-90°
-180°
-270°
-360°
1
2
3
4
5
6
7
8
9
ω
最小相移[对应图4-50(a)]
非最小相移[对应图4-50(b)]

4.14 系统的稳定性

1、稳定系统

- 有限（界）激励，产生有限（界）激励
- 有限（界）激励，产生无限（界）激励，为不稳定系统

$$r(t)=h(t)*e(t)=\int_{-\infty}^{\infty}h(\tau)e(t-\tau)d\tau$$

$$e(t)\le Me$$

$$r(t)\le Me\int_{0}^{\infty}h(\tau)d\tau$$

要使$r(t)$有界，则$\int_{0}^{\infty}h(\tau)d\tau<\infty$

$$\therefore \lim_{t\to\infty}h(t)=0$$

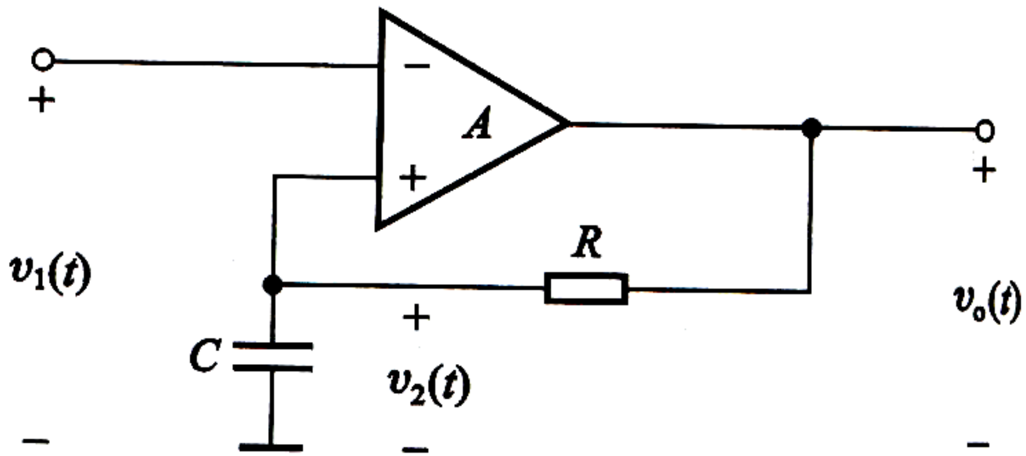
2、系统稳定的条件

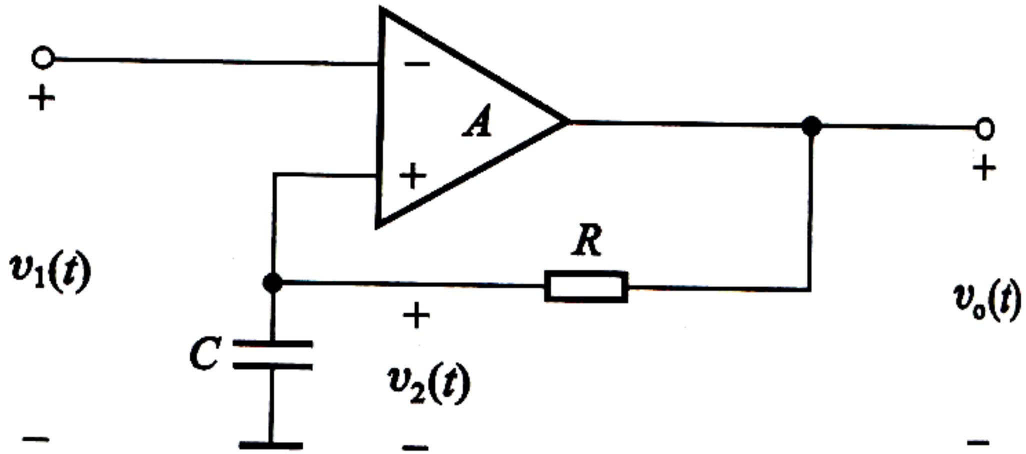
- H(s)全部极点在s左半开平面，稳定
- H(s)的极点在右半开平面，或虚轴上有二阶极点，不稳定
- H(s)虚轴上单极点，边界稳定

