第五章

FT的应用

5.1 引言

1、已学习过的内容

- FT、LT、连续时间系统的时域及复频域分析
  - FT：无数等幅振荡之和($e^{j\omega t}$)
  - LT：无数变幅振荡之和(e^{st})
- FS,FT,信号频谱，FT的性质

2、本章研究内容

- 利用H(jω)求系统响应
- 无失真传输条件
- 理想LP滤波器及物理可实现性

5.2 利用H(jω)求系统响应

- 对于稳定系统：

$$H(j\omega) = H(s)|_{s=j\omega}$$

- 典型例子讨论信号通过系统的响应变化

例：矩形脉冲通过RC低通网络

S域求解：

$$H(s)=\frac{1/sc}{R+1/sc}=\frac{\alpha}{s+\alpha}$$

$$V_1(s)=\frac{E}{s}(1-e^{-st})$$

$$V_2(s)=V_1(s)H(s)=E(\frac{1}{s}-\frac{1}{s+\alpha})(1-e^{-st})$$

$$\therefore v_2(t)=E[u(t)-u(t-\tau)]$$
$$-E[e^{-\alpha t}u(t)-e^{-\alpha(t-\tau)}u(t-\tau)]$$

$$F[u(t)]=\frac{1}{j\omega}+\pi\delta(\omega)$$

$$F[u(t-\tau)]=[\frac{1}{j\omega}+\pi\delta(\omega)]e^{-j\omega\tau}=\frac{1}{j\omega}e^{-j\omega\tau}+\pi\delta(\omega)$$

$$F[u(t)-u(t-\tau)]=\frac{1}{j\omega}(1-e^{-j\omega\tau})$$

频域求解：

$$H(j\omega)=\frac{1/j\omega c}{R+1/j\omega c}=\frac{\alpha}{j\omega+\alpha}$$

$$V_1(j\omega)=F[v_1(t)]=E\tau Sa(\frac{\omega\tau}{2})e^{-j\omega\tau/2}$$

$$V_2(j\omega)=V_1(j\omega)H(j\omega)=|V_2(j\omega)|e^{j\varphi_2(\omega)}$$

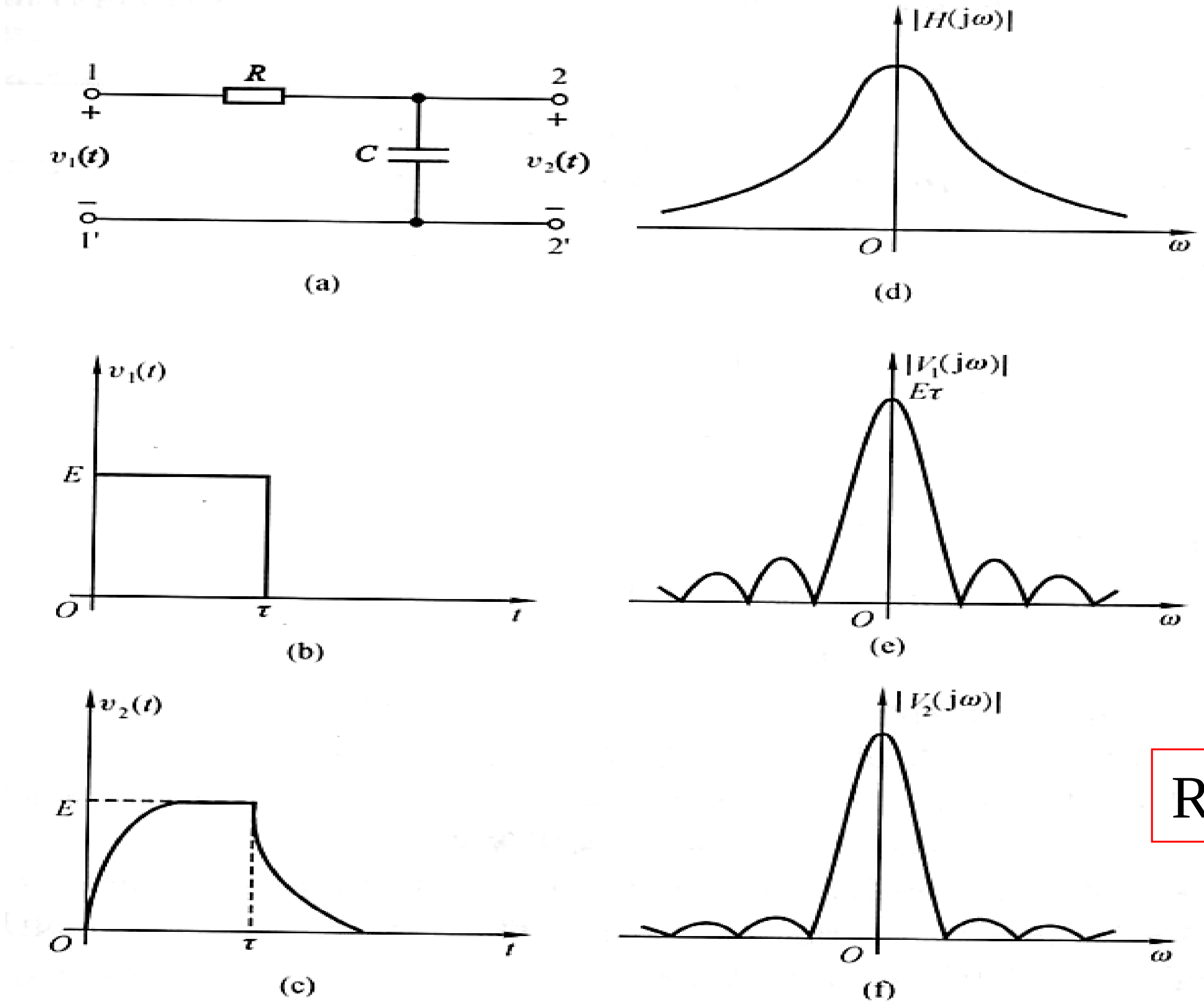
$$|V_2(j\omega)|=\frac{2\alpha E}{\omega}\frac{|\sin(\omega\tau/2)|}{\sqrt{\alpha^2+\omega^2}}$$

$$\varphi_2(\omega)=-\frac{\omega\tau}{2}-tg^{-1}(\frac{\omega}{\alpha})\begin{matrix}+(\pm\pi) & \sin(\omega\tau/2)\text{为负}\\ 0 & \sin(\omega\tau/2)\text{为正}\end{matrix}$$

$$V_1(j\omega)=\frac{E}{j\omega}(1-e^{-j\omega\tau})$$

$$V_2(j\omega)=\frac{\alpha}{\alpha+j\omega}\frac{E}{j\omega}(1-e^{-j\omega\tau})$$
$$=\frac{E}{j\omega}(1-e^{-j\omega\tau})-\frac{E}{j\omega+\alpha}(1-e^{-j\omega\tau})$$

$$\therefore v_2(t)=E[u(t)-u(t-\tau)]$$
$$-E[e^{-\alpha t}u(t)-e^{-\alpha(t-\tau)}u(t-\tau)]$$



RC变小

经过系统后，方波信号的前后沿发生了变化，陡峭的前后沿发生了指数上升及指数下降——LP滤除高频成分

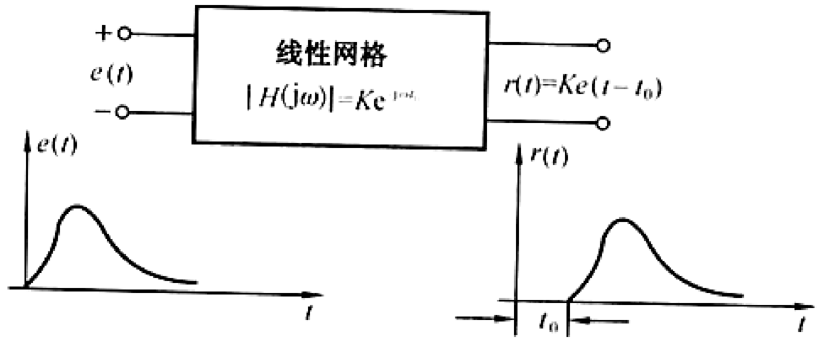
5.3　无失真传输

1、失真－系统的响应波形与激励波形不同

- 非线性失真：高次谐波
- 线性失真：不产生新的频率成份
  - 幅度失真
  - 相位失真

2、无失真传输

- 系统的响应是激励信号的精确再现



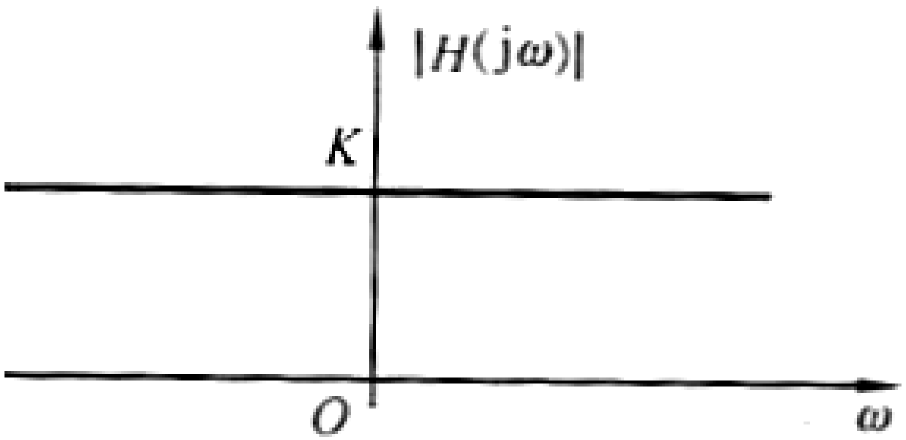
– 无失真传输的条件