例：如图放大器的输入阻抗为无限大，输出信号$V_O(s)$与差分输入信号$V_1(s)$和$V_2(s)$之间满足关系式：

$V_o(s) = A[V_2(s) - V_1(s)]$，求：

（1）$\mathrm{H}(s) = \dfrac{V_o(s)}{V_1(s)}$ (2)A满足什么条件，系统稳定？





$$\frac{V_2(s)}{V_o(s)}=\frac{1/sc}{R+1/sc}$$

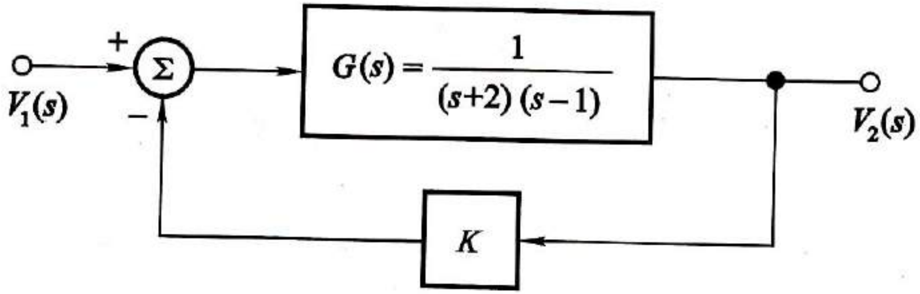
$$V_o(s)=A[V_2(s)-V_1(s)]=\frac{1/sc}{R+1/sc}AV_o(s)-AV_1(s)$$

$$\therefore H(s)=\frac{V_o(s)}{V_1(s)}=-\frac{(s+1/Rc)A}{s+\dfrac{1-A}{Rc}}$$

要使系统稳定,$(1-A)/Rc>0$

$\therefore A<1$

例：如图线性反馈系统，讨论当K从0增长时，系统稳定性的变化？



$$V_2(s)=[V_1(s)-kV_2(s)]G(s)$$

$$\frac{V_2(s)}{V_1(s)}=\frac{G(s)}{1+kG(s)}=\frac{1}{s^2+s-2+k}$$

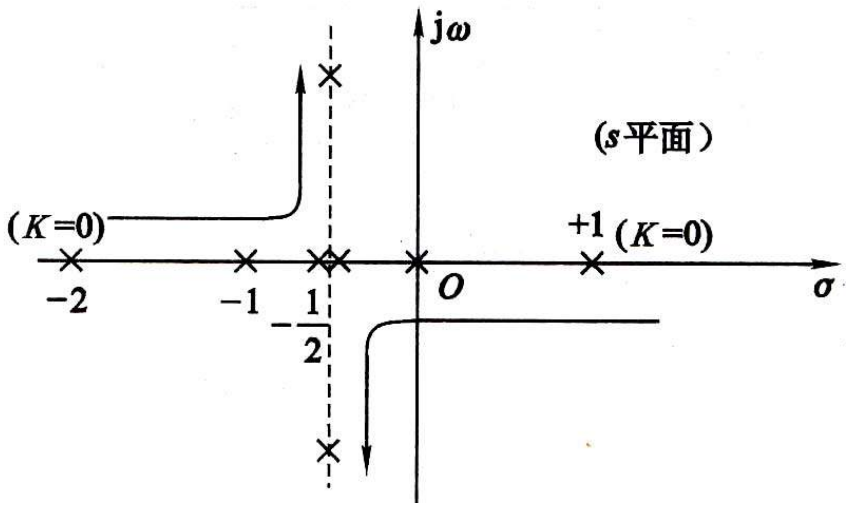
$$p_{1,2}=-\frac{1}{2}\pm\sqrt{\frac{9}{4}-k}$$