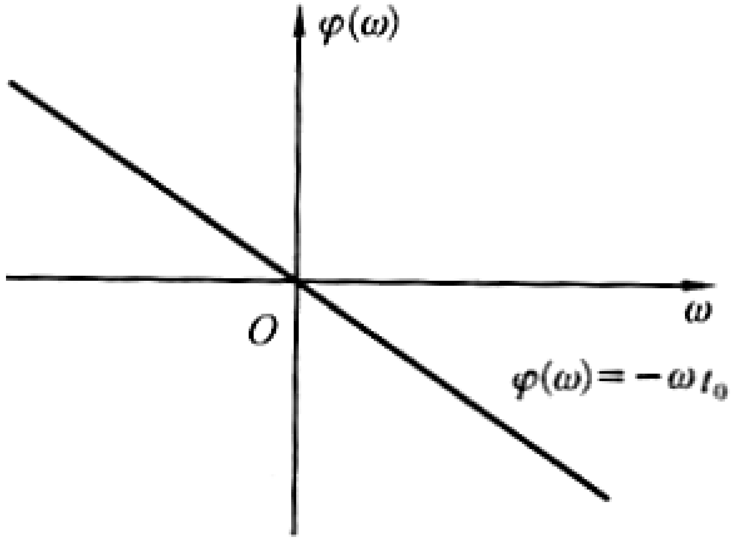
$$r(t) = Ke(t - t_0)$$

$$R(j\omega) = kE(j\omega)e^{-j\omega t_0}, R(j\omega) = E(j\omega)H(j\omega)$$

$$\therefore H(j\omega) = ke^{-j\omega t_0}$$

$$\therefore \left|H(j\omega)\right| = k, \varphi(\omega) = -\omega t_0$$





例：无失真传输与失真传输的比较

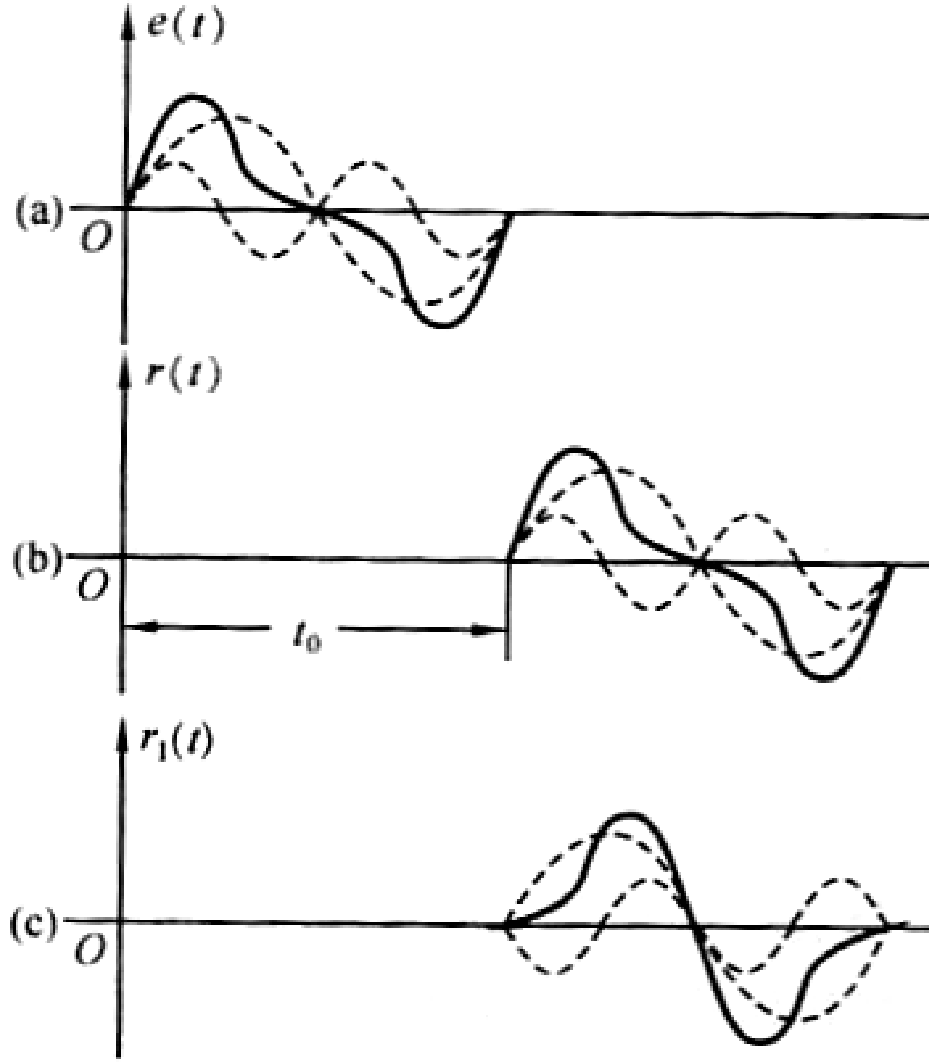
$$e(t)=E_1\sin(\omega_1 t)+E_2\sin(\omega_2 t)$$

$$r(t)=kE_1\sin(\omega_1 t-\varphi_1)$$
$$+kE_2\sin(\omega_2 t-\varphi_2)$$

$$\therefore\frac{\varphi_1}{\omega_1}=\frac{\varphi_2}{\omega_2}=-t_0$$

$$\therefore\varphi(\omega)=-\omega t_0$$

群延时$\tau=-\dfrac{d\varphi}{d\omega}=t_0$

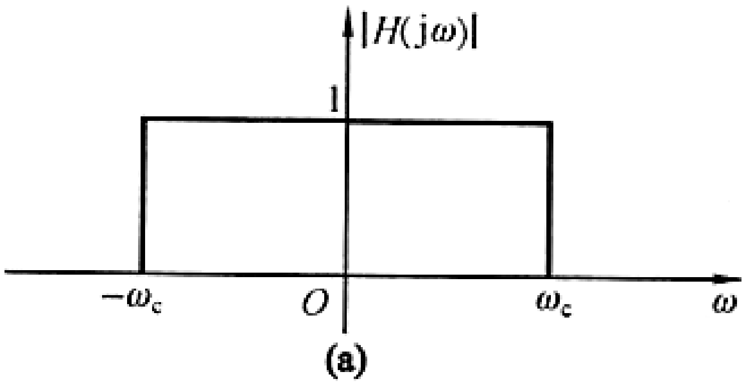


5.4 理想低通滤波器

1、理想低通滤波器及其冲激响应

$$|H(j\omega)| = \begin{cases} 1 & -\omega_c < \omega < \omega_c \\ 0 & other \end{cases}$$

$$\varphi(\omega) = -\omega t_0$$



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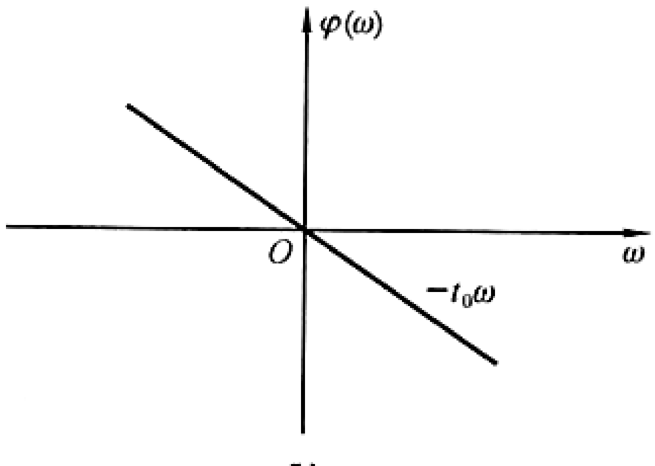

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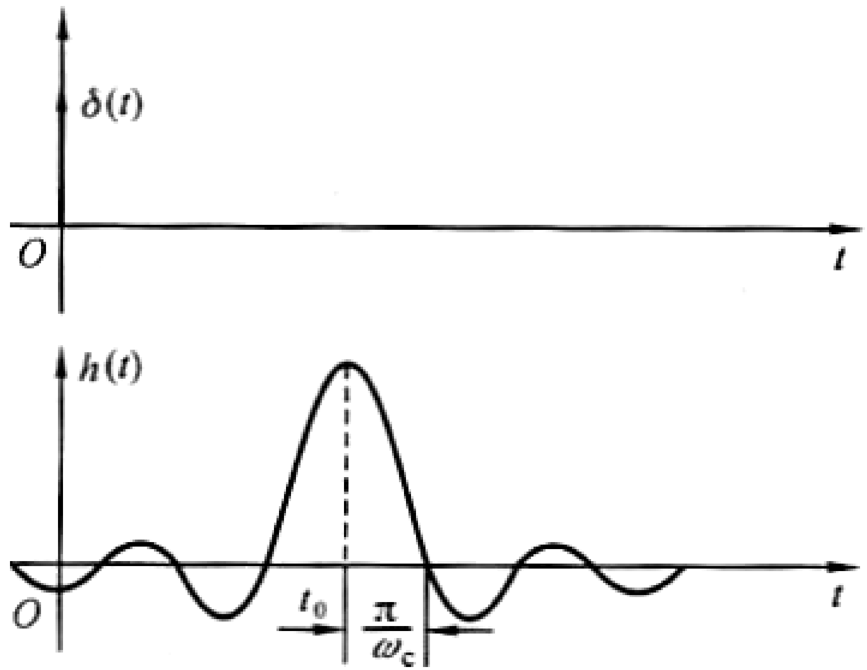
冲激响应：

$$h(t)=\frac{\omega_c}{\pi}Sa(\omega_c(t-t_0)]$$



冲激响应的特点：

1、与输入波形相比，波形产生严重失真

2、t<0时有响应，非因果系统

3、延时作用

2、理想低通滤波器的阶跃响应

$$R(j\omega)=H(j\omega)E(j\omega)$$

$$=e^{-j\omega t_0}[1/(j\omega)+\pi\delta(\omega)] \qquad (-\omega_c<\omega<\omega_c)$$

$$r(t)=F^{-1}[R(j\omega)]$$

$$g(t)=\int_{-\infty}^{t}h(\tau)d\tau=\int_{-\infty}^{t}\frac{\omega_c}{\pi}Sa[\omega_c(\tau-t_0)]d\tau$$

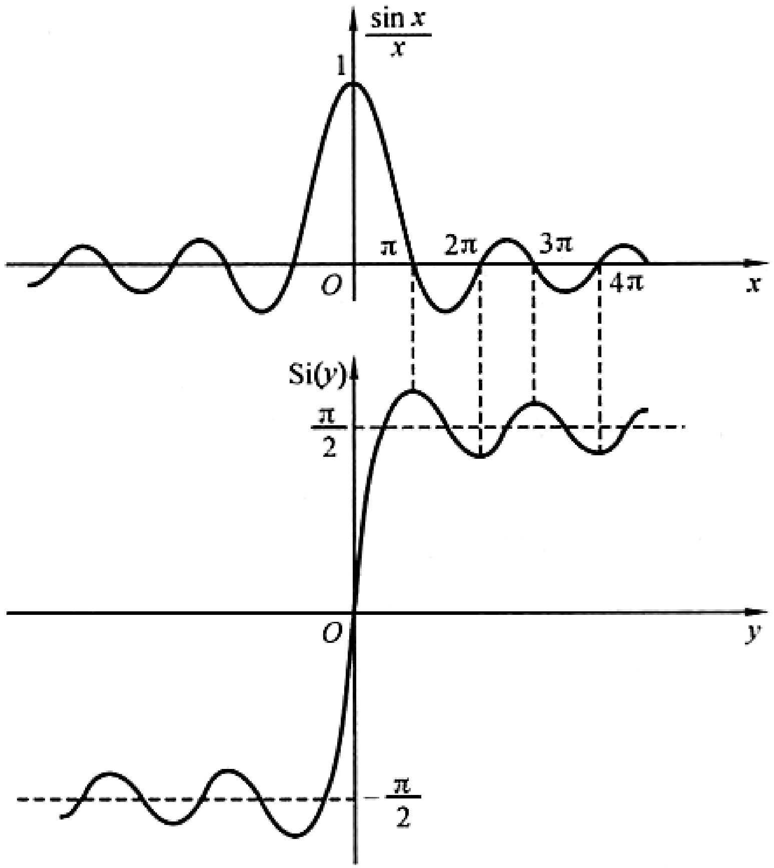
$$x=\omega_c(\tau-t_0),d\tau=\frac{1}{\omega_c}dx$$