$$k=0, p_1=-2, p_2=1$$

$$k=2, p_1=-1, p_2=0$$

$$k=9/4, p_1=p_2=-1/2$$

$\therefore k>2$稳定$,k=2$边界稳定$,k<2$不稳定



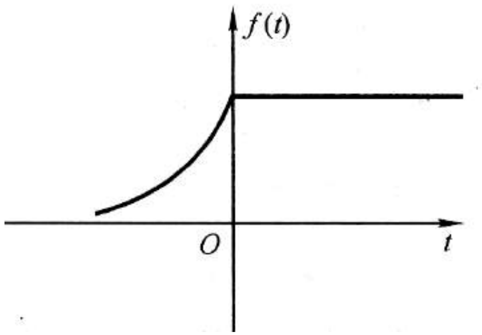
4.15　　双边LT及LT与FT关系

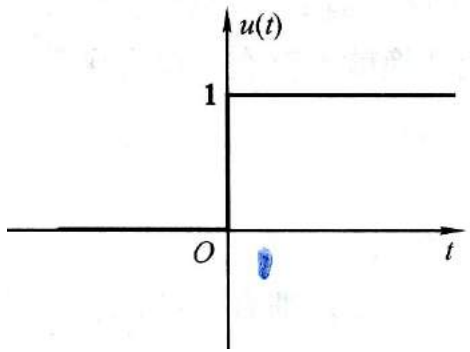
1、双边LT定义及收敛域

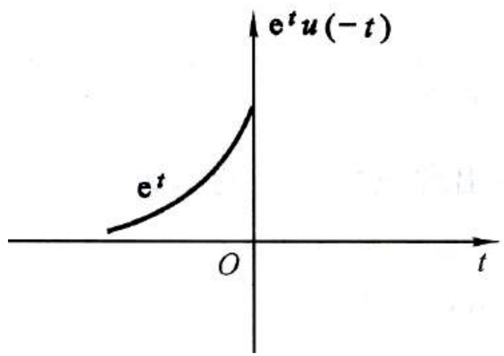
$$F_B(s)=\int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$BLT\text{存在的条件}:\int_{-\infty}^{\infty} f(t)e^{-st}dt<\infty$$

例：$f(t)=u(t)+e^{t}u(-t)$,试求BLT



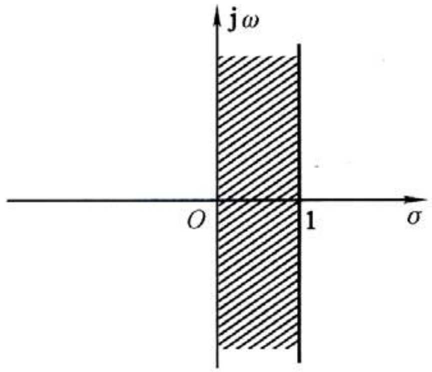




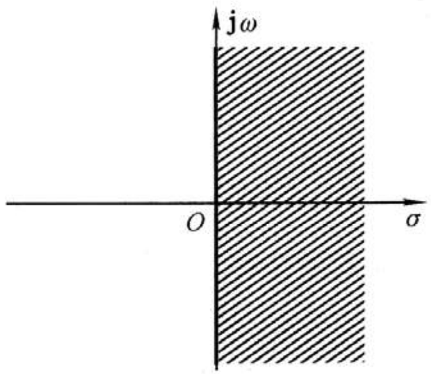
$$F_B(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$= \int_{0}^{\infty} e^{-st}dt + \int_{-\infty}^{0} e^{(1-s)t}dt$$

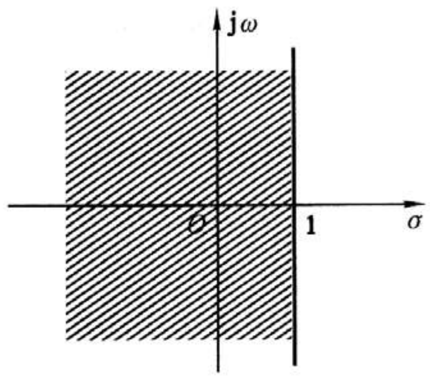
$$= \frac{1}{s} + \frac{1}{1-s}$$



(d)



(e)



(f)

同样的F(s),收敛域不同，其ILT不同

F(s)	ROC	f(t)
1/s	σ>0	u(t)
1/s	σ<0	-u(-t)
1/(s+a)	σ>-a	e^{-at} u(t)
1/(s+a)	σ<-a	$-e^{at}$ u(-t)

$$F(s)=\frac{1}{s}+\frac{1}{s+\alpha}$$

$$-\alpha<\sigma<0 \qquad f(t)=-u(-t)+e^{-\alpha t}u(t)$$

$$\sigma>0 \qquad f(t)=u(t)+e^{-\alpha t}u(t)$$

$$\sigma<-\alpha \qquad f(t)=-u(-t)-e^{-\alpha t}u(-t)$$

2、LT与FT关系

– 双边LT的ROC包括虚轴

$$F(j\omega)=F(s)|_{s=j\omega}$$

– t<0,f(t)=0,双边LT->单边LT， ROC包括虚轴

$$F(j\omega)=F(s)|_{s=j\omega}$$

– 若收敛边界在虚轴上，F(s)极点在虚轴上

$$F(j\omega)=F(s)|_{s=j\omega}+\pi\sum_{n}k_n\delta(\omega-\omega_n)$$

$$k_n=(s-j\omega_n)F(s)|_{s=j\omega_n}$$

例：

$$L[\cos\omega_0 t u(t)] = \frac{s}{s^2+\omega_0^2}$$

$$F[\cos\omega_0 t u(t)] = \frac{j\omega}{\omega_0^2-\omega^2} + \frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$$

作　　业

4-1 (5) (8) (13) (17) (20)		4-2
4-3 (4) (5)		4-4 (14) (16) (20)
4-5 (1) (2)	4-11	4-12
4-13 (C)	4-15	4-20
4-24 (b) (c)	4-26 (a) (d)	4-29
4-30	4-35	4-37
4-39 (c) (e)	4-42	4-45
4-48	4-50	