$$g(t)=\frac{1}{\pi}\int_{-\infty}^{\omega_c(t-t_0)}Sa[x]dx$$

$$=\frac{1}{\pi}[\int_{-\infty}^{0}\frac{\sin x}{x}dx+\int_{0}^{\omega_c(t-t_0)}\frac{\sin x}{x}dx]$$

$$=\frac{1}{2}+\frac{1}{\pi}Si(\omega_c(t-t_0)) \qquad Si(y)=\int_{0}^{y}\frac{\sin x}{x}dx$$

← 正弦积分函数



正弦积分函数的特性：

1）奇函数，Si(y)=Si(-y)

2）Si(0)=0

3）Si(∞)=π/2, Si(-∞)=-π/2

4）从y=0开始随y增长而增长，然后围绕π/2起伏

5）Si(y)的极值与sinx/x的零点对应

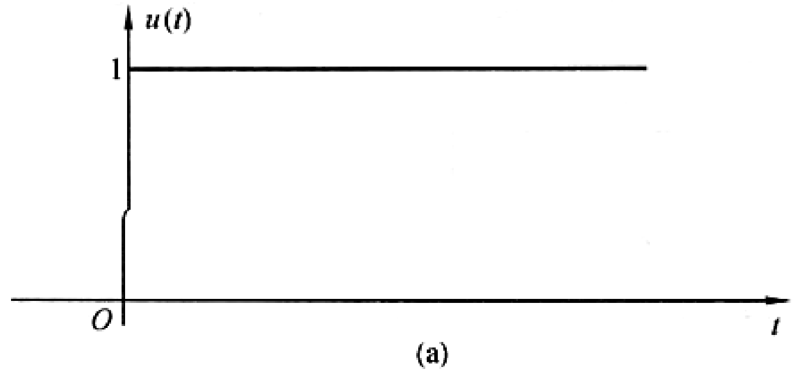
阶跃响应的特点：

1）阶跃响应是逐渐上升的，上升时间取决于滤波器的截止频率，$\omega_c\downarrow, t_r\uparrow$

2)上升时间t_r（最小值上升到最大值的时间）

$t_r = 2\pi/\omega_c = 1/f_c$, f_c为低通滤波器的带宽$(1/B)$

滤波器的通带愈宽，t_r愈小。



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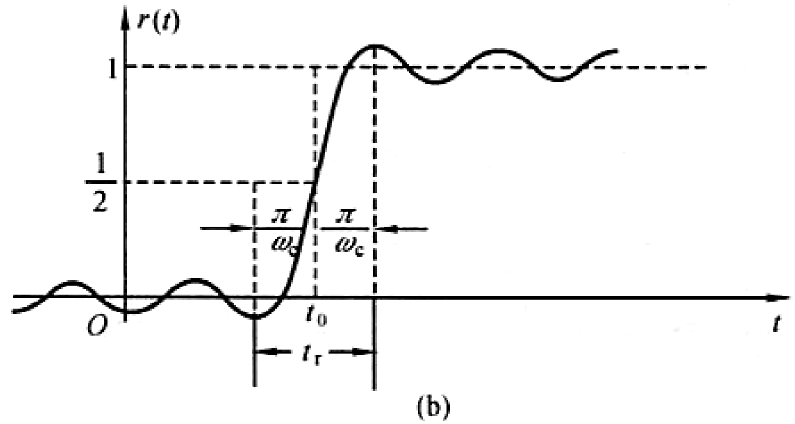

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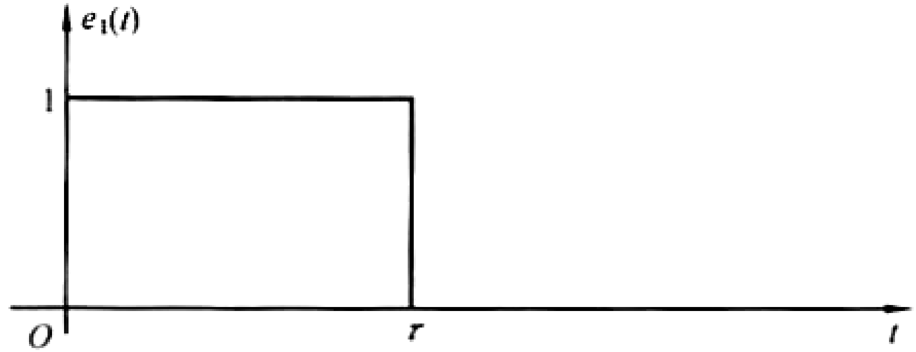
3、理想低通对矩形脉冲的响应

$$e_1(t) = u(t) - u(t-\tau)$$

$$r_1(t) = \frac{1}{\pi}\{Si[\omega_c(t-t_0)] - Si[\omega_c(t-t_0-\tau)]\}$$

$$\therefore \frac{\pi}{\omega_c} << \tau$$



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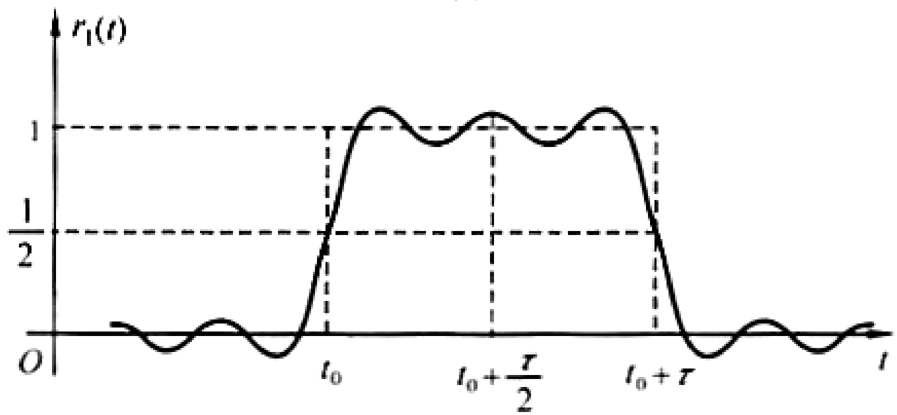
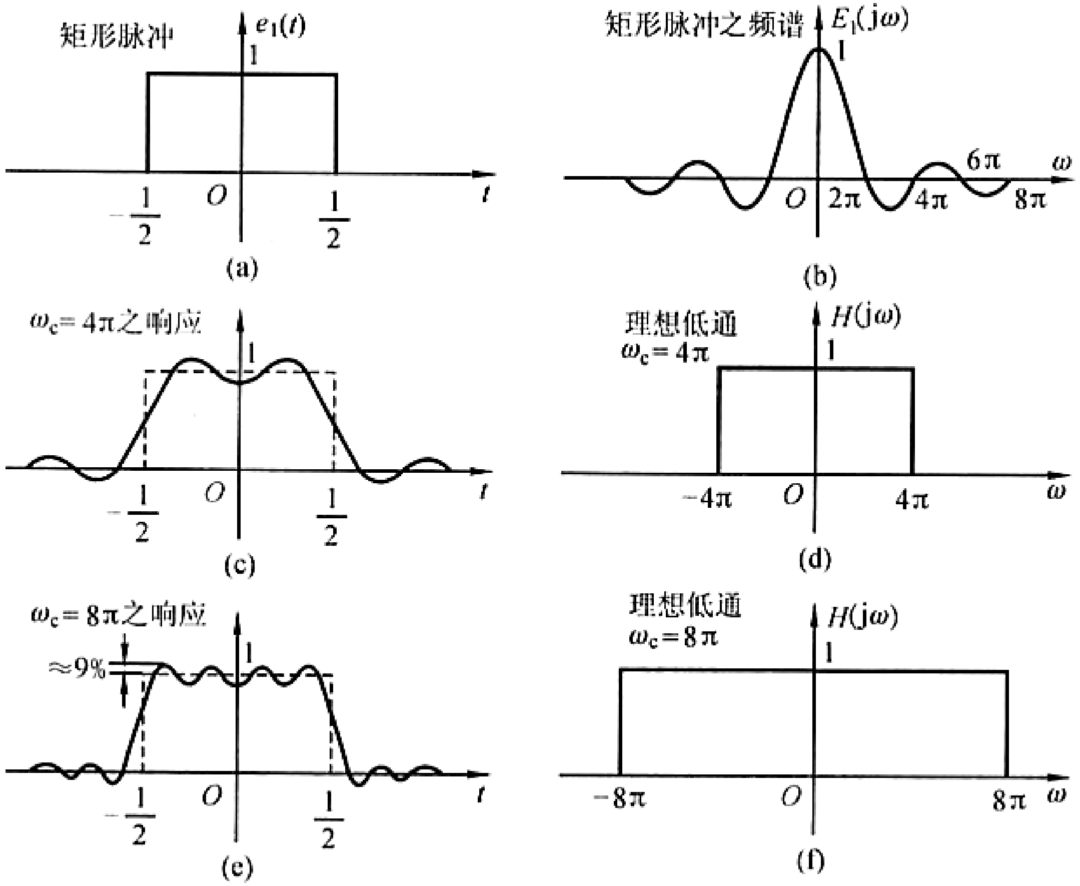


(b)

4、Gibbs现象

$$Si(y)|_{y=\pi} = \pi(1/2 + 9\%) = 1.8535$$

ω_c增大，t_r减小，但峰仍在$y = \pi$处，高度仍为9%



5.5　调制与解调

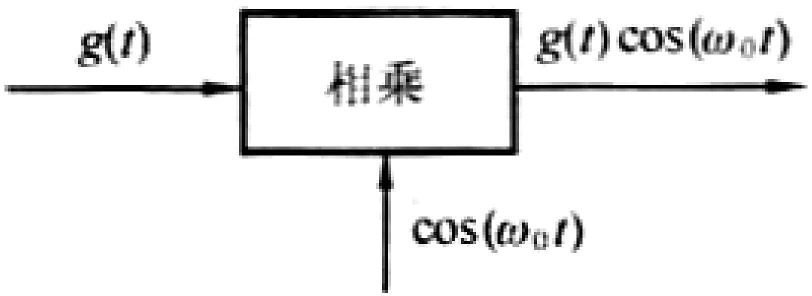
1、调制

载波信号 $\cos\omega_0 t \Leftrightarrow \pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$

$$f(t)=g(t)\cos\omega_0 t$$

$$F(\omega)=\frac{1}{2\pi}G(\omega)*\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$$

$$=\frac{1}{2}[G(\omega+\omega_0)+G(\omega-\omega_0)]$$



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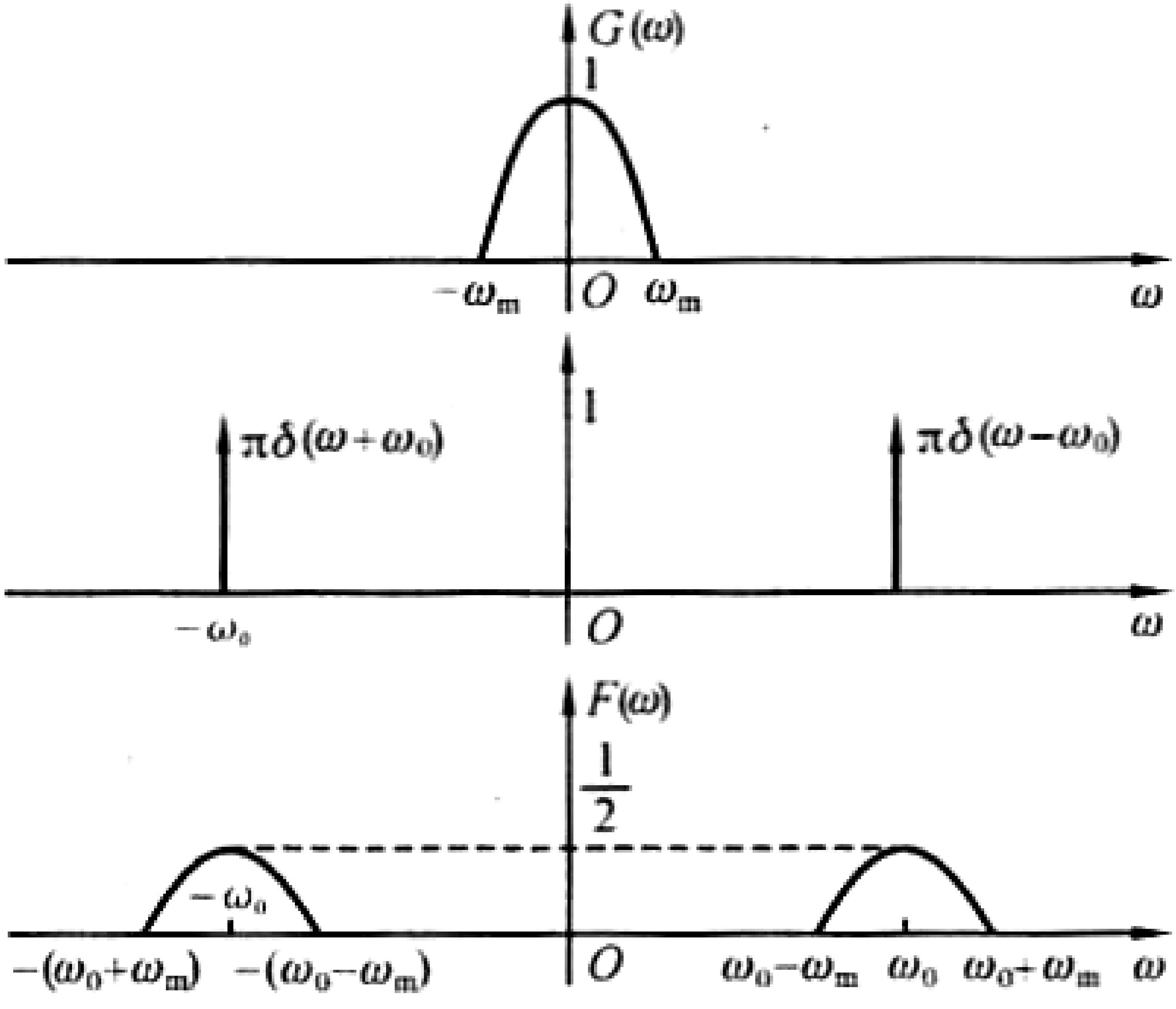

G(ω)
1
−ωm
O
ωm
ω
1
πδ(ω+ω0)
πδ(ω−ω0)
−ω0
O
ω
F(ω)
1/2
−ω0
−(ω0+ωm)
−(ω0−ωm)
O
ω0−ωm
ω0
ω0+ωm
ω

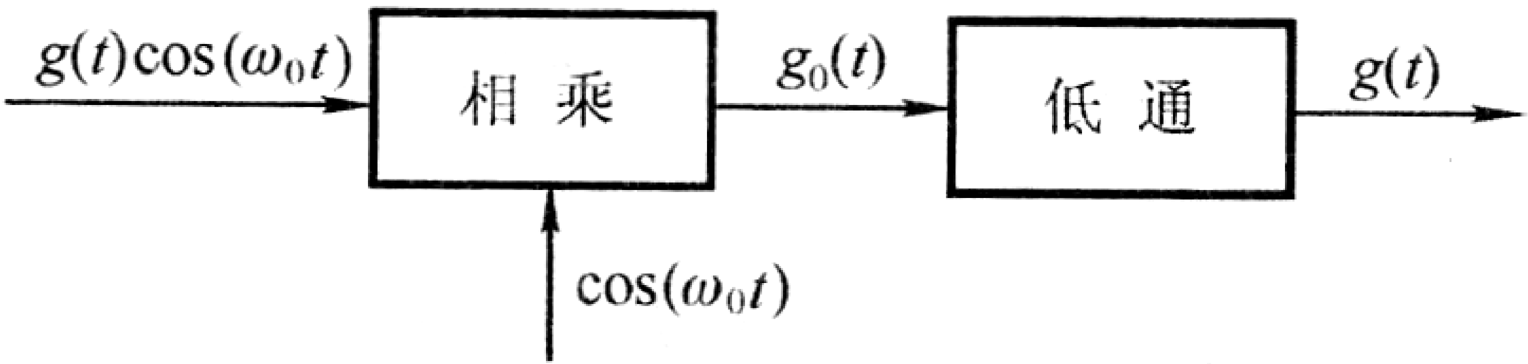
(b)

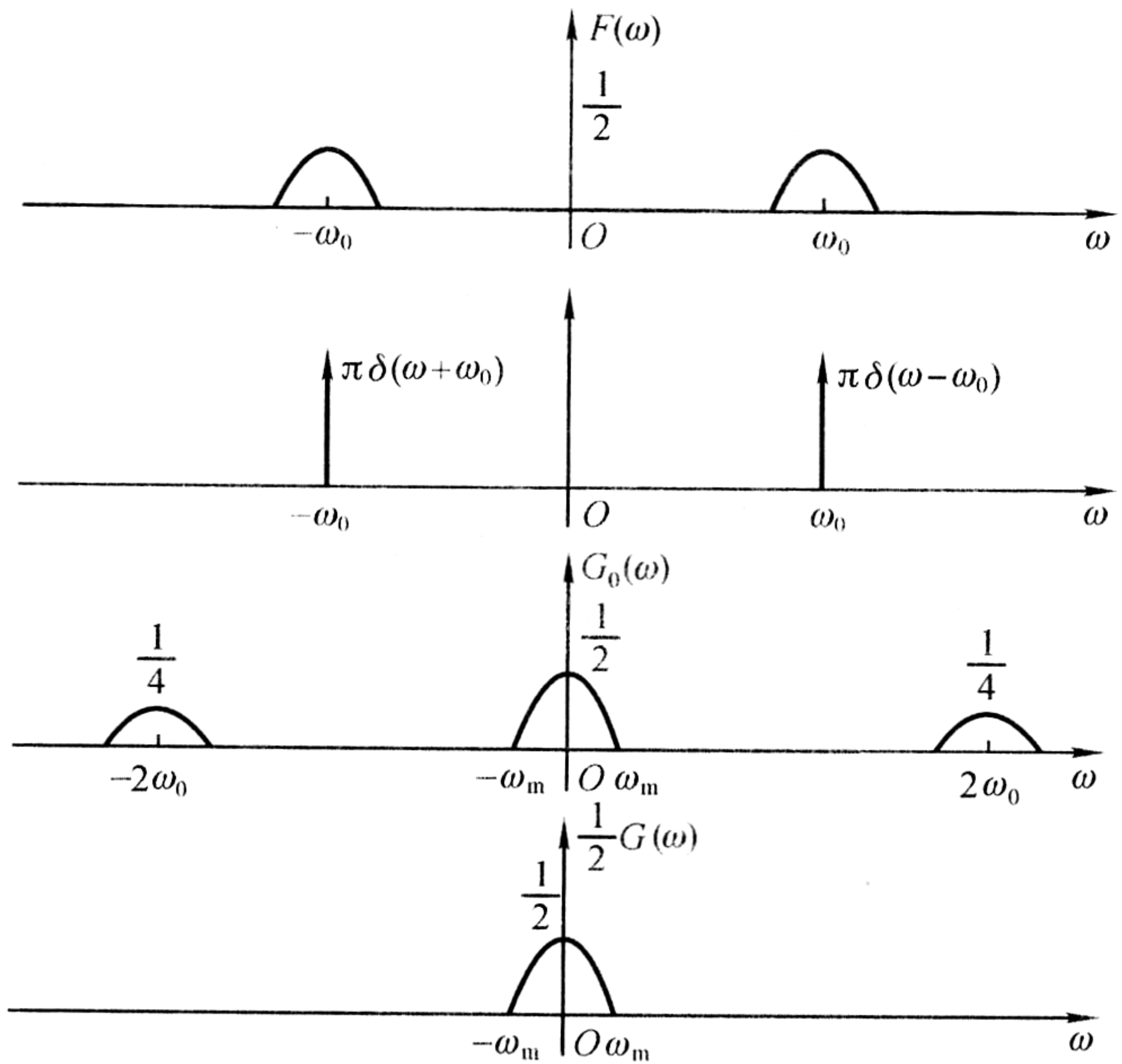
2、解调

– 由已调信号f(t)恢复原始信号g(t)的过程

$$g_0(t) = [g(t)\cos(\omega_0 t)]\cos(\omega_0 t) = \frac{1}{2}g(t)[1+\cos(2\omega_0 t)]$$

$$G_0(\omega) = \frac{1}{2}G(\omega) + \frac{1}{4}[G(\omega+2\omega_0) + G(\omega-2\omega_0)]$$





$F(\omega)$
$\frac{1}{2}$
$-\omega_0$
O
ω_0
ω
$\pi\delta(\omega+\omega_0)$
$\pi\delta(\omega-\omega_0)$
$-\omega_0$
O
ω_0
ω
$G_0(\omega)$
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{1}{4}$
$-2\omega_0$
$-\omega_m$
O ω_m
$2\omega_0$
ω
$\frac{1}{2}G(\omega)$
$\frac{1}{2}$
$-\omega_m$
O ω_m
ω

作　　业

5-4

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